Instructions

- Enter your *Registration Number* here: **CMI PG–** ☐☐☐☐☐☐

  or here: **C–** ☐☐☐☐

  Enter your *Examination Centre* here: ☐☐☐☐☐☐☐☐☐☐☐☐

- This examination has two parts Part A and Part B.
  - The time allowed is 3 hours.
  - Total Marks: 100
- Answer all questions.
- Answer questions for Part A in the special answer sheet provided for it.
- *Rough Work:* The coloured blank pages are to be used for rough work only.

For office use only

Part A

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Part B

1	
2	
3	
4	
5	
6	
7	
Total	

Overall Marks

Part A	
Part B	
Total	

Answer Sheet for Part A

Instructions

- Answer all questions for Part A on this sheet only.
- Tick on the appropriate box to indicate your answer.

	a	**b**	**c**	**d**
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

CHENNAI MATHEMATICAL INSTITUTE

M.Sc. / Ph.D. Programme in Computer Science

Entrance Examination, 15 May 2014

This question paper has 4 printed sides. Part A has 10 questions of 3 marks each. Part B has 7 questions of 10 marks each. The total marks are 100. Answers to Part A must be filled in the answer sheet provided.

Part A

1. For the inter-hostel six-a-side football tournament, a team of 6 players is to be chosen from 11 players consisting of 5 forwards, 4 defenders and 2 goalkeepers. The team must include at least 2 forwards, at least 2 defenders and at least 1 goalkeeper. Find the number of different ways in which the team can be chosen.

   (a) 260 (b) 340 (c) 720 (d) 1440

2. The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers strictly greater than 14.

   (a) $\frac{7}{11}$ (b) $\frac{5}{12}$ (c) $\frac{4}{11}$ (d) $\frac{5}{22}$

3. In the code fragment on the right, `start` and `end` are integer values and `prime(x)` is a function that returns `true` if `x` is a prime number and `false` otherwise.

```
i := 0; j := 0; k := 0;
for (m := start; m <= end; m := m+1){
  k := k + m;
  if (prime(m)){
     i := i + m;
  }else{
     j := j + m;
  }
}
```

   At the end of the loop:

   (a) `k < i+j` (b) `k = i+j` (c) `k > i+j` (d) Depends on `start` and `end`

4. Alan's task is to design an algorithm for a decision problem P. He knows that there is an algorithm A that transforms instances of P to instances of the Halting Problem such that *yes* instances of P map to *yes* instances of the Halting Problem, and *no* instances of P map to *no* instances of the Halting problem. Which of the following is true.

   (a) The existence of A implies the existence of an algorithm for P.

   (b) The existence of A implies that there is no algorithm for P.

   (c) The existence of A says nothing about whether there is an algorithm for P.

   (d) The Halting Problem can be solved using A.

5. Let $\Sigma = \{a, b\}$. For a word $w \in \Sigma^*$, let $n_a(x)$ denote the number of a's in w and let $n_b(x)$ denote the number of b's in w. Consider the following language:

$$L := \{xy \mid x, y \in \Sigma^*,\ n_a(x) = n_b(y)\}$$

What can we say about L?

(a) L is regular, but not context-free.
(b) L is context-free, but not regular.
(c) L is Σ^*.
(d) None of these.

6. Suppose we are working with a programming language that supports automatic garbage collection. This means that:

(a) Uninitialized variables are assigned null values.
(b) Unreferenced dynamically allocated memory is added back to free space.
(c) Unreachable `if-then-else` branches are pruned.
(d) Expressions where array indices exceed array bounds are flagged.

7. Let M be the maximum number of unit disks (disks of radius 1) that can be placed inside a disk of radius 10 so that each unit disk lies entirely within the larger disk and no two unit disks overlap. Then:

(a) $M < 25$
(b) $25 \leq M < 40$
(c) $40 \leq M < 55$
(d) $M \geq 55$

8. What are the possible values of $\gcd(7p + 94, 7p^2 + 97p + 47)$, where p is an arbitrary integer?

(a) Either 1 or 94
(b) Either 94 or 47
(c) Either 1 or 47
(d) None of these

9. A company is due to send a shipment to a client and the CEO has resigned. To select a new CEO, some candidates have been interviewed. One of them will be chosen through a vote. If the workers union resort to a strike and the candidates have to be interviewed again, then the shipment deadline will be missed. If there are more abstainers than voters in the vote to choose the new CEO, then the candidates have to be interviewed again. Suppose that the shipment was sent on time. Which of the following is a valid conclusion?

