ELECTIVE COURSES IN MATHS FOR AUG-NOV 2021

Note: This list is subject to changes.

(1) **Stochastic processes I**
  (a) Instructor: S Ramasubramanian
  (b) Syllabus:
    (i) Review of probability, conditional probability.
    (ii) Markov chains: Examples, transition probability matrix, recurrence, transience, stationary distribution, limit theorem, exponential convergence, eigenvalues, absorbing chains, time reversible chains, applications.
    (iii) Special topics: Markov decision processes, martingales.
  (c) Prerequisite: first course in probability.
  (d) Target audience: BSc 3rd year, MSc Maths/CS/DS.
(2) **Measure-Theoretic Probability**
  (a) Instructor: B V Rao
  (b) Syllabus:
    (i) Probability modelling, sigmafields, construction of probabilities/measures.
    (ii) Random variables/measurable functions, modes of convergence, integration.
    (iii) Product spaces.
    (iv) Riemann and Lebesgue integrals
    (v) Regularity, Egoroff, L p -spaces
    (vi) Kolmogorov zero-one law, Three series theorem, Weak/Strong Law.
    (vii) If time permits: Weak Convergence and Central Limit theorem, Conditional expectation
  (c) Prerequisite: Real Analysis.
  (d) Remarks: Home Assignments are an essential part of the course.
  (e) Target audience: BSc 3rd year, MSc Maths
(3) **Commutative Algebra**
  (a) Instructor: Mandira Mondal
  (b) Syllabus: see `https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf`
  (c) Target audience: BSc 3rd year, MSc Maths
(4) **Algebraic Geometry I**
  (a) Instructors: Pramathanath Sastry & Rupam Karmakar
  (b) Syllabus:
    (i) Pre-sheaves and sheaves of abelian groups (rings, algebras, modules over sheaves of algebras). Ringed spaces. Locally ringed spaces. Affine schemes. Schemes and quasi-coherent sheaves on them. Morphisms. Closed sub-schemes characterised by quasi-coherent ideal sheaves.
    (ii) Separated and proper maps.
    (iii) Elementary examples with curves and surfaces. Blowing up.
    (iv) Cohomology of sheaves. Derived functors, Čech cohomology.
    (v) Divisors, line bundles, linear systems.
    (vi) Proj(R) for a graded A-algebra R.
    (vii) Smooth varieites.
    (viii) Serre duality
    (ix) Curves, Riemann-Roch
    (x) Anything else, if time permits.
  (c) Pre-requisites and co-requisites:
    (i) *Commutative Algebra* The prime spectrum of a ring, flatness, Krull dimension, primary decomposition of modules, noether normalisation, nullstellensatz.

[git] • Branch: master @ 16994fb • Release: (2021-09-13)

    (ii) *Homological algebra:* Derived functors on abelian categories via injective and projective resolutions, Ext^i, Tor_i.
  (d) Textbooks/References:
    (i) Hartshorne, *Algebraic Geometry*, GTM 52, Springer, New York, 1977.
    (ii) Kempf, *Algebraic Varieties*, LMS Lecture Notes Series 172, Cambridge University Press, Cambridge UK, 1993.
    (iii) Kempf, Some elementary proofs of basic theorems in cohomology of quasi-coherent sheaves, *Rocky Mountain Journal of Mathematics*, vol 10, Number 3, Summer 1990.
    (iv) Matsumura, *Commutative Ring theory*, Cambridge studies in advanced mathematics 8, Cambridge University Press, Cambridge, 1980.
    (v) The Stacks Project Authors, *Stacks Project*, https://stacks.math.columbia.edu, 2018
  (e) Target audience: BSc 3rd year, MSc Maths

(5) **Homological Algebra**
  (a) Instructors: Clare D'Cruz & S Selvaraja
  (b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf
  (c) Target audience: BSc 3rd year, MSc Maths

(6) **Representation Theory of Finite Groups**
  (a) Instructor: Arpita Nayek
  (b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf
  (c) Target audience: BSc 3rd year, MSc Maths

