Neighbourhood Structure in Games

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Chennai

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Second level

- Third level
  - Fourth level
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Neighbourhood Sturcture in Games

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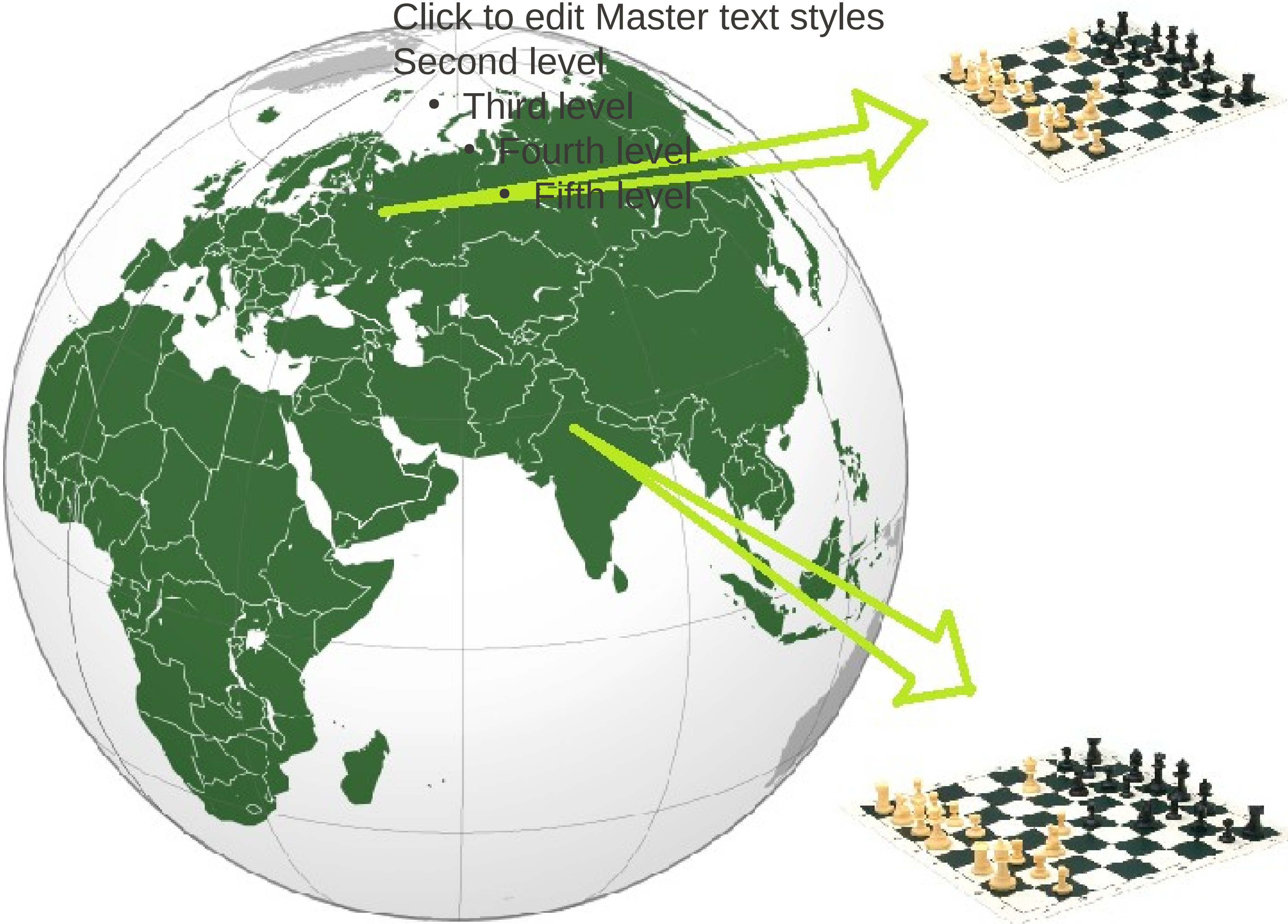

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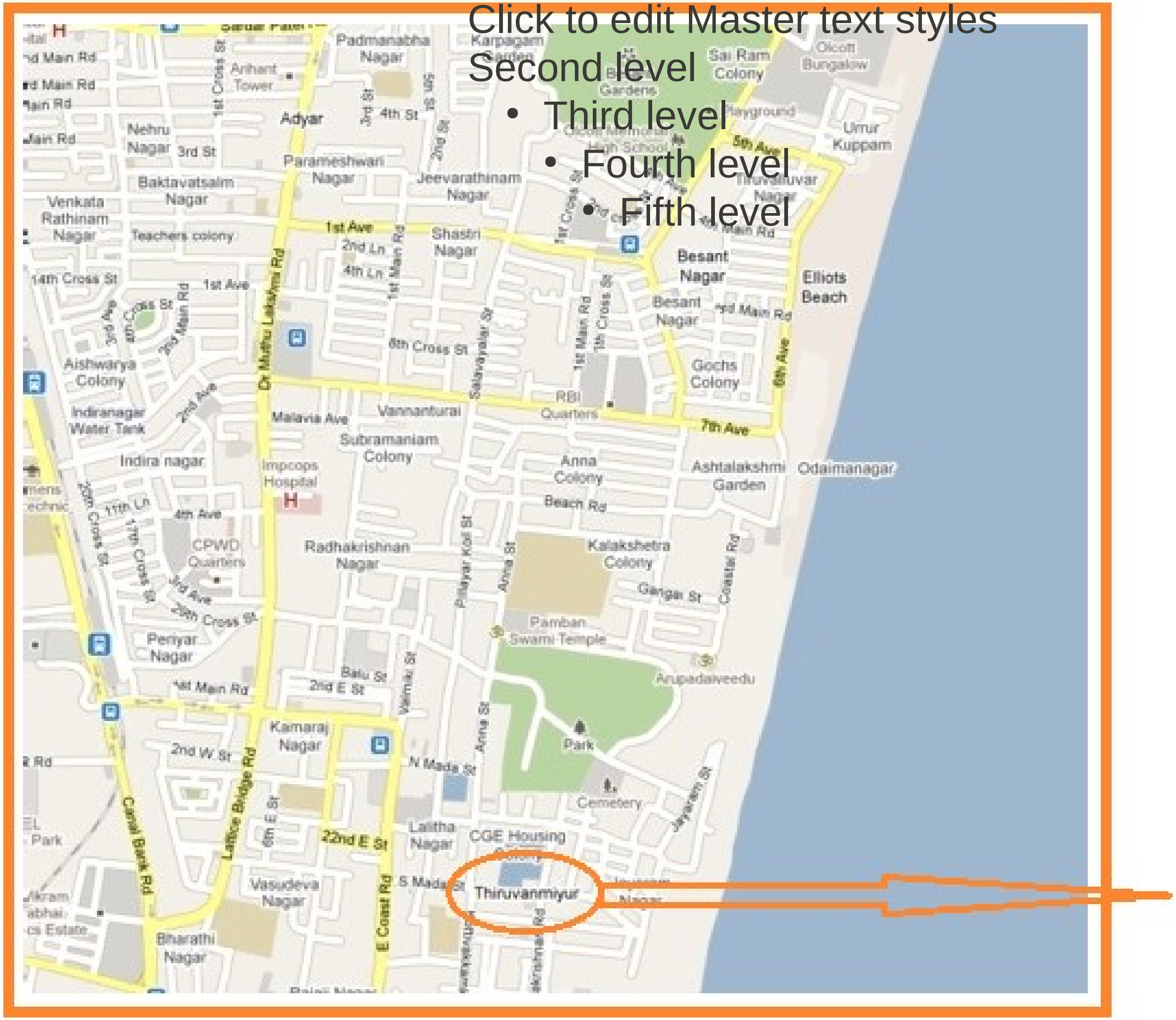

