SYLLABUS OF CORE COURSES FOR BSC MATHEMATICS & COMPUTER SCIENCE

EFFECTIVE FROM 2025-26

Contents

1. Algebra I

Throughout the course the emphasis will be on linear algebra over real and complex fields.

(1). Systems of linear equations, row reduction, vector spaces over any field, bases and dimension, change of basis, linear transformations, dimension formula, matrix of a linear transformation and change of bases

(2). Linear operators, similarity, determinants and invertibility, eigenvalues and eigenspaces, characteristic polynomial, triangular and diagonal forms

(3). Euclidean/Hermitian spaces with standard inner product, orthogonal projection, orthonormal bases and Gram-Schmidt procedure, spectral theorem for Hermitian/symmetric operators andfor normal operators.

Date: 2025-08-01.

(4). *If time permits*: one or more of the following topics: (a) minimal polynomial and diagonalizability, (b) generalized eigenspace decomposition and Jordan canonical form, (c) Cayley-Hamilton theorem, (d) dual spaces, quotients, (e) applications of linear algebra.

References.

(1). Artin, M., Algebra, second edition, PHI, 2010
(2). Additional sources as recommended by the instructor

2. Analysis I

(1). Axioms of the real number system without construction, applications of the least-upper-bound property, Archimedean principle, existence of nth roots of positive real numbers, a^x for $a > 0$ and $x > 0$, cardinality, countability of rational numbers, uncountability of real numbers.

(2). Convergence of sequences, uniqueness of limit, Sandwich lemma, examples, monotonic sequences, subsequences, Bolzano-Weierstrass theorem, $\limsup$ and $\liminf$, Cauchy sequences, completeness of $\mathbb{R}$.

(3). Infinite series, convergence and divergence, absolute convergence, conditional convergence, convergence/divergence of $\sum_{k=1}^{\infty} \frac{1}{k^p}$, comparison test, root test, ratio test, Leibniz test.

(4). complex numbers, power series, radius of convergence of power series.

(5). Continuous functions on intervals of $\mathbb{R}$, intermediate value theorem, boundedness of continuous functions on closed and bounded intervals. Uniform continuity, uniform convergence, uniform limit of continuous functions is continuous, counter example for point wise convergence.

(6). Differentiation, mean value theorem, Taylor's theorem, application of Taylor's theorem to maxima and minima, L'Hôpital rule.

(7). Construction of e^z using power series, proof of the periodicity of sin and cos.

(8). Riemann Integration: Riemann integrals, Riemann integrability of continuous functions, fundamental theorem of calculus. Improper integrals

References.

(1). R. Goldberg, *Methods of Real Analysis.*
(2). Bartle and Sherbett, *Introduction to Real Analysis.*
(3). Rudin, *Principles of Mathematical Analysis.*
(4). L. Cohen and Ehrlick, *Structure of the Real Number System.*
(5). T. Apostol, *Calculus*, vols I and II
(6). Ajit Kumar, S. Kumeresan: *A basic course in Real analysis.*

3. Introduction to Programming

The course will be based on the programming language Haskell. Function definitions: pattern matching, induction; Basic data types, tuples, lists; Higher order functions; Polymorphism; Reduction as computation, lazy evaluation; Measuring computational complexity; Basic algorithms: sorting, backtracking, dynamic programming; User-defined datatypes: enumerated, recursive and polymorphic types; Input/output.

References.

(1). R. Bird and P. Wadler, *Introduction to Functional Programming* Prentice Hall, 1988.

(2). R. Bird, *Introduction to Functional Programming using Haskell*, Prentice Hall, 1998.

(3). Paul Hudak, *The Haskell school of expression*, Cambridge University Press, 2000.

(4). Graham Hutton, *Programming in Haskell*, Cambridge University Press, 2007.

(5). Bryan O'Sullivan, John Georzen and Don Stewart, *Real World Haskell*, O'Reilly, 2007.