(7) **Introduction to Ergodic Theory**
  (a) Instructor: Keshab Chandra Bakshi
  (b) Syllabus:
    (i) Basic Notions: Measure-preserving and non-singular transformations, Recurrence, Ergodicity, Strong Mixing, Weak mixing.
    (ii) Ergodic theorems: The mean Ergodic theorem and the Pointwise ergodic theorem.
    (iii) Spectral theory of measure preserving transformations, Irreducible Koopman representations.
    (iv) Entropy: Entropy of a partition, conditional entropy, topological entropy, Entropy of a measure-preserving transformation, Kolmogorov-Sinai Theorem.
  (c) Prerequisites: Graduate Analysis I
  (d) Textbooks:
    (i) Halmos, Lectures on Ergodic Theory
    (ii) Karl Petersen, Ergodic Theory
  (e) Target audience: MSc Maths

(8) **Algebraic Number theory**
  (a) Instructors: Purusottam Rath & Jyothsnaa Sivaraman
  (b) Syllabus: Dedekind domains. Ring of Integers (Quadratic and Cyclotomic fields). Geometry of numbers. Class number (Finiteness). Unit theorem. Ramification, Discriminant and different. Introduction to Frobenius elements and Artin Symbol.
  (c) Prerequisites: Algebra IV / Graduate Algebra I
  (d) Textbooks:
    (i) Neukirch, Algebraic number theory
    (ii) Marcus, Number fields
  (e) Target audience: BSc 3rd year, MSc Maths

(9) **Algebraic Curves and Riemann Surfaces**
  (a) Instructor: Tanya Kaushal
  (b) Syllabus:
    (i) Basic definition and examples of Riemann Surfaces, Affine curves and Projective curves.
    (ii) Functions on Riemann surfaces
    (iii) Holomorphic maps between Riemann surfaces
    (iv) Monodromy group of a holomorphic map
    (v) Abel Ruffini theorem
    (vi) Integration on Riemann surfaces

(vii) Divisors and meromorphic functions
(viii) Algebraic curves and Riemann-Roch theorem
(ix) Applications of Riemann -Roch theorem to classification of Riemann surfaces.

(c) Textbooks:
  (i) Main textbook: R. Miranda, Algebraic curves and Riemann surfaces. (Chapters 1-7, omitting the last section of Ch. 7)
  (ii) Lecture notes will also be provided.
  (iii) Further references:
    (A) Sam Raskin, The Weil conjectures for curves, https://math.uchicago.edu/~may/VIGRE/VIGRE2007/REUPapers/FINALFULL/Raskin.pdf
    (B) Mircea Mustata, Zeta functions in Algebraic Geometry http://www.math.lsa.umich.edu/~mmustata/zeta_book.pdf
    (C) Henryk ??o????dek, The topological proof of Abel-Ruffini theorem, Topol. Methods Nonlinear Anal. 16(2): 253-265 (2000).
    (D) Hannah Santa Cruz, A survey on the monodromy of algebraic functions, http://math.uchicago.edu/~may/REU2016/REUPapers/SantaCruz.pdf
    (E) W. Fulton, Algebraic Curves: An Introduction to Algebraic Geometry http://www.math.lsa.umich.edu/~wfulton/CurveBook.pdf
    (F) Historical Fun reading: Popescu-Pampu, Patrick, What is the genus?, History of Mathematics Subseries, 2162, Springer, 2016.

(d) Prerequisites: Complex analysis in one variable, abstract topology, abstract algebra (rings, modules, ideals). (Optional Pre-requisites, which may render slight advantage) Some knowledge of manifolds and fundamental groups.

(e) Target audience: BSc 3rd year, MSc Maths

(10) **Introduction to Generating Functions**
  (a) Instructor: Shuchita Goyal
  (b) Syllabus:
    (i) Counting problems in sets, multisets, permutations, partitions, trees. (To be covered from GF: Ch-1, Ch-2 and EC Ch-1.)
    (ii) Ordinary and exponential generating functions. (To be covered from GF Ch-3.)
    (iii) Posets and sieve methods. (To be covered from EC Ch-2.)
    (iv) Lattices and their Mobius functions. (To be covered from EC Ch-3.)
    (v) Combinatorial identities and the WZ (Wilf-Zielberger) method. (To be covered from GF Ch-3, Ch-4.)
  (c) Prerequisites: Discrete Mathematics
  (d) Textbooks:
    (i) GF = generatingfunctionology by Herbert Wilf
    (ii) EC = Enumerative combinatorics vol 1 by Richard Stanley.
  (e) Target audience: BSc 3rd year, MSc Maths

