M.Sc. in Data Science
Sample Questions for Admission Test

Exam pattern

The exam will have 40 questions to be solved in 3 hours. Of these, 20 questions will be objective type (multiple choice, or fill in the blank with a single number/expression) and 20 questions will be short answer type (calculate an answer and give a brief justification). The objective type problems carry 2 points each (with **no partial credit**) and the short answer type problems carry 3 points each (partial credit possible). The total number of points is 100.

Here are some questions indicative of the level and scope of the exam.

Notation

- A function f from a set A to a set B is said to be **injective (or one-to-one)** if $f(x) = f(y)$ implies $x = y$ for all $x, y \in A$;
- f is said to be **surjective (or onto)** for every $y \in B$ there exists $x \in A$ such that $f(x) = y$;
- f is said to be **bijective** if it is both injective and surjective;
- f is said to be **invertible** if there exists a function g from B to A such that $f(g(y)) = y$ for all $y \in B$ and $g(f(x)) = x$ for all $x \in A$.
- For a matrix A, $|A|$ denotes the determinant of A and A^T denotes the transpose of A.

Objective type questions

In multiple choice questions, there may be multiple correct choices. You have to select all to get full marks. For questions that ask you to calculate a value, you will be assessed based on the answer you provide. No explanation is required.

1. If $A = \begin{vmatrix} a & b \\ b & a \end{vmatrix}$ and $A^2 = \begin{vmatrix} \alpha & \beta \\ \beta & \alpha \end{vmatrix}$, then which of the following is/are true?

   (a) $\alpha = a^2 + b^2$
   (b) $\beta = 2ab$
   (c) $\alpha = a^2 - b^2$
   (d) $\beta = -2ab$

2. A function $y = f(x)$ is said to be even if $f(-x) = f(x)$ for every x. For example, $f(x) = x^2$ is an even function. Which of the following are even functions?

   (a) $\dfrac{e^x - 1}{e^x + 1}$
   (b) $\dfrac{a^x + a^{-x}}{2}$, where $a > 0$
   (c) $x^2 - |x|$

(d) $\log\left(\frac{1-x}{1+x}\right)$

3. Which of the following statements is/are true?

   (a) $f : \mathbf{R} \to \mathbf{R}$, $f(x) = \cos x$, is neither injective nor surjective function.
   (b) $f : \mathbf{R} \to \mathbf{R}$, $f(x) = x^3$, is an injective but not a surjective function.
   (c) $f : \left[\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$, $f(x) = \sin x$, is bijective function.
   (d) $f : \mathbf{N} \to \mathbf{N}$, $f(n) = 2n - 1$, is an injective but not a surjective function

4. Sushma has 2 children. Let A be the event that the elder child is a girl, B be the event that at least one child is a girl and C be the event that both children are girls. Which of the following are correct statements?

   (a) $P(C \mid A) = \frac{1}{2}$.
   (b) $P(C \mid B) = \frac{1}{2}$.
   (c) $P(C \mid A) = \frac{1}{3}$.
   (d) $P(C \mid B) = \frac{1}{3}$.

5. Let X represent the lifetime of a mobile phone model. The probability that a phone will have a lifetime of more than a years is given by the following formula.

$$P(X > a) = \int_a^\infty \lambda e^{-\lambda x} dx, \text{ where } \lambda = 0.5$$

   Which of the following statements are correct?

   (a) The probability that a phone has a lifetime between 2 and 3 years $\int_3^\infty \lambda e^{-\lambda x} dx - \int_2^\infty \lambda e^{-\lambda x} dx$.
   (b) The probability that a phone has a lifetime less than 5 years is $\int_5^\infty \lambda e^{-\lambda x} dx$.
   (c) The probability that a phone has a lifetime less than 7 years is $1 - \int_7^\infty \lambda e^{-\lambda x} dx$.
   (d) The probability that a phone has a lifetime between 3 and 7 years is $\int_3^\infty \lambda e^{-\lambda x} dx - \int_7^\infty \lambda e^{-\lambda x} dx$.