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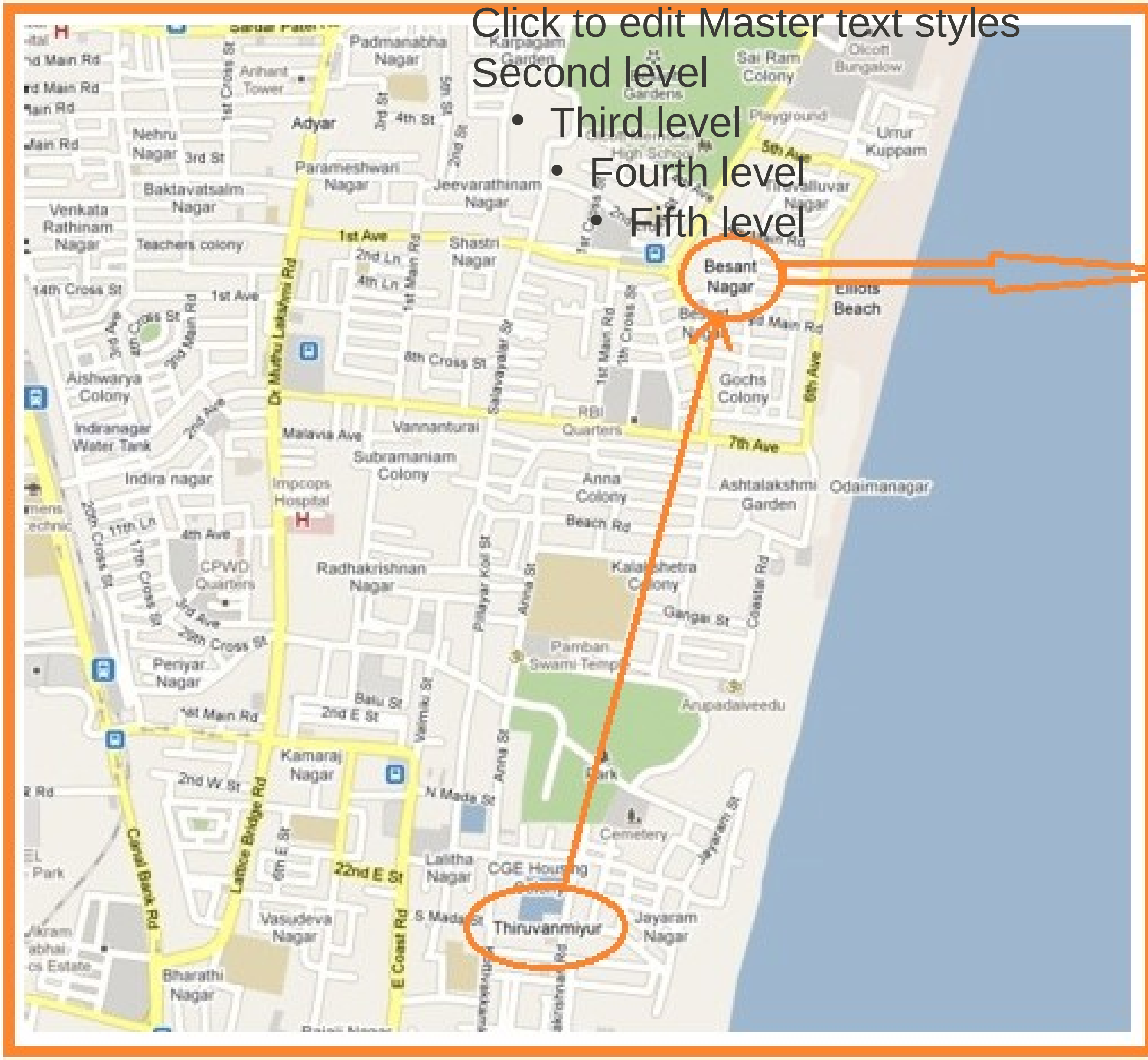

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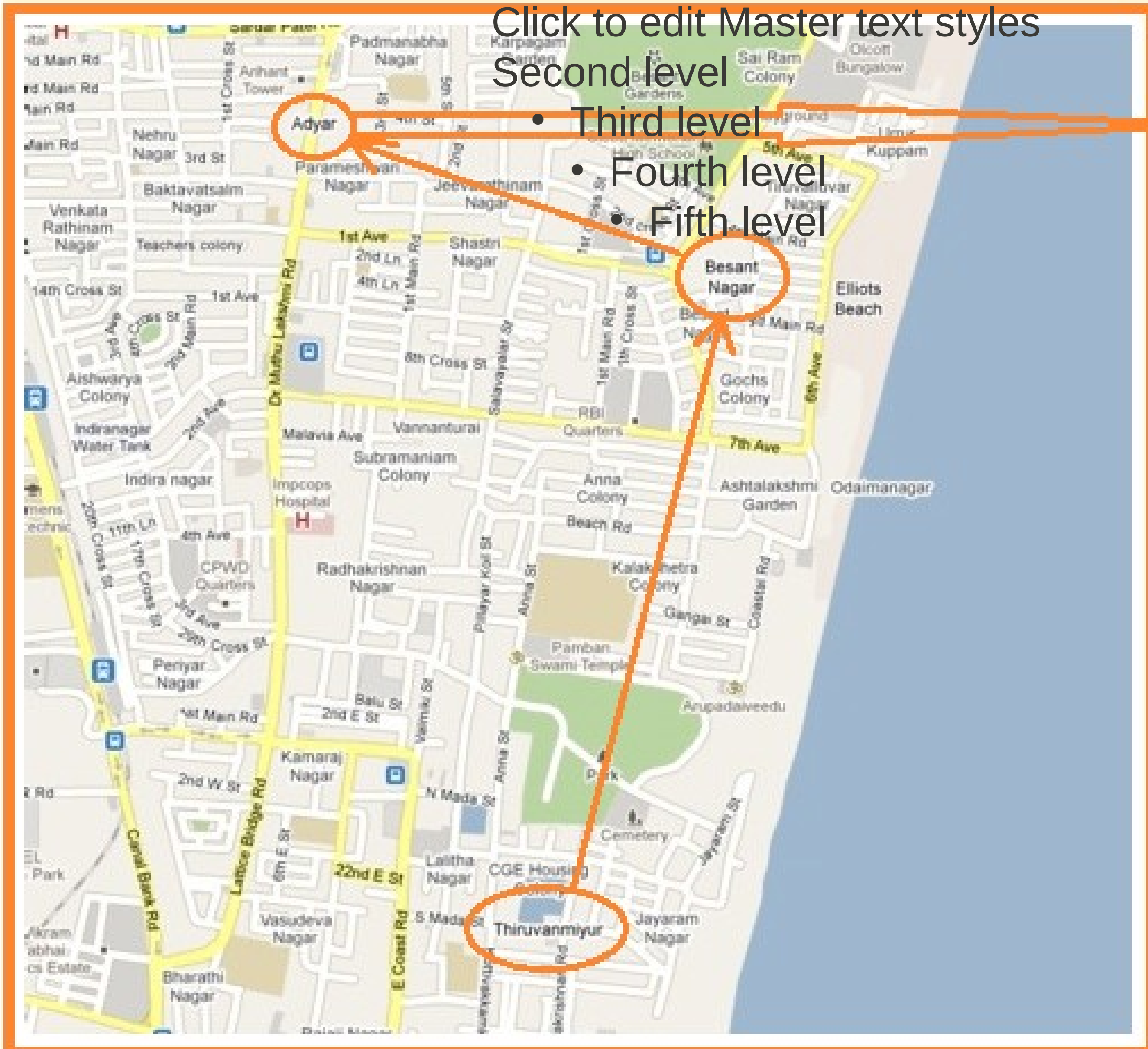

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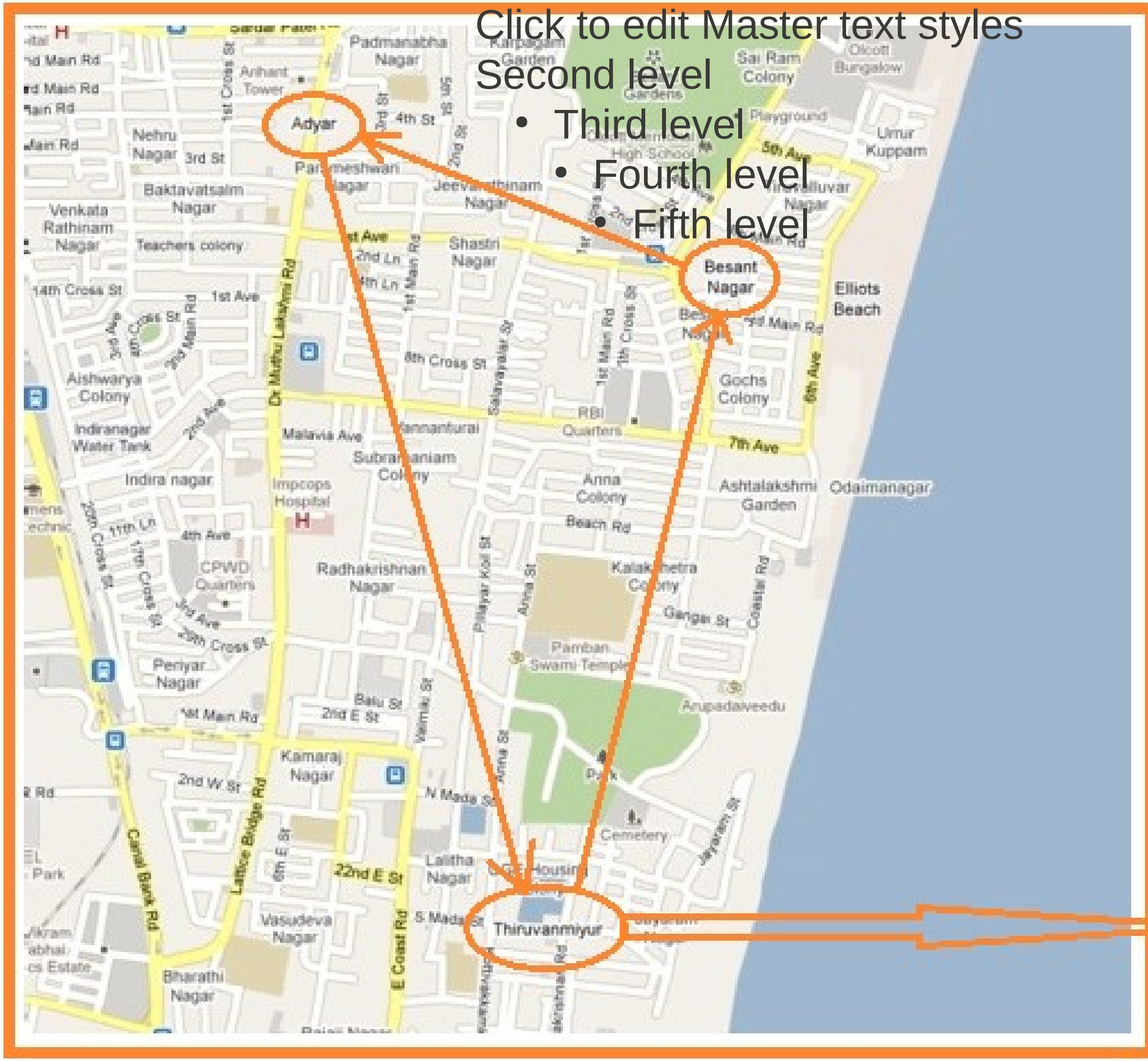