(11) **Optimization Techniques**
  (a) Instructor: T Parthasarathy / Sujatha Babu
  (b) Syllabus: The topics covered are as follows:
    (i) Linear programming
      (A) Introduction, formulation of a linear program (LP), definitions and theorems, Simplex algorithm + examples
      (B) Duality (weak and strong duality theorem, Farkas lemma, complementary slackness theorem, KKT conditions for optimality)
      (C) Transportation problem formulated as a LP (including theorems, revised simplex algorithm + example, unbalanced transportation problem, degeneracy)
      (D) Linear Complementarity Problem (LCP)
      (E) Solving large scale LPs using Dantzig-Wolfe decomposition technique
    (ii) Nonlinear programming (NLP)

- (A) Introduction to NLP
- (B) 4 types of NLPs being considered
  - Minimization problem (MP) - Local minimization problem (LMP) - Fritz John saddlepoint problem (FJSP) - Kuhn Tucker saddlepoint problem (KTSP)
- (C) Connection between MP and LMP, MP and FJSP/KTSP
- (D) Optimality conditions: Sufficiency Theorems + example to find optimal for FJSP; Constraint qualifications (CQ)- Slater's CQ, Karlin's CQ, Strict CQ; KTSP necessary optimality theorem + when there are linear equality constraints.

(iii) Dynamic programming
- (A) Discounted Dynamic programming (DP)
- (B) Discounted DP for finite state space: Operators $L(f)$ and T; Theorems leading to existence of optimal policies and (p, ϵ) optimal policies; Examples.
- (C) Brief mention of existence of optimal policies relating to countable state space

(c) Prerequisites:
- (i) Undergrad Real analysis
- (ii) Undergrad algebra
- (iii) Basic knowledge of Markov chains for the portion relating to Dynamic programming

(d) References:
- (i) For LP, there are way too many excellent resources online and lots of books. No prescribed textbook.
- (ii) For Non-linear programming: Chapter 5 from the book Non-linear Programming by Mangasarian
- (iii) For Dynamic programming:
  - (A) Maitra's notes (ISI Calcutta): for topics relating to finite state space.
  - (B) D.Blackwell, Discrete dynamic programming(1962).
  - (C) D.Blackwell, Discounted dynamic programming(1965).

(e) Target audience: BSc 3rd year, MSc Maths

(12) **Coding Theory**
- (a) Instructor: Sharad Sane
- (b) Syllabus: (This is the syllabus from an earlier offering. Need to check.)
  - (i) Basic coding theory, Generator and Parity Check matrices, Maximum likelihood decoding and Shannon's noisy channel theorem
  - (ii) Some basic interesting codes and their properties, Hamming and Cyclic codes, Reed-Solomon codes, BCH codes, QR cdoes, Binary and Ternary Golay codes
  - (iii) Weight enumerators and MacWilliams identities, Self-dual codes and their classficiation
  - (iv) Bounds on codes, Gilbert-Varshamov bound, Hamming and Griesmer bounds, Orthogonal polynomials and linear programming bound
  - (v) Hadamard matrices, Plotkin bound and Levenshtein theorem
  - (vi) Reed-Muller codes of higher orders and connections with Hadamard matrices of maximum excess and the Menon type Hadamard matrices
  - (vii) Lloyd's theorem on perfect codes
  - (viii) Codes and designs: Assmus-Mattson theorem
  - (ix) Lattices and codes
- (c) Textbooks:
  - (i) J.H. van Lint, An Introduction to Coding Theory, Springer Graduate Texts in Mathematics
  - (ii) J. MacWilliams and N.J.A. Sloane, Theory of error correcting codes, North-Holland
  - (iii) J. Birbrauer, An Introduction to coding theory, CRC Press
- (d) Prerequisites:
- (e) Target audience: BSc 3rd year, MSc Maths