4. Classical Mechanics I

(1). Space and Time, Newton's Laws, Conservation Laws, Harmonic, Damped, Forced, and Kicked Oscillators, Rocket Motion, Collision Problems, Projectiles, Central Forces, Inverse Square Law, Rutherford Scattering, Centrifugal and Coriolis Forces, Potential Theory.

(2). Principle of Least Action, Constraints and Generalised Coordinates, Lagrange's Equations, Noether's Theorem and Symmetries, Applications, Hamilton's Equations, Small Oscillations, Stability, Normal Modes.

(3). Lorentz Transformations, Space-Time Diagrams, Length Contraction, Time Dilation, Kinematics and Dynamics of a Particle, Composition of Velocities - Proper Time, Equations of Motion in Absolute Form and Relative Form.

References.

(1). C. Kittel, W. D. Knight, M. A. Ruderman, C. A. Helmholz, and B. J. Moyer, *Mechanics: Berkeley Physics Course*, Vol. 1. Tata-McGraw Hill.

(2). T.W.B. Kibble, F. H. Berkshire, *Classical Mechanics*, World Scientific.

(3). J. L. Synge and B. A. Griffith, *Principle of Mechanics*, Nabu Press, 2011.

5. English

(1). *Poetry:* Selections from Indian and English poets. (a) Tagore, (b) Keats, (c) Hopkins.

(2). *Prose:* Three Short Stories and one Novel. (a) Sharatchandra, (b) R. K. Narayan, (c) Somerset Maugham, (d) Hemmingway's *Old Man & The Sea.*

(3). *Drama:* Shakespeare's *The Tempest* (selected portions)

(4). *Interactive Communication:* (a) Public Speaking (b) Field-work: interviews with selected individuals

(5). *Self-Expression:* Paper to be submitted on a subjective topic.

(6). *Effective Language:* Use of language in different contexts — technical, informal, literary etc.

6. Algebra II

(1). Bilinear forms, symmetric and Hermitian forms, Sylvester's law, skew-symmetric forms (at most 25% of the course)

(2). Groups, subgroups, homomorphisms and normal subgroups, equivalence relations and partitions, cosets, quotient groups and quotient vector spaces, isomorphism theorem and correspondence theorem for groups and vector spaces, product groups, group actions, counting using orbits and stabilizers, class equation, Sylow theorems, free groups, generators and relations

(3). Examples along the way to illustrate the theory, including cyclic groups, dihedral groups, symmetric groups and their conjugacy classes, simplicity of alternating groups, p-groups, general linear groups, orthogonal groups in small dimensions and Euler's theorem on SO(3).

(4). *If time permits*: One or more of the following topics: (a) unitary and symplectic groups, (b) linear groups, (c) solvable and nilpotent groups, (d) tensor products of vector spaces (using explicit bases).

References.

(1). M. Artin, *Algebra*, second edition, PHI, 2010
(2). D. Dummit and R. Foote, *Abstract Algebra*
(3). I. Herstein, *Topics in Algebra*
(4). N. Jacobson, *Basic Algebra I*
(5). E. Vinberg, *A course in algebra*
(6). Additional sources as recommended by the instructor

7. Calculus I

(1). Metric spaces: examples of metric spaces, convergence of sequences in metric spaces. Topology of $\mathbb{R}^n$: Euclidean, ℓ^1 and ℓ^∞ norms on $\mathbb{R}^n$ and the equivalence of convergence of sequences in $\mathbb{R}^n$.

(2). Open and closed sets in metric spaces, properties, Continuous functions on metric spaces, equivalent properties of continuity (using sequences and open sets), boundedness of continuous functions defined on closed and bounded subsets of $\mathbb{R}^n$.

(3). Inner product and linear maps on $\mathbb{R}^n$.

(4). Vector valued functions — space curves, derivative as a linear map, Chain rule, Matrix representation and partial derivatives, sufficient condition for differentiability, equality of mixed partial derivatives. Taylor's formula and its application to maxima and minima.