6. Let S denote the set of all employees in a large corporation. For $i, j \in S$, let $A(i, j) = 1$ if j reports to i and is 0 otherwise. $A(i, i) = 0$ by definition. Which of the following statements is/are true?

   (a) $\sum_{i \in S} A(i, j) \leq 1 \quad \text{for all } j \in S$

   (b) $\sum_{i \in S} A(i, j) \leq 1 \quad \text{for all } i \in S$

   (c) $\sum_{i \in S} A(i, j) \geq 1 \quad \text{for all } j \in S$

   (d) $\sum_{i \in S} \sum_{j \in S} A(i, j) = n.$

7. You are told the following facts:

   - All slithy toves are mimsy.
   - All mimsy creatures are borogove.

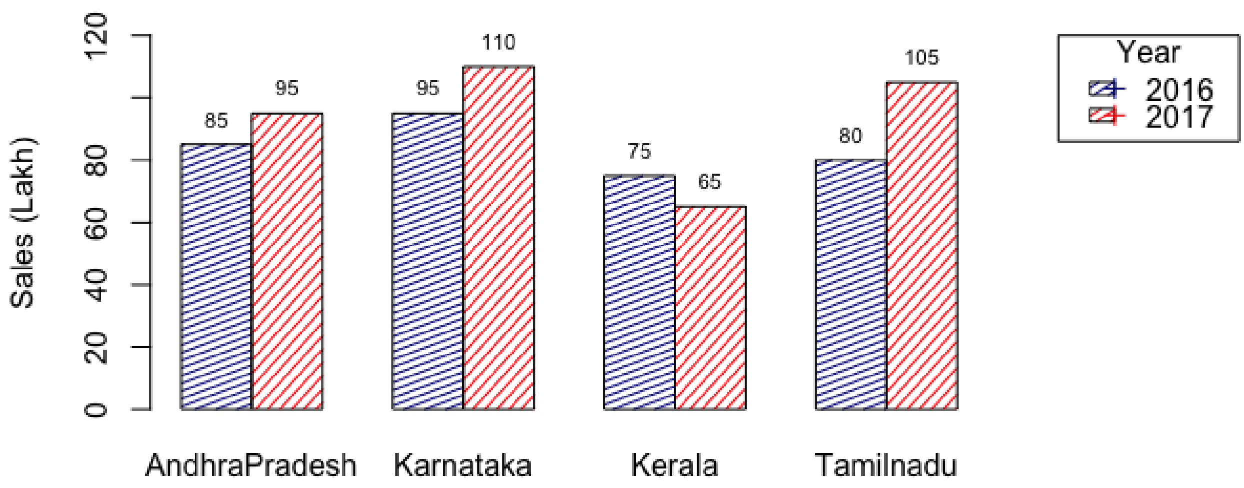
   It turns out the jabberwock is not mimsy. Therefore it must be the case that

(a) The jabberwock is not borogove.
(b) The jabberwock is not a slithy tove.
(c) The jabberwock is neither borogove nor a slithy tove.
(d) The jabberwock may be either borogove or a slithy tove.

8. Avinash is taller than Abhay. Bharat is taller than Vinu and Vinay is taller than Bharat. Which of the following additional pieces of information can determine the tallest person? (Select all that apply.)

(a) Avinash is taller than Vinay
(b) Bharat is shorter than Avinash
(c) Vinay is shorter than Abhay
(d) Abhay is shorter than Vinu

Description for next two questions: The bar graph given below shows the sales of non-fiction books of an on-line store, during two consecutive years 2016 and 2017, for four states of India.



9. Which of the following statements are correct?

(a) The ratio of the total sales of Karnataka for both years to the total sales of TamilNadu for both years is 41:37.
(b) The total sales for the year 2017 is Rs. 335/-
(c) The average sales in 2016 is 89.33% percent of the average sales of 2017.
(d) The average sales in southern India over two years is Rs 355/-.

10. Which of the following statements are correct?

(a) The sales of Kerala is dropped by 13%
(b) The sales of TamilNadu has increased by 25%.
(c) The growth in book sales in Andhra Pradesh is larger than the Karnataka.