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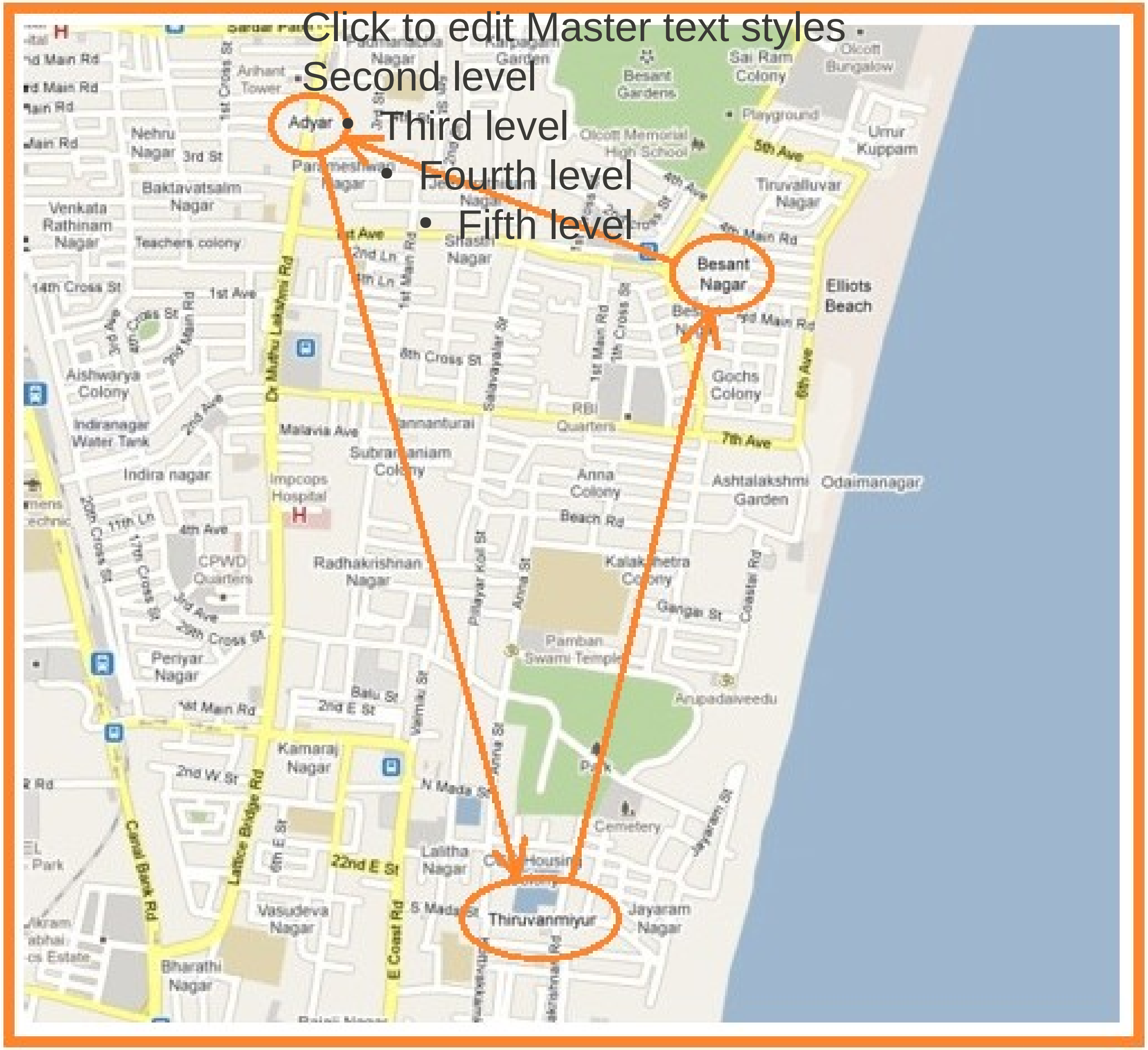
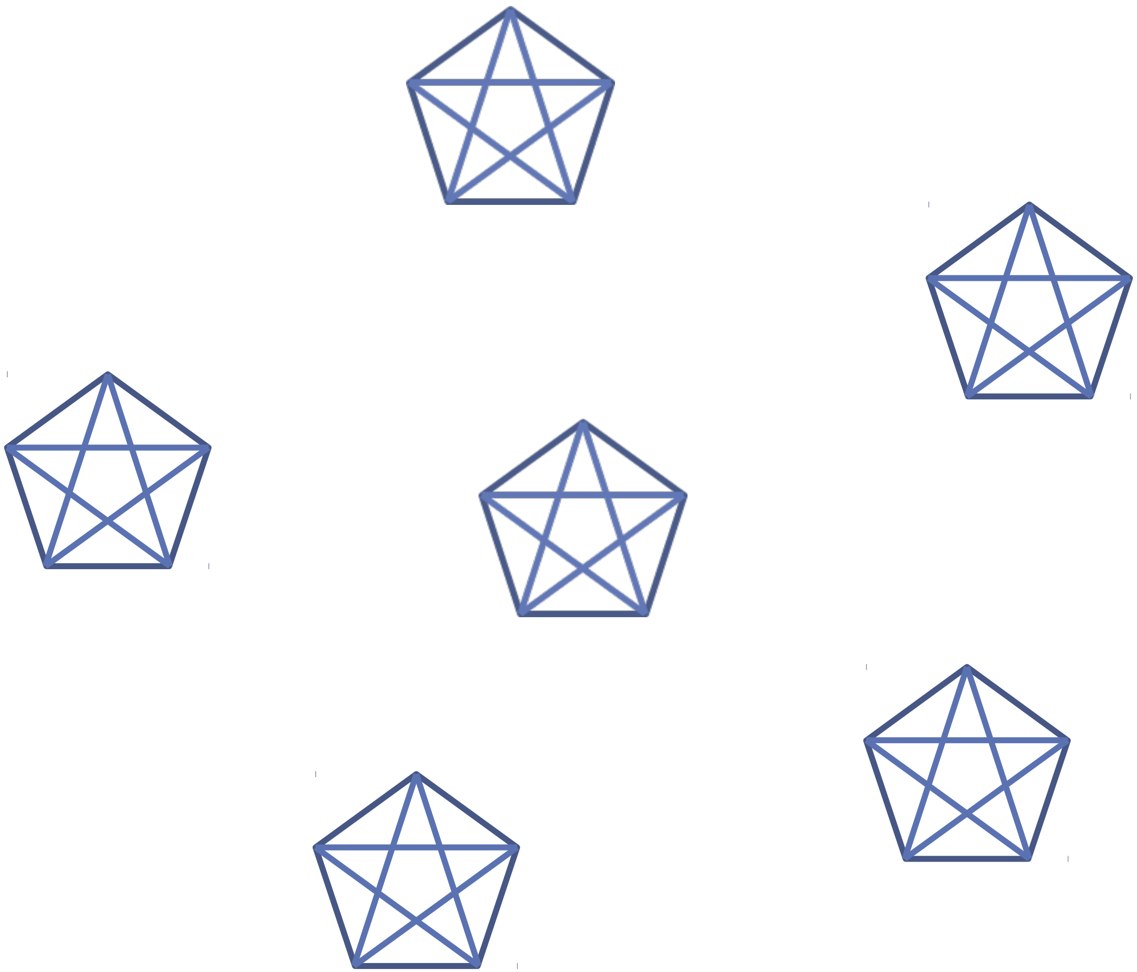