(5). Inverse function theorem, Implicit function theorem, tangent spaces of level surfaces, directional derivatives, gradient, Jacobian, critical points and regular values, Lagrange multipliers.

(6). Focussed on computations and examples: (iterated) double and triple integrals – applications (surface area, moment and centre of mass), change of variables. Vector calculus – Green's theorem – divergence and curl, surface integrals, Stokes' theorem, the divergence theorem.

References.

(1). T. Apostol, *Calculus*, vols I and II
(2). James Stewart, *Single and Multivariable Calculus –Early transcendentals* (Chapters 13, 14, 15, 16.)
(3). Rudin, *Principles of Mathematical Analysis.*
(4). Additional sources as recommended by the instructor

8. Advanced Programming

Imperative programming using Python and C.

References.

(1). Brian W. Kernighan and Dennis M. Ritchie, *The C Programming Language*, Prentice-Hall, 1999.

(2). Jon Kleinberg and Eva Tardos, *Algorithm Design*, Pearson/Addison-Wesley, 2006.

(3). Mark Lutz, *Programming Python*, O'Reilly, 2001.

(4). Mark Pilgrim, *Dive into Python*, available online, `http://www.diveintopython.org`

9. Discrete Mathematics

Principles of Counting, binomial coefficients, generating functions, partitions, Graph Theory: paths, degree sequences, trees, minimum spanning trees, shortest path, bipartite matching, Tutte's theorem, connectivity, flows, graph colouring.

References.

(1). N. L. Briggs, *Discrete Mathematics*, Oxford Science Publications, 1989.

(2). Douglas B. West, *Introduction to Graph Theory*, Prentice-Hall India, 2001.

10. Probability Theory

Discrete probability, conditional probability and independence, Baye's theorem, random variables, distributions, expectations and moments, notion of dependence, joint distributions, covariance, standard examples like Binomial, multinomial, geometric, Poisson etc, sums of independent variables, expectations and moments , Chernoff bounds, moment-generating functions, sampling distribution, the law of large numbers, central limit theorem,

Continuous probability: densities, independence, joint distributions, covariance, characteristic functions, standard examples like uniform, normal, exponential, Gamma etc., sums of independent variables, convolutions, moment-generating functions, sampling distribution, law of large numbers, central limit theorem, Markov chains

References.

(1). W. Feller, *An Introduction to Probability Theory and its Applications*, Vol.1, John Wiley.

(2). G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes*, Oxford Science Publications.

(3). K. S. Trivedi, *Probability and Statistics with Queuing, Reliability and Computer Science Applications*. Prentice-Hall.

(4). N. Alon and J. H. Spencer, *The probabilistic method*

(5). M. Mitzenmacher and E. Upfal, *Probability and computing*

11. Algebra III

(1). Rings, homomorphisms and ideals, quotient rings, isomorphism and correspondence theorems, examples of noncommutative rings including quaternions, ring of square matrices/linear operators and its 2-sided ideals

(2). polynomial rings and long division, products and idempotents, Chinese remainder theorem, fraction fields, maximal ideals, Nullstellensatz over complex numbers

(3). Factorization in domains, PID, UFD, Gauss Lemma, factoring integer polynomials, Gaussian integers

(4). Fields, field extensions, multiplicativity of degree, algebraic and transcendental elements, minimal polynomial, adjoining roots, splitting field, multiple roots, finite fields: construction, uniqueness, subfields, cyclicity of the multiplicative group, factoring $x^{p^n} - x$ over $\mathbb{F}_p$.

(5). *If time permits*: one or more of the following topics: (a) quadratic number fields (b) basic algebraic geometry (c) free algebras (tensor, symmetric, exterior algebras with explicit basis).

References.

(1). M. Artin, *Algebra*, second edition, PHI, 2010
(2). D. Dummit and R. Foote, *Abstract Algebra*
(3). I. Herstein, *Topics in Algebra*
(4). N. Jacobson, *Basic Algebra I*
(5). E. Vinberg, *A course in algebra*
(6). Additional sources as recommended by the instructor

12. Analysis II

(1). Construction of the real number system: assuming $\mathbb{N}$ and induction, construct $\mathbb{Z}$, $\mathbb{Q}$ and the Cauchy construction of $\mathbb{R}$.