(d) Overall growth in book sales is 11.9%

Description for next four questions: In an exam 43% passed in Mathematics, 52% passed in Physics and 52% passed in Chemistry. Only 8% students passed in all the three. 14% passed in Mathematics and Physics and 21% passed in Mathematics and Chemistry and 20% passed in Physics and Chemistry. The number of students who took the exam is 200.

11. How many students passed in only in Mathematics?

12. What is the ratio of students passing only in Mathematics to the students passing only in Chemistry?

13. What is the ratio of the number of students passing only in Physics to the students passing in either Physics or Chemistry or both?

14. A student is declared to have passed in the exam only if he/she passes at least two subjects. How many students were declared to have passed in the exam?

15. In the code fragment below, `i0`, `j0`, `k0`, `start` and `end` are integer values and `prime(x)` is a function that returns `True` if `x` is a prime number.

```
i := i0; j := j0; k := k0;
for m in [start,start+1, ..., end] {
  if (prime(m)){
     i := i - 1;
     j := j - 1;
     k := k + 2;
  }else{
     i := i - 1;
     j := j + 2;
     k := k - 1;
  }
}
```

At the end of the loop which of the following are valid statements about the relationship between the different values in the program for any choice of `i0`, `j0`, `k0`, `start` and `end`? Note that `%` denotes the modulus operator: `a%b` is the remainder when integer `a` is divided by integer `b`.

(a) `(i - k)%3 == (i0 - k0)%3`

(b) `(i - j) == (i0 - j0)`

(c) `(i - k)%3 == (j - k)%3`

(d) `(i + j + k) == (i0 + j0 + k0)`

16. What does the following function compute in terms of n and d, where n and d are both assumed to be integers?

```
function f(n,d) {
  if (d == 0) {
     return(1)
  } else if (d < 0) {
     return(f(n,d+1)/n))
  } else {
     return(n*f(n,d-1))
  }
}
```

(a) $\log_d n$ if $d < 0$, n^d if $d > 0$

(b) n^d for all values of d

(c) $n \times d$ if $d > 0$, $n \div d$ if $d < 0$

(d) $n \times d$ for all values of d

Short answer type questions

You have to provide a brief explanation with each answer. You will be assessed both on the answer and the explanation. Partial credit is possible.

Description for the next 3 questions: a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ defines a function from $\mathbb{R}^2$ to $\mathbb{R}^2$ given by -

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} as + bt \\ cs + dt \end{bmatrix},$$

for each point $\begin{bmatrix} s \\ t \end{bmatrix} \in \mathbb{R}^2$.

1. Determine the image of the x- axis under the function defined by the matrix $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

2. Determine the image of the y- axis under the function defined by the matrix $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

3. Determine the image of the line $y = x$ under the function defined by the matrix $C = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$.

4. Find the value of $f(\pi/6)$, where $f(\theta) = \begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.

5. Given the sets $A = \{1, 2, -3, 4\}$ and $B = \{1, 3, 5, 7\}$, is $f(x) = 2x - 1$ a well-defined function from A to B? Give reasons for your answer.

6. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = (e^x - e^{-x})$. Is f an invertible function? Give reasons for your answer.

7. In a group of 50 students, what is the probability that at least two students share the same birthday? (You can assume that none of the students is born on Feb 29). Give explanation for your answer.

8. Hospital records show that 75% of patients suffering from a disease die due to that disease. What is the probability that 4 out of 6 randomly selected patients recover?

9. A group of 60 students is randomly split into 3 classes of equal size. All partitions are equally likely. What is the probability that Jack and Jill will end up in the same class?

10. Do the three lines $2x_1 + 3x_2 = 1$, $x_1 + x_2 = 0$ and $2x_1 - 5x_2 = 7$ intersect in a common point? Give reasons for your answer.

11. For what value(s) of h can the vector v be expressed as a linear combination of $\{v_1, v_2, v_3\}$ where
$v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}$, $v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$, and $v = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$

12. Is the following statement true: Let A be an $n \times n$ matrix. If A^T is not invertible then A is also not invertible. Give justification for your answer.