(a) The workers union did not resort to a strike.
(b) The number of voters was more than the number of abstainers.
(c) (a) or (b).
(d) If the workers union resorted to a strike, then the number of voters was greater than or equal to the number of abstainers.

10. Avinash is taller than Abhay. Bharat is taller than Vinu and Vinay is taller than Bharat. Which of the following is a minimal set of additional information that can determine the tallest person?

(a) Vinay is taller than Avinash and Abhay is taller than Bharat.
(b) Avinash is taller than Vinay.
(c) Abhay is shorter than Vinay.
(d) None of the above.

Part B

1. Let A be a regular language. Consider the following operations on A:

$$2A := \{\ xy \mid x, y \in A \text{ and } x = y\}$$
$$A^2 := \{\ xy \mid x, y \in A\}$$

   One of these operations necessarily leads to a regular language and the other may not. Identify which is which. For the regular operation, give a proof that it is regular. For the non-regular operation, give an example of an A such that applying the operation on it results in a non-regular language.

2. There are n students in a class. The students have formed k committees. Each committee consists of more than half of the students. Show that there is at least one student who is a member of more than half of the committees.

3. *Air CMI* operates direct flights between different cities. For any pair of cities A and B, there is at most one flight in a day from A to B. The online site *FakeTrip* is trying to compile a list of all routes available through *Air CMI*.

   (a) *FakeTrip* has a table of all direct flights operated by *Air CMI*. From this, it wants to prepare a list of all pairs of cities connected by a sequence of flights. Describe an algorithm for this and analyze the complexity of your algorithm.

   (b) *CheapTricks* is trying to enter the market by improving on *FakeTrip*. *CheapTricks* has realized that not all connections listed by *FakeTrip* are feasible because of arrival and departure constraints. A connection from A to B to C is possible if the scheduled arrival of the flight from A to B is at least one hour before the scheduled departure of the flight from B to C.

   Given a table of direct flights with scheduled arrival and departure times, describe an algorithm for *CheapTricks* to list all pairs of cities connected by a route on which all connections are feasible within the same day. Analyze the complexity of your algorithm.

4. The frequency of a number in an array is the number of times it appears in the array. Describe an algorithm that finds the most frequent number in an array of n numbers. If there are multiple numbers with highest frequency then list them all. Analyze the complexity of your algorithm.

5. At the end of its fifth successful season, the Siruseri Premier League is planning to give an award to the Most Improved Batsman over the five years. For this, an Improvement Index will be computed for each batsman. This is defined as the longest sequence of increasing scores by the batsman among all his scores over the five seasons. For example, if the scores for a batsman over the five seasons are $(20, 23, 6, 34, 22, 52, 42, 67, 89, 5, 100)$, his Improvement Index is 7 based on the sequence $(20, 23, 34, 52, 67, 89, 100)$. Describe an efficient algorithm based on dynamic programming to compute the Improvement Index for a batsman with an overall sequence of n scores. Analyze the complexity of your algorithm.

6. (a) Let A be an array of n integers, sorted so that $A[1] \leq A[2] \leq \cdots \leq A[n]$. You are given a number x. The aim is to find out if there are indices k and l such that $A[k] + A[l] = x$. Design an algorithm for this problem that works in time $O(n)$.

(b) Let A be array of n integers that is not assumed to be sorted. You are given a number x. The aim is to find out if there are indices k, l and m such that $A[k] + A[l] + A[m] = x$. Design an algorithm for this problem that works in time $O(n^2)$.

7. Consider the code below, defining the function A:

```
A(m, n, p) {
    if (p == 0) return m+n;
    else if (n == 0 && p == 1) return 0;
    else if (n == 0 && p == 2) return 1;
    else if (n == 0) return m;
    else return A(m, A(m,n-1,p), p-1);
}
```

(a) Express $A(m, n, 1)$ as a function of m and n.

(b) Express $A(m, n, 2)$ as a function of m and n.

(c) Compute $A(2, 2, 3)$ and $A(2, 3, 3)$.