(2). Metric spaces: Characterisation of open sets in R as countable union of disjoint open intervals. Various equivalent formulations of Compactness, Consequences of compactness, open balls not totally bounded in infinite dimensions.

(3). Completeness - Completions, examples of completeness : compact sets, all bounded functions, $C(X)$ (with X compact), $C_0(X)$ (with X locally compact), ℓ^p, ℓ^∞, c_0 sequence spaces.

(4). Connectedness, Path connectedness, properties of Cantor set Baire category theorem, (optional: applications like nowhere differentiable functions are of second category, set of discontinuities of point wise convergence of continuous functions are countable), Cantor intersection theorem

(5). Dini's theorem, Stone-Weierstrass theorem and applications, Arzela-Ascoli theorem and their applications to ODEs, Contraction mapping theorem.

(6). Fourier series: Fourier transform for 2π-periodic functions, Dirichlet and Fejer Kernels, convolutions, Fejer's theorem, Riemann-Lebesgue lemma.

(7). *If time permits*: Tychonoff's theorem, Hilbert cube, locally compact, one-point compactification.

References.

(1). S. Kumeresan: Topology of metric spaces.
(2). S. Kesavan Lectures on Fourier series.
(3). Additional sources as recommended by the instructor

13. Calculus II

(1). Definition of Riemann-Darboux integrals, Jordan-measurability, Fubini theorem, area/volume under graph as application.

(2). Quick review of multivariable differentiation, Jacobian matrix.

(3). Change of variables in integration: Formulation (but no proof) and examples, especially polar, spherical coordinates. Proof in the case of a linear bijection.

(4). (On $\mathbb{R}^2$ and $\mathbb{R}^3$) Curl and divergence of vector fields, gradient of a function. Poincaré lemma for curl-free vector fields. Stokes'/Green's theorem on $\mathbb{R}^2$ and $\mathbb{R}^3$. Examples and applications possibly to solutions of Laplacian.

The next two sections to be all done on open subsets of a finite-dimensional real vector space.

(5). Multilinear algebra, exterior algebra Vector fields, 1-forms, exterior forms and exterior differentiation. Pull-back of forms.

(6). Smooth cubical chains, boundary, Stokes' Theorem statement. Proof in dimensions one and two.

References.

(1). M. Spivak, *Calculus on Manifolds*.
(2). C. C. Pugh, *Real Mathematical Analysis*.
(3). Rudin, *Principles of Mathematical Analysis*.
(4). Additional sources as recommended by the instructor.

14. Design and Analysis of Algorithms

Big O notation, sorting and searching, algorithm analysis techniques, recurrences, graph algorithms: DFS, BFS, shortest paths, spanning trees, divide and conquer, greedy algorithms, dynamic programming, data structures: heaps, binary search trees, union-find — advanced topics: LP, network flows

References.

(1). T.H. Cormen, C.E. Leiserson, and R.L. Rivest: *Introduction to algorithms*, Prentice-Hall (1998).
(2). J. Kleinberg and E. Tardos: *Algorithm design*, Pearson/Addison-Welsey (2006).

15. Theory of Computation

Finite automata, regular languages, pumping lemma, stack automata, context free languages, applications to compilers, Turing machines, universal Turing machines, halting problem, non deterministic Turing machines, complexity classes, P v/s NP

References.

(1). J. E. Hopcroft and J. D. Ullman, *Introduction to Automata theory, Languages and Computation* Narosa.
(2). D. Kozen: *Automata and Computability*, Springer.

16. Complex Analysis

Complex numbers and geometric representation, analytic functions, power series, exponential and logarithmic functions, conformality, Mobius transformations, complex integration, Cauchy's theorem, Cauchy's integral formula, singularities, Taylor's theorem, The maximum principle, The residue theorem and applications.