13. Is the following statement true: If $A = \begin{bmatrix} P & Q \\ R & S \end{bmatrix}$, then the transpose of A is $A^T = \begin{bmatrix} P^T & Q^T \\ R^T & S^T \end{bmatrix}$. Give justification for your answer.

14. Is the following statement true: If A is an invertible matrix, then $A + A^T$ is invertible. Give justification for your answer.

15. Is the following statement true: Suppose A and B are $n \times n$ matrices such that AB is invertible. Then both A and B are invertible. Give justification for your answer.

16. Is the following statement true: let I be a $k \times k$ identity matrix, 0 is the $k \times k$ matrix with all 0's and A is an arbitrary $k \times k$ matrix. Then the $2k \times 2k$ matrix $\begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$ is invertible. Give justification for your answer.

17. Suppose A and B are $n \times n$ matrices such that all elements in the last column of B are zeroes. Compute the last column of the matrix AB.

18. A touring cricket team has 15 players, comprising 6 specialist batsmen, 5 specialist bowlers, 2 all-rounders and 2 wicket-keepers. It is time to select the team for the next match. A team consists of 11 players and the desired composition is 5 specialist batsmen, 4 specialist bowlers, 1 all-rounder and 1 wicket-keeper. In how many ways can such a team be selected?

19. There are 60 students in the class and each knows at least one programming language among Java, C++ and Python. 40 students know at least two of the languages and 10 know all three. How many students know exactly one programming language?

20. Varsha lives alone and dislikes cooking, so she goes out for dinner every evening. She has two favourite restaurants, *Dosa Paradise* and *Kababs Unlimited*, to which she travels by local train. The train to *Dosa Paradise* runs every 10 minutes, at 0, 10, 20, 30, 40 and 50 minutes past the hour. The train to *Kababs Unlimited* runs every 20 minutes, at 8, 28 and 48 minutes past the hour. She reaches the station at a random time between 7:15 pm and 8:15 pm and chooses between the two restaurants based on the next available train. What is the probability that she ends up eating in *Kababs Unlimited*?

21. An advertisement for a tennis magazine states, "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, I'm reading about tennis." We can assume that the speaker can do at most one of these activities at a time. What is the speaker doing? Give reasons for your answer.

    (a) Playing tennis.

(b) Watching tennis.
(c) Reading about tennis.
(d) None of the above.

22. Ball Mart has 10^7 different items in stock across all its stores worldwide. The company has collected billing data for 10^{10} customer transactions. Each individual bill has at most 10 distinct items in it.

    Ball Mart's CEO wants to optimize the company's inventory and has asked for a list of those items that appear in at least 2% of the billed transactions. Which of the following is the most precise upper bound one can compute for the number of such items, given the data?

    (a) 500
    (b) 1000
    (c) 5000
    (d) 20000

23. An ant starts at the origin (0,0), facing north. On day 1, it moves 1 unit forward to (0,1) and turns right 90 degrees. On day 2, it moves 2 units forward to (2,1) and turns right 90 degrees. On day 3, it moves 3 units forward to (2,-2) and turns right 90 degrees. It keeps doing this—on day i, it moves i units forward and turns right 90 degrees. After 65 days, where is the ant?

24. You have k feet of fence to make a rectangular play area alongside the wall of your house. The wall of the house bounds one side. What is the largest size possible (in square feet) for the play area?

25. Suppose a new airline is offering promotional air fares for travel between various cities are as follows:

	To Mumbai	To Delhi	To Chennai	To Kolkata	To Pune
From Mumbai	-	4000	5000	7000	2500
From Delhi	4500	-	6000	3200	5000
From Chennai	3000	4000	-	4800	5400
From Kolkata	3800	4500	3300	-	6000
From Pune	3000	3000	6000	6000	-

    If a passenger living in Delhi wishes to visit Mumbai, Pune, Kolkata and Chennai and return to Delhi. What route should she follow and what would be her cost. Justify your answer.

26. Assume that the relation "friend" is symmetric. Show that if $n \geq 2$, then in any group of n people there are at least two with the same number of friends in the group.