Neighbourhood Sturcture in Games

The Model



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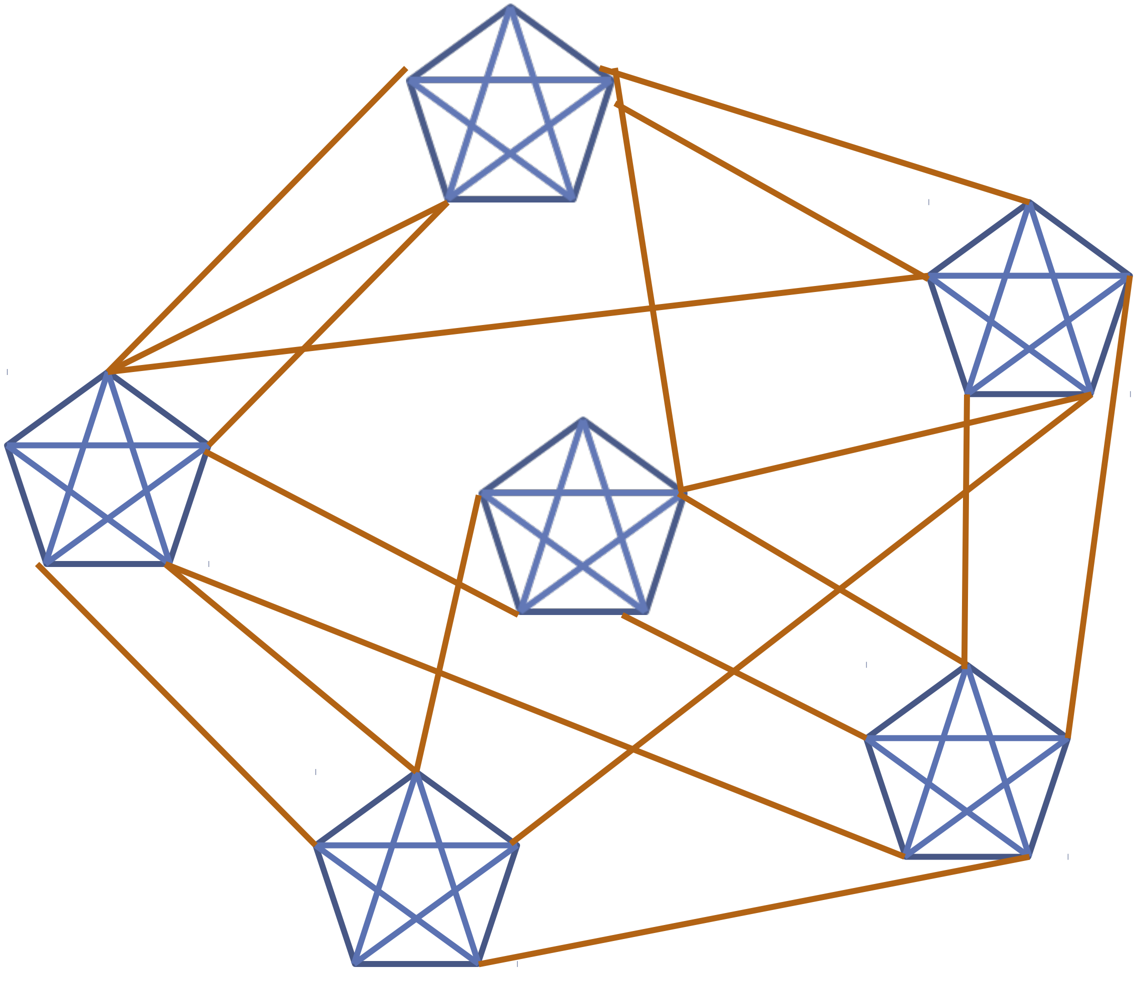


Neighbourhood Sturcture in Games

Related Work

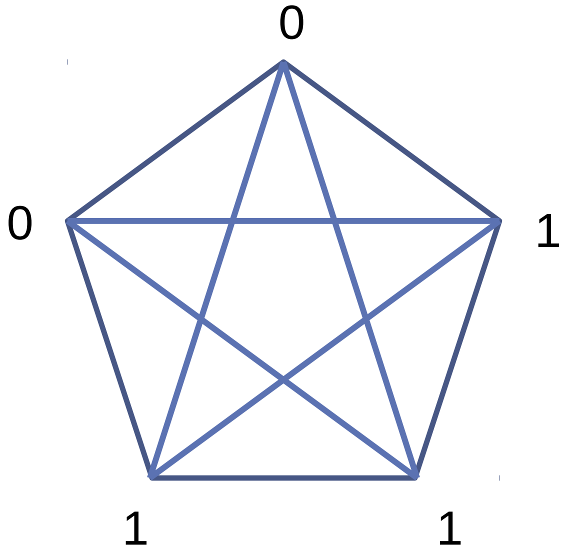
Michael J. Kearns, Michael L. Littman, and Satinder P. Singh. *An efficient, exact algorithm for solving tree-structured graphical games*. In NIPS, pages 817–823, 2001

Michael J. Kearns, Michael L. Littman, and Satinder P. Singh. *Graphical models for game theory*. In UAI, pages 253–260, 2001

H. Peyton Young. *The evolution of conventions*. In Econometrica, volume 61, pages 57–84. Blackwell Publishing, 1993

H. Peyton Young. *The diffusion of*

Weighted Coordination Games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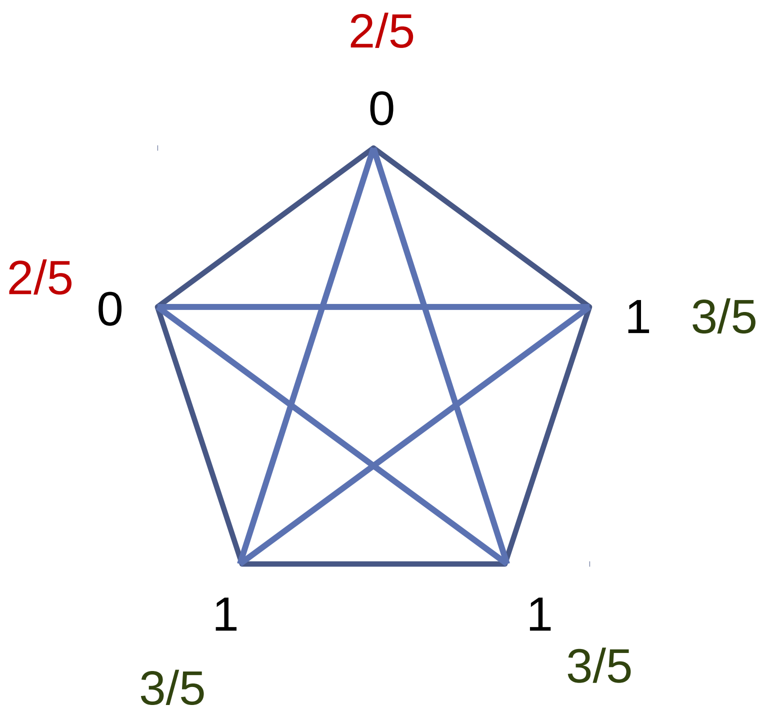
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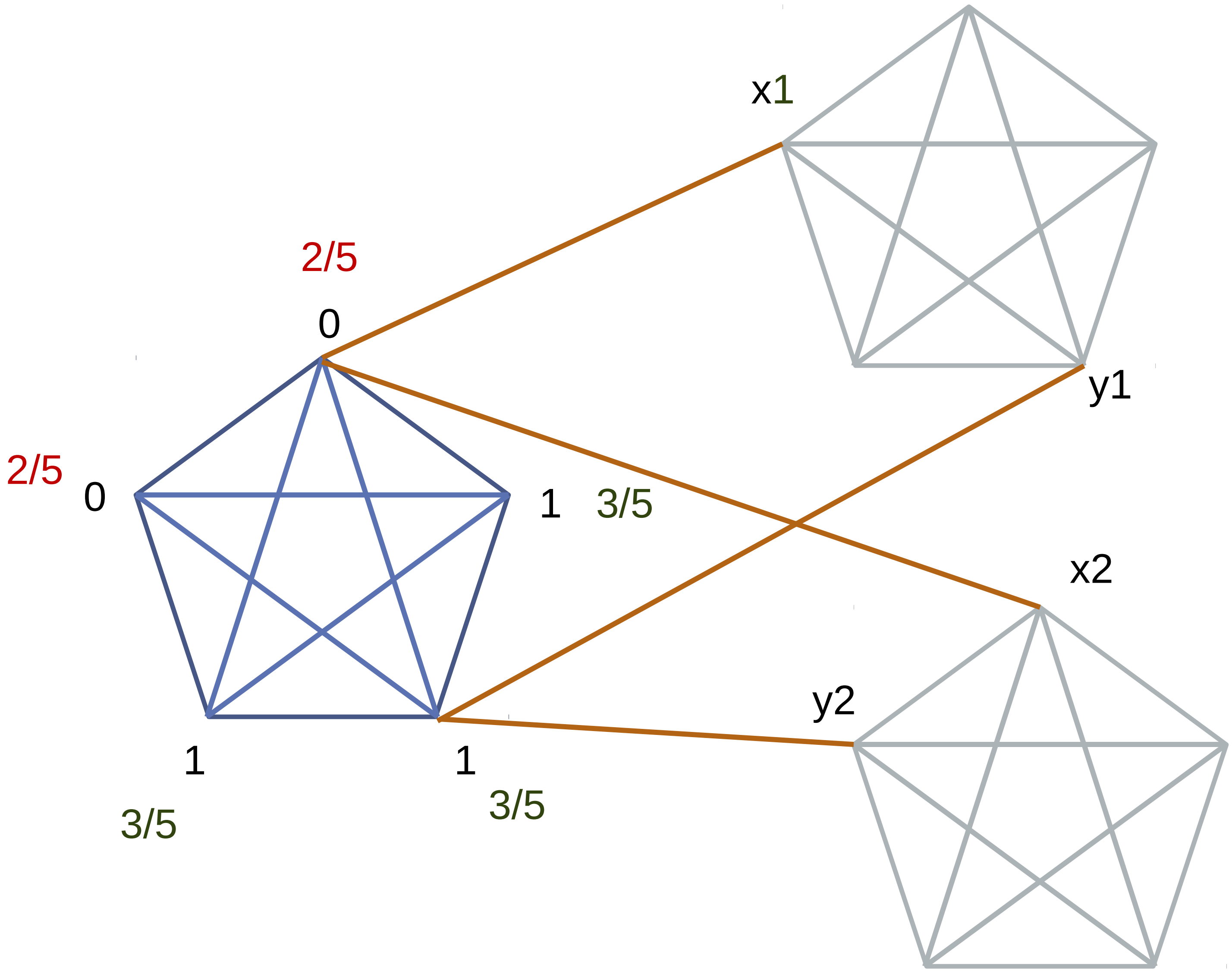

Neighbourhood Sturcture in Games

Static Neighbourhoods

Description of type t

If payoff in round k > 0.5 then

play same action a in round k+1

else if all players with the maximum payoff in round k played a different action 1-a

play 1-a in round k+1

Else play a in round k+1

EndIf

Theorem:

Let G be a neighbourhood graph and let m be the number of neighbourhoods (cliques) and let M be the maximum size of a clique. If all the players are of the same type t then the game stabilises in at most mM steps.