References.

(1). L. V. Ahlfors, *Complex analysis*, Tata Mc-Graw Hill.
(2). Stein and Shakarchi, *Complex Analysis*
(3). Hille, *Analytic Function Theory.*
(4). Alpay, *A Complex Analysis Problem Book*
(5). Nevanlinna and Paatero, *Introduction to Complex Analysis.*

17. Differential Equations

This is a course on ordinary differential equations, concentrating on equations of the type $\dot{\mathbf{x}} = v(t, \mathbf{x})$, where v is a suitable map from an open subset of $\mathbb{R}^{n+1}$ to $\mathbb{R}^n$.

(1). Computational techniques: *Basic techniques:* Separable DEs, linear first order DEs, exact DEs. Linear DEs with constant coefficients. *Special techniques:* instructor's choice (e.g. Bernoulli Equations, reduction of order, Bessel equations, hypergeometric equations and special functions).

(2). Existence and uniqueness: phase space, phase curves, integral curves, Autonomous and non-autonomous differential equations, Lipschitz and locally Lipschitz functions, Picard's theorem, Intervals of existence for the initial value problems (IVP), Maximal interval of existence.

(3). Linear DEs: First order vector valued linear differential equations. Reduction of scalar nth order linear differential equation to a first order $\mathbb{R}^n$ valued linear DE. Wronskians. Variation of parameters for first order vector DEs. The exponential of a linear endomorphism. Real and complex Jordan forms, general solutions of homogeneous first order vector valued linear DEs with constant coefficients.

(4). Continuity and differentiability with respect to initial conditions. The statement of the rectification theorem.

(5). *If time permits*: One or more of the following topics. Proof of the rectification theorem. Existence of phase curves on compact manifolds. Solutions via Laplace and Fourier transforms. Power series solutions. First integrals. Lyapunov stability.

References.

(1). V. I. Arnold, *Ordinary Differential Equations*, translated by Richard A. Silverman, MIT Press (also Prentice-Hall, India), Cambridge,MA, U.S.A., 1973.
(2). V. I. Arnold, *Ordinary Differential Equations*, Third Edition, translated by Roger Cooke, Universitext, Springer-Verlag,
(3). W. E. Boyce and R. C. DiPrima *Elementary Differential Equations and Boundary Value Problems*, Ninth Edition, Wiley, 2009.
(4). C. Chicone, *Ordinary Differential Equations with Applications*, Springer, 2006.
(5). E. A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, McGraw-Hill, New York, 1955 (republished Dover, New York, 2006).
(6). G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGraw-Hill, 1991.
(7). L. Perko, *Differential Equations and Dynamical Systems*, Springer.
(8). Additional sources as recommended by the instructor.

18. Topology

(1). Topological spaces, connected and path connected spaces, connected sets in the real line, compact spaces, compact sets in the real line, Tychonoff theorem, quotient spaces, locally compact spaces,

(2). countability and separation axioms, Urysohn lemma and at least one application from the three listed below, compactifications.

(3). homotopy of paths and fundamental groups, covering spaces, deformation retracts, fundamental groups of the circle, the punctured plane and S^n.

(4). *If time permits*: Applications of Urysohn lemma: (a) partitions of unity (b) Urysohn metrization theorem and (c) Tietze extension theorem. More examples of calculating fundamental groups (figure eight, simple cases of van Kampen theorem).

References.

(1). J. Munkres, Topology, Second Edition, Prentice Hall
(2). Additional sources as recommended by the instructor

19. Programming Language Concepts

(1). Object oriented-programming
(2). Event driven programming
(3). Exception handling
(4). Concurrent programming
(5). Foundations of functional programming: λ-calculus, type checking
(6). Logic programming
(7). Scripting languages

References.

(1). John C Mitchell, *Concepts in Programming Languages*, Cambridge University Press, 2003.
(2). Ravi Sethi, *Programming Languages*, Addison Wesley, 1996.