27. The function below takes three integer arguments. It is supposed to return the minimum of these three values.

```
function min3(x,y,z) {
   if (x <= y) {
      if (x <= z) {
         minimum := x
      }
   }
   else {
      if (y <= z) {
         minimum := y
      }
      else {
         minimum := z
      }
   }

   return(minimum)
}
```

There is an error in the function. Find values `x`, `y`, `z` for which `min3(x,y,z)` fails to return the correct value and explain the error.

Solutions

Objective type questions

Explanations are provided for some questions to aid understanding. In the exam, no explanations are required for these type of questions.

1. **(a) and (b)**.
2. **(b) and (c) are even functions**.
3. **(a), (c) and (d) are true**.
4. **(a) and (d)**.
5. **(a) and (b) are False, (c) and (d) are True**

   (a) would be correct if the terms were reversed. (b) is the probability that the lifetime is more than 5 years.
6. **Only (a) is true**.

   (a) Is true since every employee can report to at most one manager. (b) is false since multiple employees can report to an employee. (c) Is false since the CEO does not report to anyone. (d) is false since everyone other than the CEO reports to exactly one manager and hence the double sum equals $n-1$.
7. **(b) The jabberworck is not a slithy tove**.

   Since all slithy toves are mimsy by the first statement, anything that is not mimsy cannot be a slithy tove. On the other hand, the second statement does not rule out non-mimsy creatures from being borogove.
8. **(a) Avinash is taller than Vinay and (c) Vinay is shorter than Abhay**.

   We are given that Avinash is taller than Abhay and Vinay is taller than Bharat who is taller than Vinu. We need to resolve which of Avinash and Vinay is taller. (a) gives us this information directly. (c) tells us that Vinay is shorter than Abhay who is known to be shorter than Avinash, so Vinay is shorter than Avinash. The other two choices do not unambiguously resolve which of Avinash and Vinay is taller.
9. (a) **True**

   $\textit{Required Ratio} = \dfrac{(95+110)}{(80+105)} = \dfrac{205}{105} = \dfrac{41}{37}.$

   (b) **False**

   Rs 335/- is the sales of year 2017.

   (c) **True**

   Average sales in 2016 $(85+95+75+80)/4 = 335/4$;
   Average sales in 2017 $(95+110+65+105)/4 = 375/4$;
   $\textit{Required percentage} = \left[\dfrac{335/4}{375/4}\times 100\right] \approx 89.33\%.$

   (d) **True**
10. (a) **True**

    $((65-75)/75)\times 100 = -13.33\%$

    (b) **False**

    $(105-80)/80\times 100 = 31.25\%$

    (c) **False**

    The converse is true.

(d) **True**

11. **32**

12. **16:19**

13. **26:84**

14. **78**

15. **(a) and (d)**

    In each iteration, the pairwise difference between `i`*,* `j` *and* `k` *changes by 0 or by 3, so (a) holds. Also, the values subtracted from two of the variables are added back to the third, so (d) holds. The absolute difference between* `i` *and* `j` *changes as the program runs, so (b) does not hold. For (c) to hold, the initial difference* `i0 - k0` *and* `j0 - k0` *should be congruent modulo 3.*

16. **(b) n^d for all values of d**

    This computes n^d by repeated multiplication. For d negative, the answer is $\frac{1}{n^d}$ by repeated division.

Short answer type questions

1. Points on the x-axis are of the form $\begin{bmatrix} x \\ 0 \end{bmatrix}$ for $x \in \mathbb{R}$. So to find the image of the x-axis under the function defined by A, we calculate:

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 0 \end{bmatrix} = \begin{bmatrix} 1(x) + 3(0) \\ 0(x) + 1(0) \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix},$$

   so the image of the x-axis is again the x-axis. Hence A maps the x-axis to itself.

2. Points on the y-axis are of the form $\begin{bmatrix} 0 \\ y \end{bmatrix}$ for $y \in \mathbb{R}$. So

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ y \end{bmatrix} = \begin{bmatrix} y \\ 0 \end{bmatrix}.$$

   Thus B maps the y-axis to the x-axis.

