## CMI Lectures on SGA3, Jan. 3-28, 2024 by Patrick Polo

Progress of the lectures

Note than in January 2024 the duration of each lecture was 75 mn, whereas it was 90 mn in August 2023. Thus the material covered in each lecture is slightly less than it was in August.

(1) Friday, Jan. 5: This was a warm-up lecture. As the contents of Lect. 11 on Weil restriction and representability of centralisers are rather technical, only the statement of Prop. 26.9 was given, and then we moved to Lect. 12. We gave the statement of Th. 27.6 (existence of maximal tori over a field), the definition of Cartan subgroups (30.3) and their known properties over an algebraically closed field (Prop. 28.6), the definition of a reductive center (Def. 28.1), stated Th. 28.5 and proved assertions (a) and (b) of this theorem.

(2) Tuesday, Jan. 9: We stated without proof Prop. 28.3, then proved assertions (c,d,e) of Theorem 28.5 and its Corollary 28.7. To finish the lecture, we moved to Section 29:  $\mathfrak{g}$  is a finite-dim. Lie algebra over an infinite field k; we give the definition of regular elements and of the nilpotent rank of  $\mathfrak{g}$  (Def. 29.2).

(3) Thursday, Jan. 11: We proved Propositions 29.3 and 29.6 and stated without proof Theorem 29.7. Assuming this theorem, we proved Prop. 30.1 and Th. 30.2. We contrasted the result 30.2 (b) for adjoint groups with Remark 29.2 about  $SL_2$  over a field of char. 2. Then we proved assertion (1) of Th. 29.7, but for uniqueness.

(4) Tuesday, Jan. 16: We restated Th. 29.7 as Theorem 31.1, reviewed the proof of (1), and proved (2) and (3). Then we came back to Lect. 11 and gave the definition of Weil restriction (Def. 25.1), the important Remark 25.7 in the case of interest in this Lecture, and gave a summarized definition of "essentially free" (see Def. 25.4).

(5) Thursday, Jan 18: We gave Remark 25.2, Def. 25.4, Example 25.5, then stated Lemma 25.6 and proved Th. 25.8. Then we proved Prop. 26.1 and its Corollary 26.4 (which uses Def. 26.30. (In a future version of the notes, this definition and corollary should be right after Prop. 26.1, whilst Prop. 26.2 and its three corollaries would be in Section 27.)

(6) Friday, Jan 19: As suggested above, the former Prop. 26.2 has become Prop. 27.1 (this induces a modification of the section numbering of Lect. 12, see the version of the notes of Jan. 20). So, in this lecture of Jan. 19, we gave Prop. 27.1, Def. 27.4 and 27.5. (To save time, Corollaries 27.2, 27.3 were not mentioned.) Then we proved the smoothness result in Prop. 27.6. Then we began Lect. 13:

(7) Monday, Jan 22: Lect. 13, proof of Th. 32.2 (aka, maximal tori exist etale-locally), assuming the difficult Th. 4.4 of SGA3, Exp. X, which we stated as Th. 34.8, explaining briefly that the crucial point was Prop. 34.6.

(8) Tuesday, Jan 23 (previsions): Recollection of Theorem 22.2 and Propositions 23.11 and 23.12 from Lecture 10, then proofs of Prop. 34.6 and Th. 34.8. Together with the sketch of proof given for Th. 24.1 of Lect. 10, this completes the proof that for a reductive S-group scheme G, maximal tori exist etale-locally on S.

(9) Wednesday, Jan 24 (previsions): Brief overview of the dynamical method of Conrad, Gabber, Prasad. Then we head for the main results of SGA3, Exp. XX: firstly, the proof of Theorem 1.5, following Conrad's lectures.

(10) Thursday, Jan 25 (previsions): completion of the proof of Th. XX 1.5. Time permitting, statement of the second main result of Exp. XX: Theorem 2.1, and overview of its proof (to be completed in the lectures of August 2024).