Proof Idea:

Associate a potential with every configuration of the graph

Show that whenever the configuration changes from round k to k+1 the potential strictly increases

The maximum possible potential of the graph is bounded

Theorem:

Let G be a neig be the number of neighbourho he maximum size of a clique same type t then the game stabil

A weight or value unique for every configuration; independent of the history

Proof Idea:

Associate a potential with every configuration of the graph

Show that whenever the configuration changes from round k to k+1 the potential strictly increases

The maximum possible potential of the graph is bounded

Theorem:

Let G be a nei ... per
of neighbourh ...
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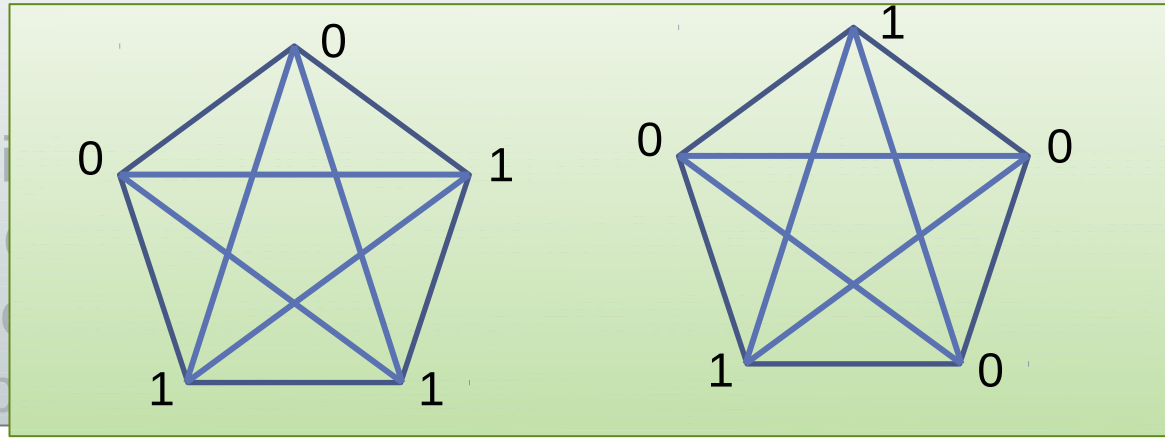

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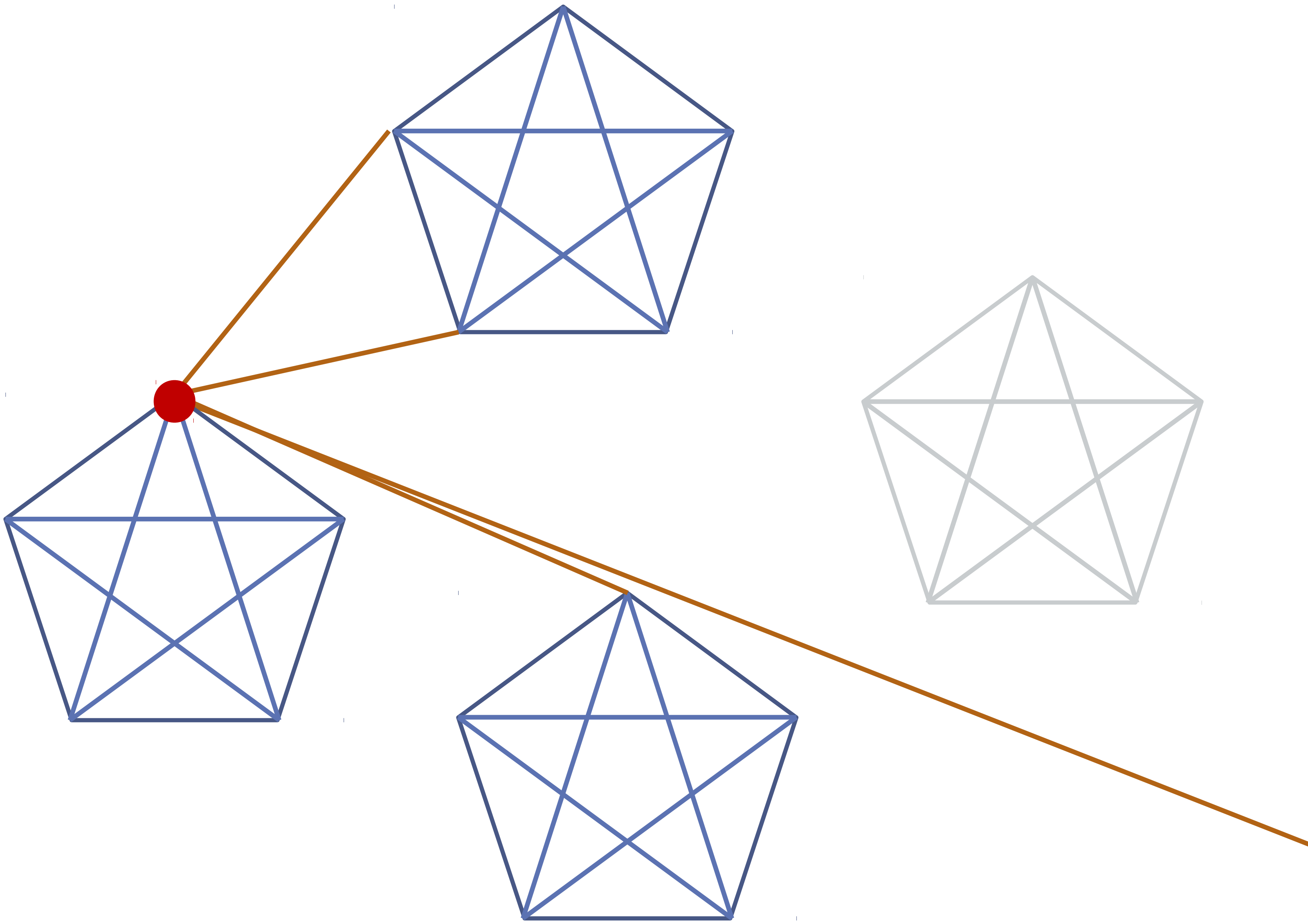
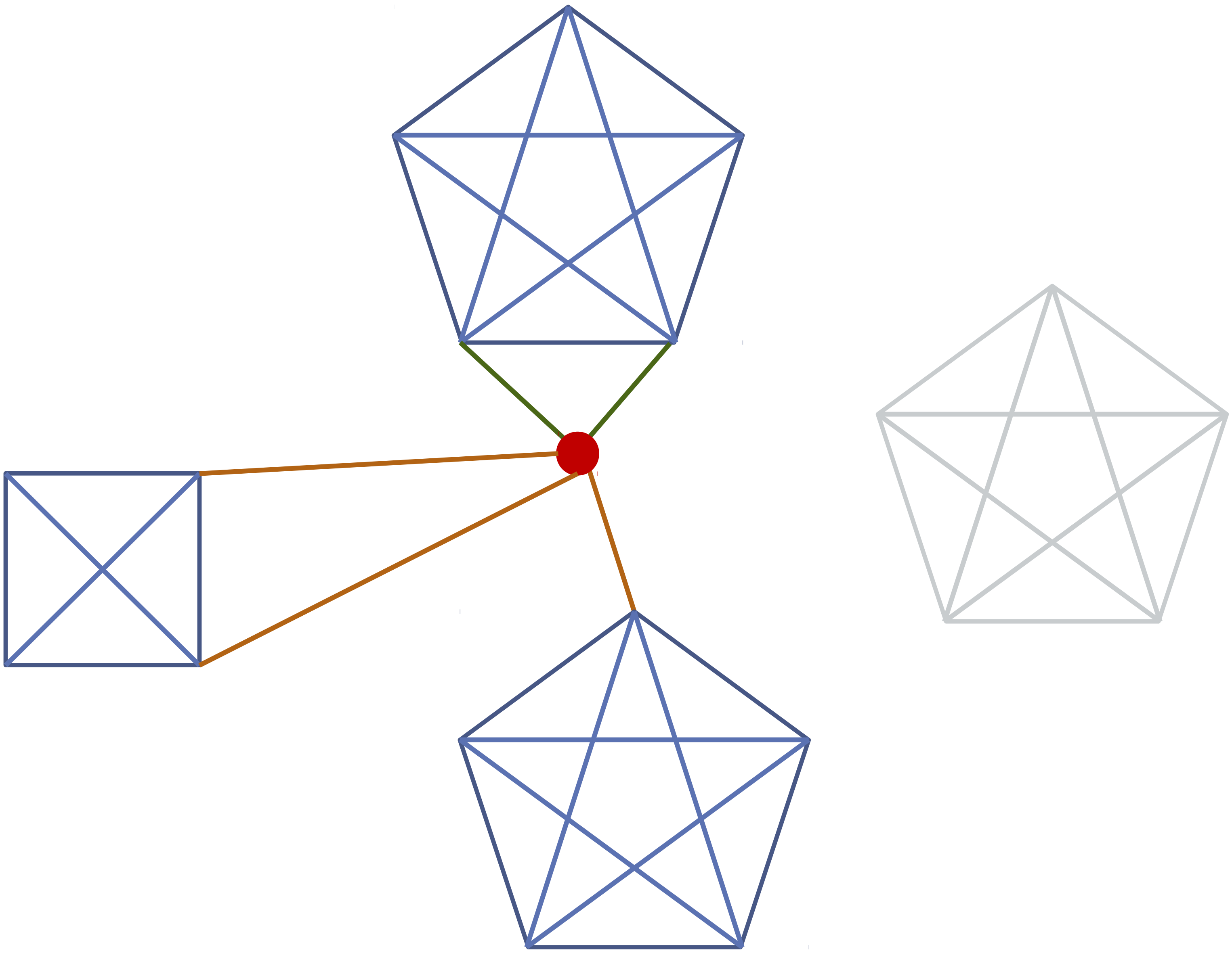
Dynamic Neighbourhoods