3. Points on the line $y = x$ are of the form $\begin{bmatrix} x \\ x \end{bmatrix}$ for $x \in \mathbb{R}$. Now

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ x \end{bmatrix} = \begin{bmatrix} x \\ 4x \end{bmatrix}.$$

   Thus the image of the line $y = x$ under the function defined by C is the line $y = 4x$.

4. $f(\pi/6) = \cos(2\pi/6) = \cos(\pi/3) = 1/2$.

   The determinant evaluates to $\cos^2(\theta) - \sin^2(\theta)$. Recall that $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$. Hence $\cos(2\theta) = \cos(\theta + \theta) = \cos^2(\theta) - \sin^2(\theta)$.

5. $f(-3) = -7$, but -7 does not belong to B, so the function is not a well defined function from A to B. For the other three values in A, we have $f(A) \in B$.

6. Here $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$. Since f is a continuous function, it follows that f is surjective. Further, since $f'(x) = e^x + e^{-x} > 0$ for all x, f is strictly increasing and hence f is injective. Thus f is a bijective function and hence admits an inverse.

7. The required probability equals 1 minus the probability that all 50 have distinct birthdays. Hence the answer is $1 - \dfrac{{}^{365}P_{50}}{365^{50}}$.

8. The required probability equals observing 4 successes (survivors) in 6 independent trials. This is a binomial distribution with $n = 6$, $x = 4$, $p = 0.25$ (probability of living (success)) and $q = 0.75$ (probability of dying (failure)).

   If X is the number number of survivors, then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$. Thus the required probability is $\binom{6}{4}(0.25)^4(0.75)^2$.

9. **19/59**. *Assign a different number to each student from 1 to 60. Numbers 1 to 20 go in group 1, 21 to 40 go to group 2, 41 to 60 go to group 3. All possible partitions are obtained with equal probability by a random assignment if these numbers, it doesn't matter with which students we start, so we are free to start by assigning a random number to Jack and then we assign a random number to Jill. After Jack has been assigned a random number there are 59 random numbers available for Jill and 19 of these will put her in the same group as Jack. Therefore the probability is 19/59.*

10. Solving the first two equations simultaneously gives $x_2 = 1$ and $x_1 = -1$. These values of x_1 and x_2 do not satisfy the third equation (since $2x_1 - 5x_2 = -7 \neq 7$); thus the system is inconsistent and does not have a common solution. Another way to solve this would be to consider the augmented matrix

$$\begin{bmatrix} 2 & 3 & | & 1 \\ 1 & 1 & | & 0 \\ 2 & -5 & | & 7 \end{bmatrix}$$

   and apply row reduction to get

$$\begin{bmatrix} 2 & 3 & | & 1 \\ 0 & -1/2 & | & -1/2 \\ 0 & 0 & | & 14 \end{bmatrix}.$$

   This system has no solution, since no value of x_1, x_2 will satisfy $0(x_1) + 0(x_2) = 14$.

11. The question is same as the following : when does the system of equations $x_1v_1 + x_2v_2 + x_3v_3 = v$ admit a solution? Now solve the system to get the answer $h = 5$.

12. True: since $\det A = \det(A^T)$.

13. False. It is $A^T = \begin{bmatrix} P^T & R^T \\ Q^T & S^T \end{bmatrix}$.

14. False: for example $\begin{bmatrix} 1 & 4 \\ 0 & 4 \end{bmatrix}$.

15. True: We know that $|AB| = |A| \cdot |B|$. If any one of A or B is not invertible, i.e. one of $|A|$ or $|B|$ equals zero, then $|AB|$ would equal zero, a contradiction to the fact that AB is invertible.

16. True: the matrix $\begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$ is upper triangular, so its determinant is just the product of its diagonal entries, hence the determinant equals 1 and hence it is invertible.

17. The last column of AB will also consist of zeroes since for all $1 \leq i \leq n$:

$$(AB)_{in} = \sum_{k=1}^{n} \left((A)_{ik} + (B)_{kn}\right).$$

   Here $(A)_{ij}$ denotes the $(i,j)^{th}$ entry of the matrix A.

