(1) A, D.
(2) B, C, D.
(3) B, D.
(4) A, B, D.
(5) A, B, D.
(6) A, C, D.
(7) A, B, C.
(8) C, D.
(9) A, B, C.
(10) 2.

Part B

(11) (A) Consider \( r : \mathbb{R} \to [0, 1] \)

\[
r(x) = \begin{cases} 
0, & r < 0; \\
x, & r \in [0, 1]; \\
1, & r > 1.
\end{cases}
\]

(B) No. Every retract \( Y \) of \( \mathbb{R} \) must be connected because the map \( r \) is continuous.

(C) Every retract \( Y \) of \( \mathbb{R} \) is closed. To see this, consider \( \phi : \mathbb{R} \to \mathbb{R} \times \mathbb{R}, x \mapsto (r(x), x) \).

Then \( Y = \phi^{-1}( \text{diagonal} ) \). Since \( \mathbb{R} \) is Hausdorff, the diagonal is closed, and so is \( Y \).

(12) Write

\[
g_n(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{\xi^n}{\xi - z} d\xi.
\]

Note that

\[
g_n(z) = \begin{cases} 
1, & n = 0, z = 0 \\
0, & n \neq 0, z = 0 \\
z^n, & z \neq 0
\end{cases}
\]

Hence

\[
f(z) = \sum_{n=-\infty}^{N} a_n g_n(z) = \begin{cases} 
a_0, & z = 0; \\
F(z), & z \neq 0.
\end{cases}
\]

(13) Write \( F_n = (-1)^n \frac{\int_0^1 \phi(t)e^{-at}dt}{n!} \). Then

\[
0 = \int_0^1 \phi(t)e^{-at}dt = \int_0^1 \phi(t) \left( \sum_{n=0}^{\infty} \frac{(-at)^n}{n!} \right) dt
\]

\[
= \lim_{N \to \infty} \int_0^1 \phi(t) \left( \sum_{n=0}^{N} \frac{(-at)^n}{n!} \right) dt
\]

\[
= \lim_{N \to \infty} \sum_{n=0}^{N} \int_0^1 \phi(t) \left( \frac{(-at)^n}{n!} \right) dt
\]

\[
= \sum_{n=0}^{\infty} F_n a^n
\]
By way of contradiction assume that $U \not\subset \mathbb{R}$. Then, since $U$ is open, $U \not\subset \overline{U}$. Pick $x \in U \setminus U$ and a sequence $\{x_n\} \subset U$ converging to $x$. Since $h$ is uniformly continuous, $\{h(x_n)\}$ is a Cauchy sequence in $\mathbb{R}$, it converges to $y \in \mathbb{R}$. Hence $\{x_n\}$ converges go $h^{-1}(y) \in U$, a contradiction.

Since $\det A = -1$, the characteristic polynomial of $A$ is of the form $X^2 + bX - 1$ for some $b \in \mathbb{R}$, so $A$ has real eigenvalues, $\lambda_1, \lambda_2$. Let $v_i$ be an eigenvector for $\lambda_i$, $i = 1, 2$. For $i = 1, 2$, $\lambda_i^2 v_i = \lambda_i v_i, v_i = v_i A^2 Av_i = v_i v_i$, so $\lambda_i$ is 1 or $-1$. Without loss of generality, $\lambda_1 = 1$ and $\lambda_2 = -1$. Then $A$ gives a reflection about the line spanned by $v_1$ sending $v_2$ to $-v_2$.

(A) For any $G$, 0 is a characteristic subgroup. Let $0 \not= H \subset \mathbb{Q}$ be a characteristic subgroup. Let $0 \not= x \in H$ and $y \in \mathbb{Q}$. Then the map $r \mapsto ry/x$ is an automorphism of $\mathbb{Q}$, and it takes $x$ to $y$. Hence $y \in H$, so $H = \mathbb{Q}$. Hence 0 and $\mathbb{Q}$ are the only characteristic subgroups of $\mathbb{Q}$.

(B) For any $g \in G$, the map $G \rightarrow G, g_1 \mapsto gg_1g^{-1}$ is an isomorphism, so, for every characteristic subgroup $H$ of $G$, $gHg^{-1} = H$. Hence $H$ is normal. The converse is false: take $H = \mathbb{Z}$ inside $G = \mathbb{Q}$.

(C) Using (B) it is enough to show that there does not exist a nonzero $v \in \mathbb{R}^3$ whose span is stable under the action of $SO_3$. This is true: if $v, w \in \mathbb{R}^3$ are non-zero vectors of the same length, there exists $A \in SO_3$ such that $Av = w$.

For positive integers $n$, write $U_n = \{z \in \mathbb{C} \setminus \{0\} : |z| < n\}$. These form an open cover of $\mathbb{C} \setminus \{0\}$. Note that for every $z_1 \sim z_2$, $|z_1| = |z_2|$, so for every $n$, $\pi^{-1}(\pi(U_n)) = U_n$. Hence $\pi(U_n), n \geq 1$ is an open cover of $X$. This does not have a finite sub-cover since the open cover $U_n, n \geq 1$ does not have a finite sub-cover.

First note that the minimal polynomial of $g$ divides $X^{[G]} - 1$. (A) If $\text{char} k = 0$ then for every $g$, $g$ has distinct eigenvalues and hence is diagonalizable, so $g = 1$, contradicting the hypothesis that $[G] > 1$. (B) Let $p = \text{char} k$. Let $g \in G$ and write its order as $p^m m$ with $m = 1$ or $m > 1$ and $p \nmid m$. The minimal polynomial of $g^m$ is $X^m - 1$ which has distinct roots, so, again, by the above argument, $g^m = 1$, so $m = 1$. (C) Hence $G$ is a $p$-group. Use class equation.

The limit is $f(1)$. This is true for $x^k, k \geq 0$, and hence also for polynomials. By Weierstrass’ theorem, it is true for all continuous functions.