Neighbourhood Sturcture in Games



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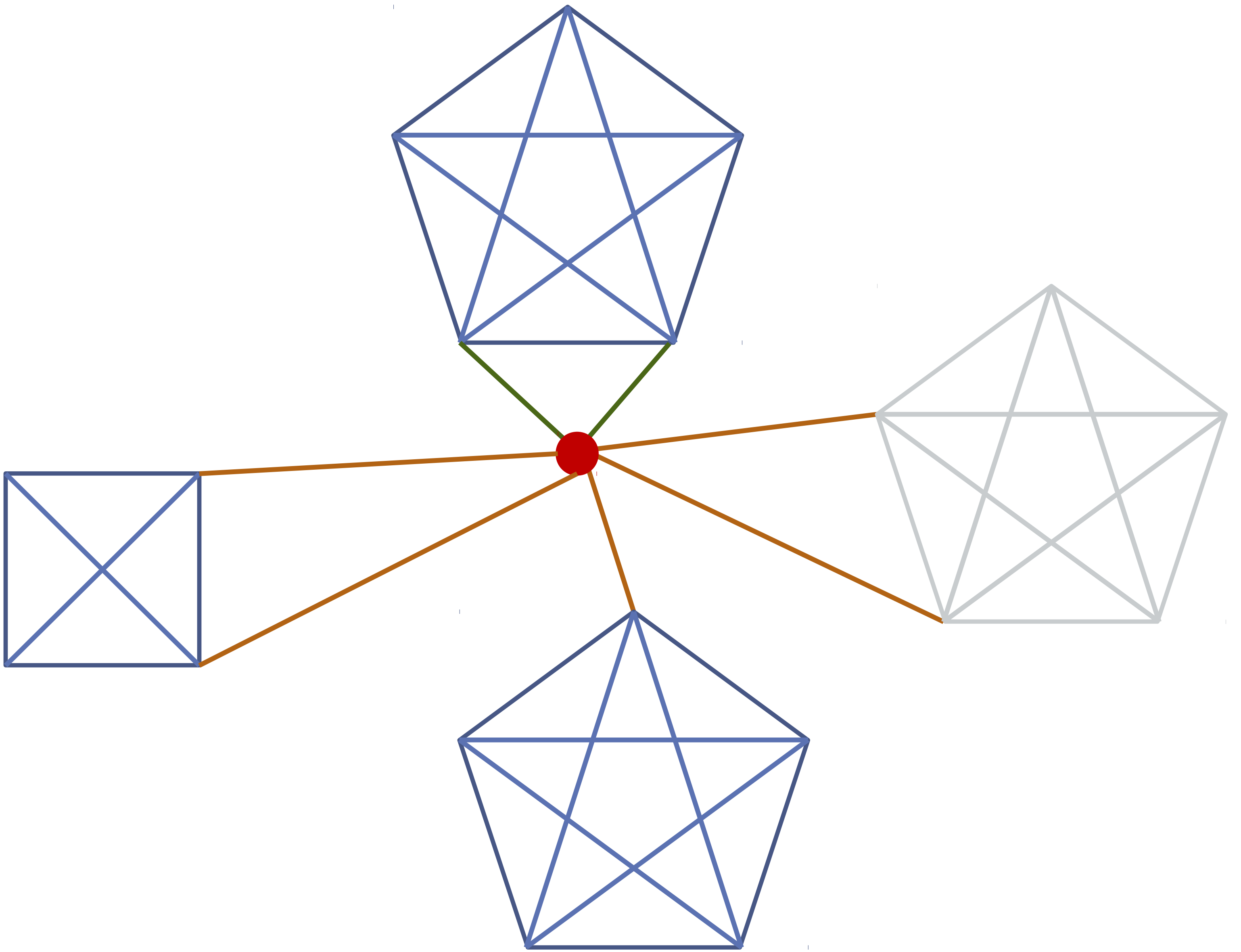


Neighbourhood Sturcture in Games

Description of type t

If payoff > 0.5 then

Stay in the same neighbourhood X

ElseIf there is a player j in a different visible neighbourhood X' who received the maximum (visible) payoff in round k and this payoff is greater than my payoff then

Join X' in round k+1

Else

Stay in X

Theorem:

Let a game have n players where the dynamic neighbourhood structure is given by a graph G. If all the players are of the same type t, then the game stabilises in at most nn(n+1)/2 steps.