18. **120**. *There are* 6 *ways to choose* 5 *out of* 6 *batsmen,* 5 *ways to choose* 4 *out of* 5 *bowlers,* 2 *ways to choose* 1 *out of* 2 *all-rounders and* 2 *ways to choose* 1 *out of* 2 *wicket-keepers. These choices are independent, so multiply* $6 \cdot 5 \cdot 2 \cdot 2$ *to get* 120. *Alternatively, there are* 6 *ways to choose one batsman to drop,* 5 *ways to choose one bowler to drop,* 2 *ways to choose* 1 *all-rounder to drop and* 2 *wayS to choose 1 wicket-keeper to drop, so again* $6 \cdot 5 \cdot 2 \cdot 2 = 120$.

19. **20**. *Since* 40 *students know at least two programming languages and each one knows at least one language, there are* 20 *who know exactly one. The fact that* 10 *know all three is irrelevant.*

20. $\mathbf{\frac{2}{5}}$.

    If Varsha arrives in the intervals 7:20–7:28, 7:40–7:48, or 8:00–8:08, the next train goes to Kababs Unlimited. *This adds up to 24 minutes out of a total of 60 minutes so* $\frac{24}{60} = \frac{2}{5}$ *is the probability that she ends up having dinner at* Kababs Unlimited

21. **(b) Watching tennis.**

    If the speaker is not watching tennis, then she is reading about tennis. This means she is not playing tennis, which implies that she is watching tennis. Thus, the assumption that she is not watching tennis leads to a contradiction. Therefore she is watching tennis.

22. **(a) 500 items.**

    An item that is in 2% of the bills must appear in 2×10^8 *bills. Across all bills, there are at most* $(10^{10}) \times 10 = 10^{11}$ *items mentioned. So at most* $(10^{11})/(2 \times 10^8) = 500$ *items can appear in 2% of the bills. The number of items in stock is irrelevant.*

23. **(-32,33)**

    Each pair of odd numbered moves $(1,3)$, $(5,7)$, $\ldots$ *results in a net y-displacement of* -2. *Each pair of even numbered moves* $(2,4)$, $(6,8)$, $\ldots$ *results in a net x-displacement of* -2. *After* 64 *days, the ant has made* 16 *pairs of odd numbered moves and* 16 *pairs of even numbered moves, so the net displacement is* $(-32,-32)$. *Move* 65 *takes it to* $(-32,33)$.

24. $k^2/8$ **square feet.**

    Let l *be the length and* b *be the breadth of the rectangular play area. The problem is to maximize the area* $A = l \times b$ *subject to the condition that* $2b + l = k$. *Now* $2b + l = k$ *implies* $l = k - 2b$, *so that* $A = l \times b = (k-2b)b = kb - 2b^2$. *The derivative* $\frac{dA}{db} = k - 4b$*; to maximize* A*, we equate* $\frac{dA}{db}$ *to zero, so that the maximum is attained when* $k = 4b$ *i.e.* $b = k/4$. *In that case,* $l = k - 2b = k/2$. *Thus the maximum area* $A = k^2/8$ *square feet.*

25. The cheapest flights in each row are - Mumbai to Pune (2500), Delhi to Kolkata (3200), Chennai to Mumbai (3000), Kolkata to Chennai (3300) and Pune to Mumbai as well as to Delhi (3000). Thus, the cheapest routing is Delhi - Kolkata - Chennai - Mumbai - Pune - Delhi with the cost 2500+3200+3000+3300+3000=15000.

26. Any person in the group can have 0 to $(n-1)$ friends. If someone has 0 friends, then no one has $(n-1)$ friends and if someone has $(n-1)$ friends then no one has 0 friends. Hence the number of possibilities for the number of friends the n people in the group have must be either $0, 1, \ldots, (n-2)$ or $1, 2, \ldots, (n-1)$. In each case there are $(n-1)$ possibilities and n people. Hence at least two people in the group must have the same number of friends.

27. The function fails when `z` < `x` < `y`. For example, `min3(8,11,4)`. In this situation, `minimum` is not assigned any value. Since `x` < `y`, the first `if` condition succeeds, and since `x` > `z`, the nested `if` fails and the function exits the outer `if` without assigning any value to `minimum`.