(13) **Linear Groups**
- (a) Instructor: Kamalakshya Mahatab
- (b) Syllabus: see https://www.cmi.ac.in/teaching/electives/maths_std_el.pdf
- (c) Target audience: BSc 3rd year, MSc Maths

(14) **Representations of algebras and quivers**
- (a) Instructor: Upendra Kulkarni

(b) Syllabus:
   (i) Semisimple modules, Jordan-Holder/Krull-Schmidt theorems, Jacobson radical, radical/socle series
   (ii) Tensor products, tensor-hom adjunction for bimodules
   (iii) Tensor/exterior/symmetric algebras, generators and relations, Lie and universal enveloping algebras, quivers/path algebras
   (iv) Induction/coinduction, important cases: Frobenius reciprocity, recollement associated with an idempotent
   (v) More material if time permits e.g. projective covers and Cartan matrix of an algebra
   (vi) Basic example of finite dimensional reps of $\mathfrak{sl}_2(\mathbb{C})$.
   (vii) Wedderburn-Artin structure theory of semisimple algebras via double centralizers/Jacobson density
   (viii) Application to group algebras of finite groups
   (ix) Symmetric group representations via Vershik-Okounkov approach
   (x) Schur Weyl duality for S_n and $\mathrm{GL}(V)$
   (xi) Gabriel's theorem on quivers of finite type
(c) Prerequisites: Algebra IV
(d) Textbooks:
   (i) Etingof, Introduction to Representation Theory.
   (ii) Morel, Princeton notes.
   (iii) Brion/Crawley-Boevey/Schiffler, notes on quivers.
(e) Target audience: BSc 3rd year, MSc Maths 2nd year.

(15) **Matrix Computations**
(a) Instructor: Kavita Sutar
(b) Syllabus: There are 3 main problems studied in numerical linear algebra: solving a system of equations, solving the least squares problem and solving the eigenvalue/eigenvector problem. NLA focuses on methods and algorithms as well as matters of conditioning and perturbation theory.
The syllabus will be as follows: (methods pertaining to the 3 problems listed above)
   (i) Preliminaries: Matrix norms, Rayleigh quotient and its properties, floating point arithmetic, the notion of stability.
   (ii) Solving systems of equations: Conditioning and perturbation theory, Gaussian elimination and LU factorization, Cholesky decomposition, QR decomposition (Gram-Schmidt, Householder, Givens' methods), SVD.
   (iii) Eigenvalue methods: Conditioning of the eigenvalue problem, perturbation theory, some basic methods (power iteration, inverse iteration, simultaneous iteration), QR iteration, methods for symmetric matrices (Jacobi's method, bisection method, divide-and-conquer, Rayleigh quotient iteration). Iterative methods such as Lanczos, Arnoldi and Krylov subspace methods (if time permits).
   (iv) Least squares methods: methods for full rank matrices, methods for rank-deficient matrices.
(c) Textbooks:
   (i) G.W. Stewart, Introduction to matrix computations.
   (ii) Golub and Loan, Matrix computations.
(d) Prerequisites: a first course in linear algebra
(e) Target audience: BSc 3rd year, MSc Maths

(16) **Commutative Algebra and Invariants of Groups**
(a) Instructor: Manoj Kummini
(b) Syllabus: Finite generation of invariants, Noether bound, Molien theorem, depth of invariant rings, Cohen-Macaulay and Gorenstein invariant rings, pseudo-reflections, Shephard-Todd-Chevalley-Serre theorem, modular invariants, Dickson invariants.
(c) Textbooks:
   (i) Benson, Polynomial invariants of finite groups, London Mathematical Society Lecture Notes No. 190, Cambridge University Press.

(ii) Stanley, Invariants Of Finite Groups and Their Applications to Combinatorics Bull. Amer. Math. Soc. (New Series), vol. 1, no. 3, 1979.
(iii) Smith, Polynomial invariants of finite groups, Birkhauser.

(d) Prerequisites: a first course in commutative algebra, with knowledge of primary decomposition, integral extensions and Krull dimension of noetherian rings.
(e) Target audience: BSc 3rd year, MSc Maths