Proof Idea: Same as before

Associate a potential with every configuration of the graph

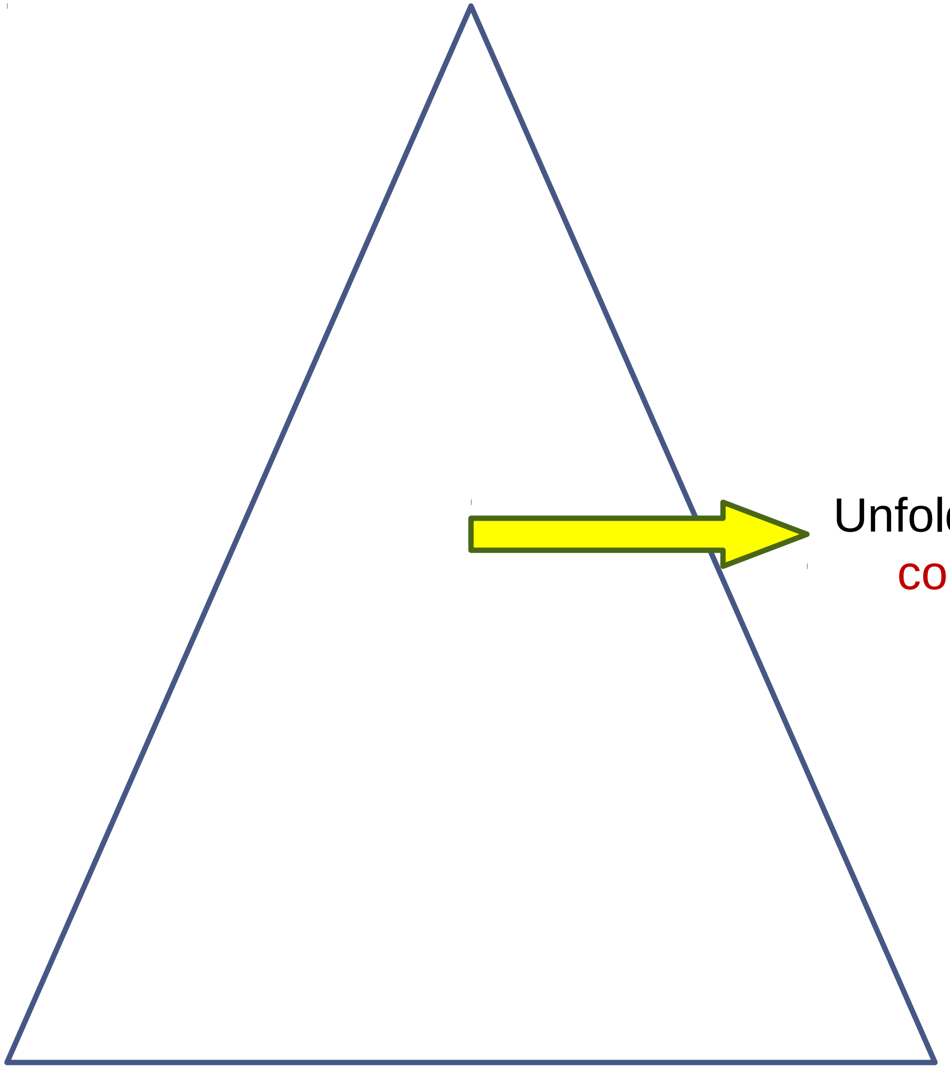
Show that whenever the configuration

General Neighbourhood Games

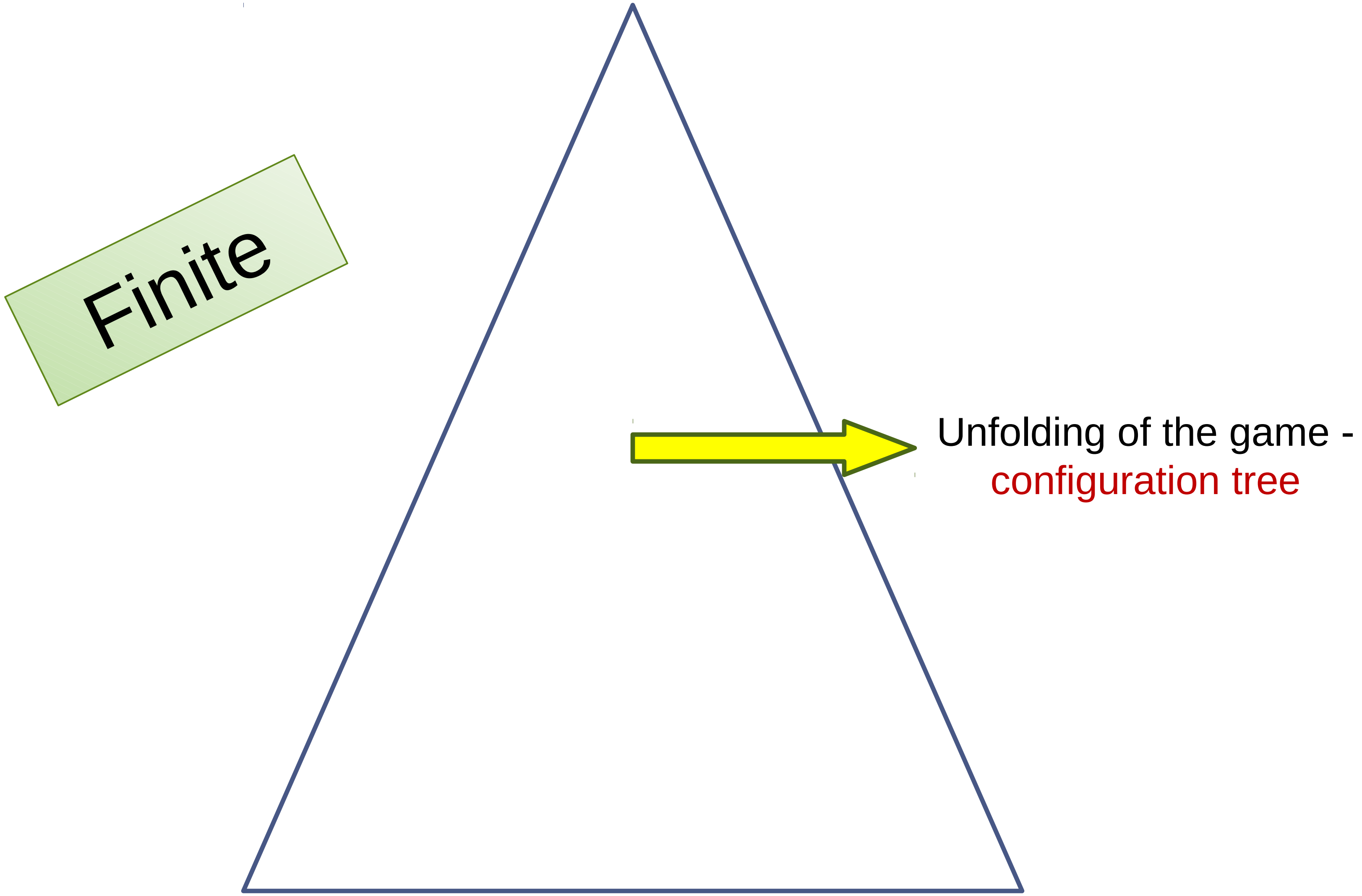
Theorem:

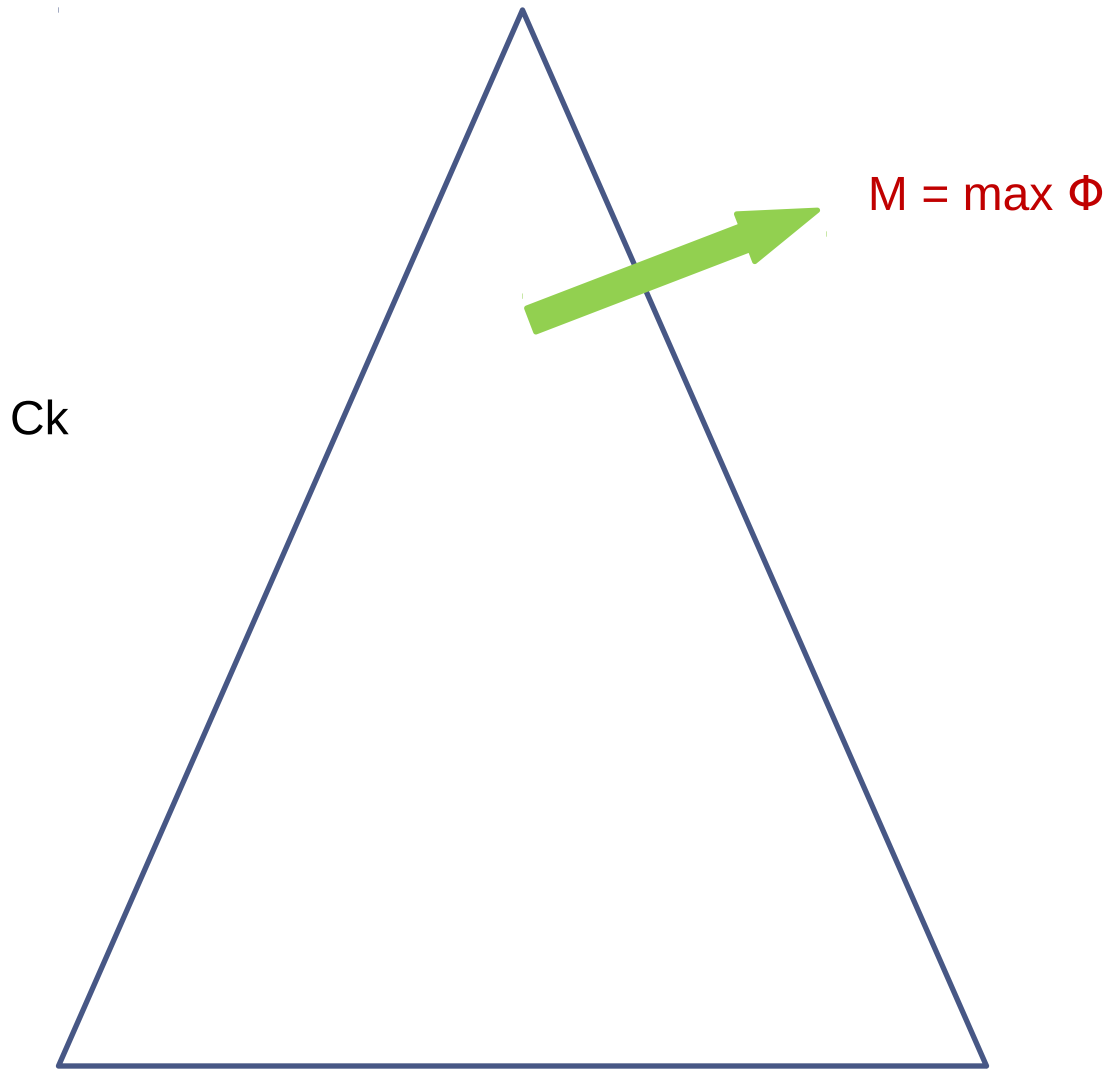
A general game with n players and with either a static or a dynamic neighbourhood structure eventually stabilises if and only if we can associate a potential Φk with every round k such that if the game moves to a different configuration from round k to round k + 1 then $\Phi k+1 > \Phi k$ and the maximum possible potential of the game is bounded.

Proof

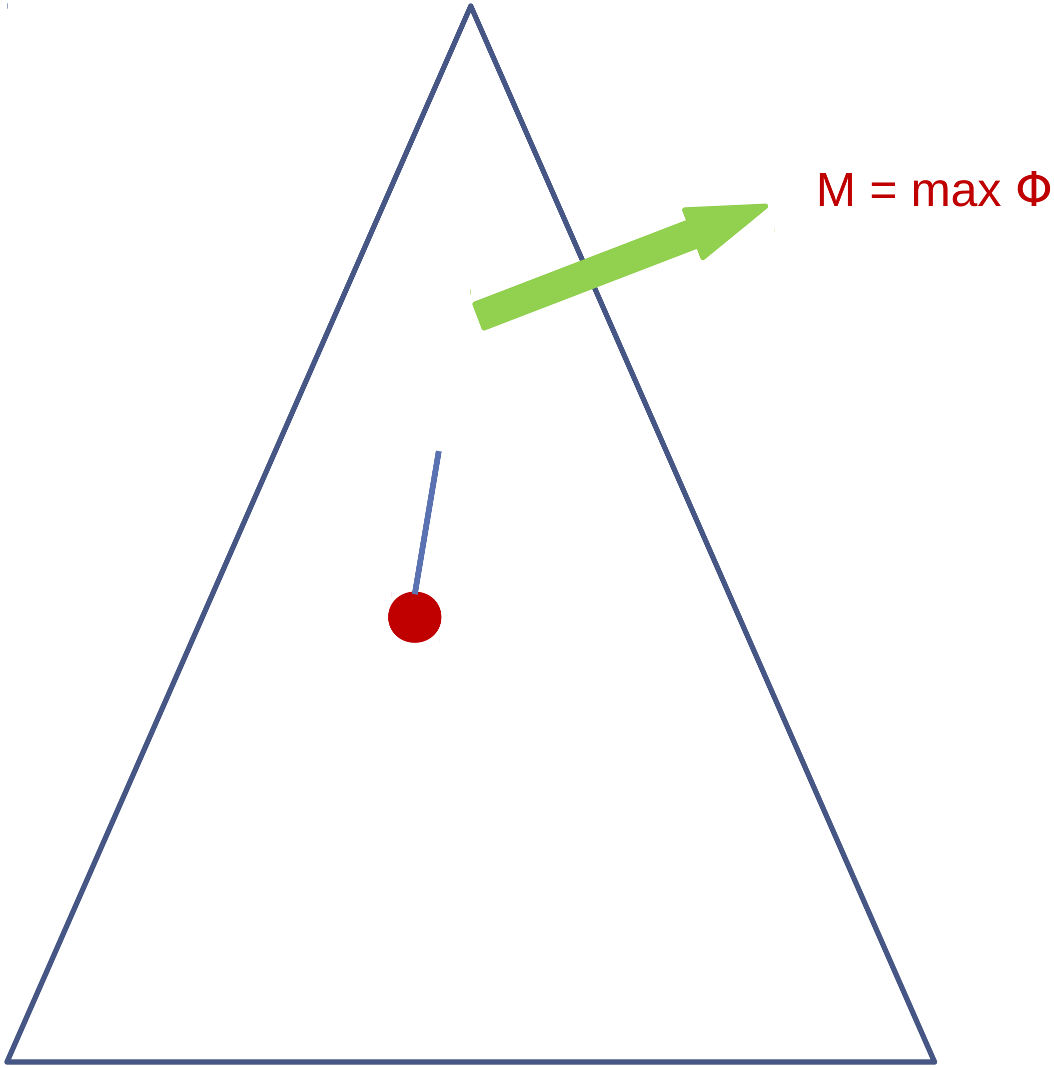


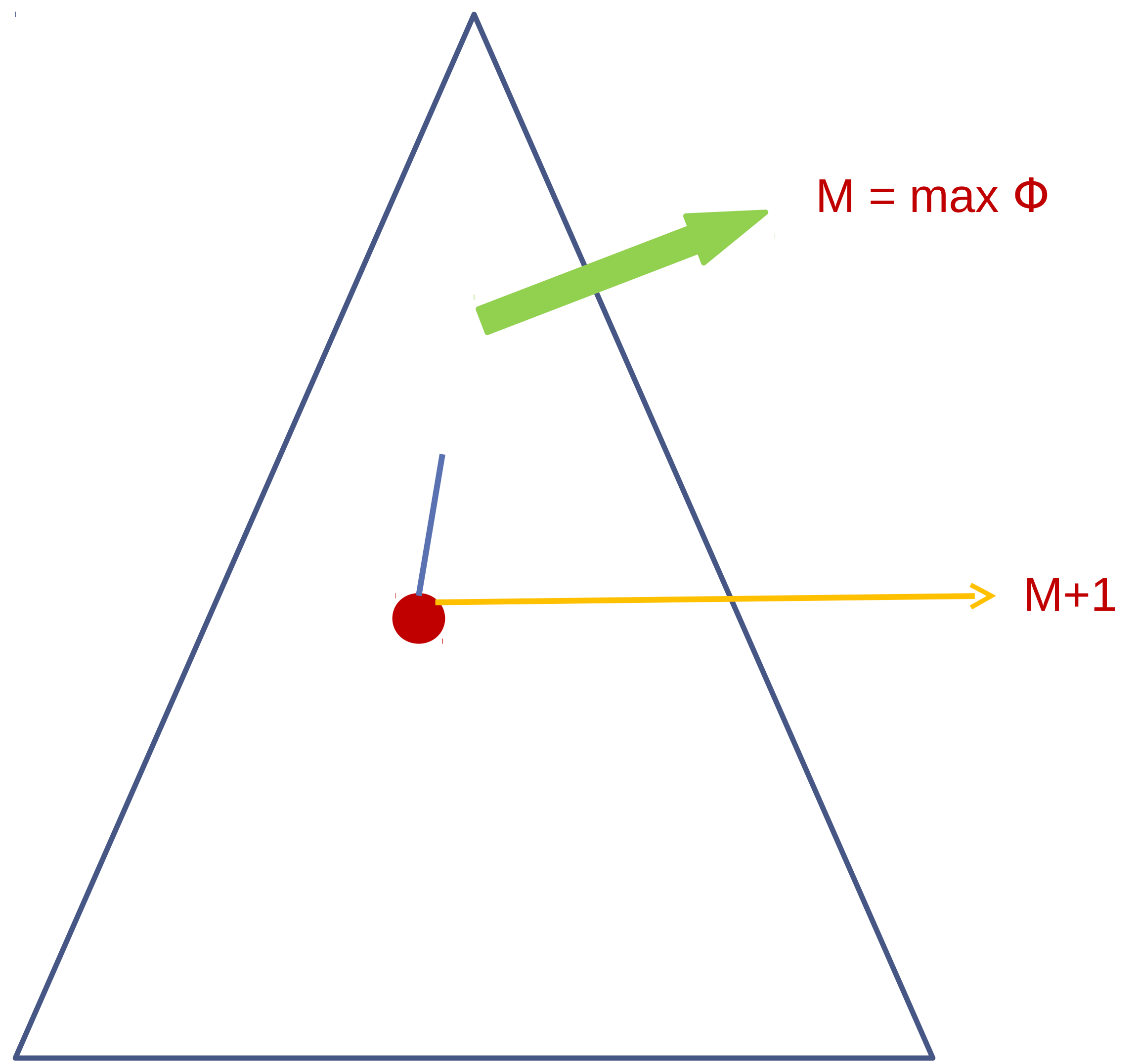
Unfolding of the game -
configuration tree

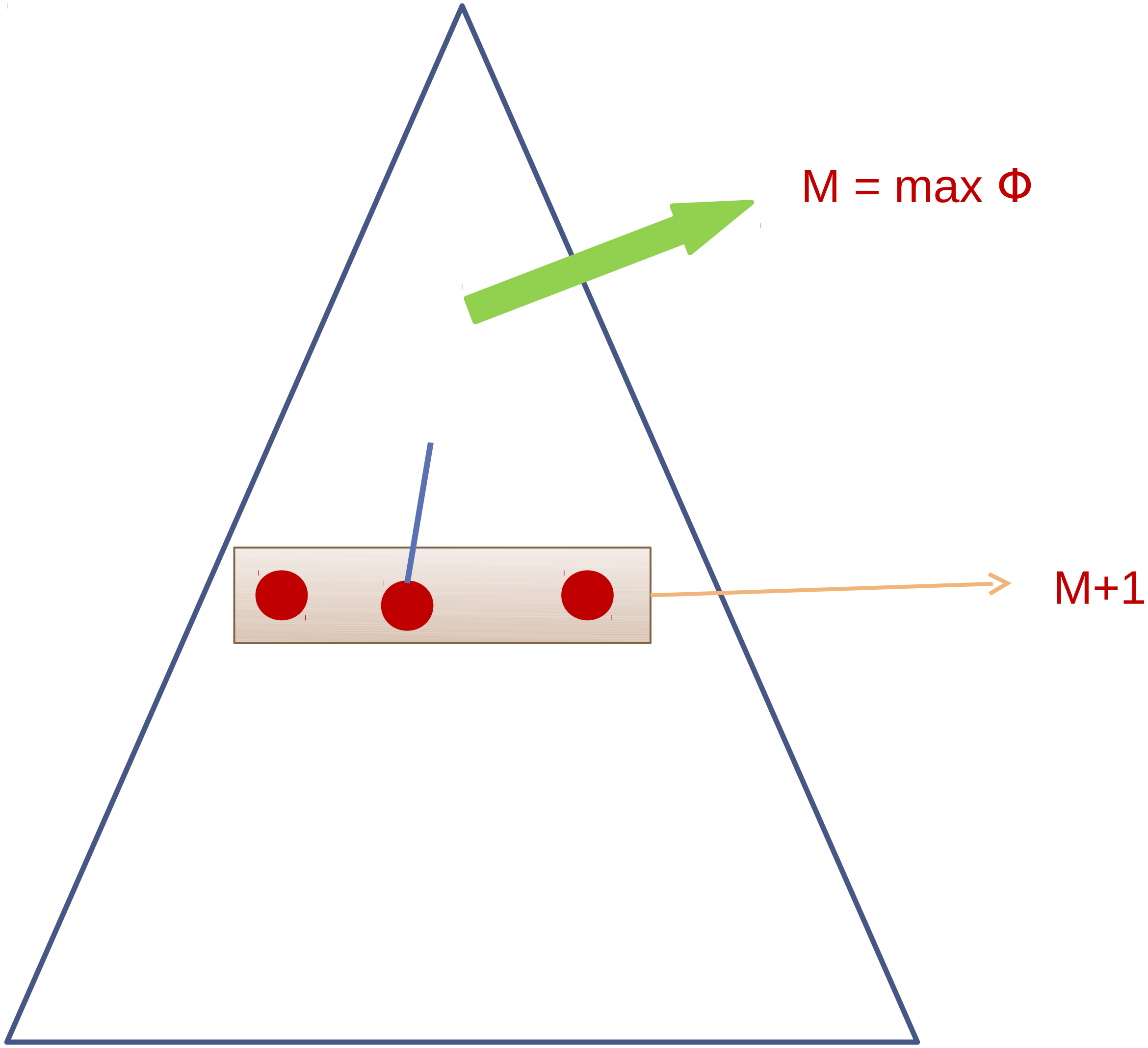

Finite
Unfolding of the game -
configuration tree



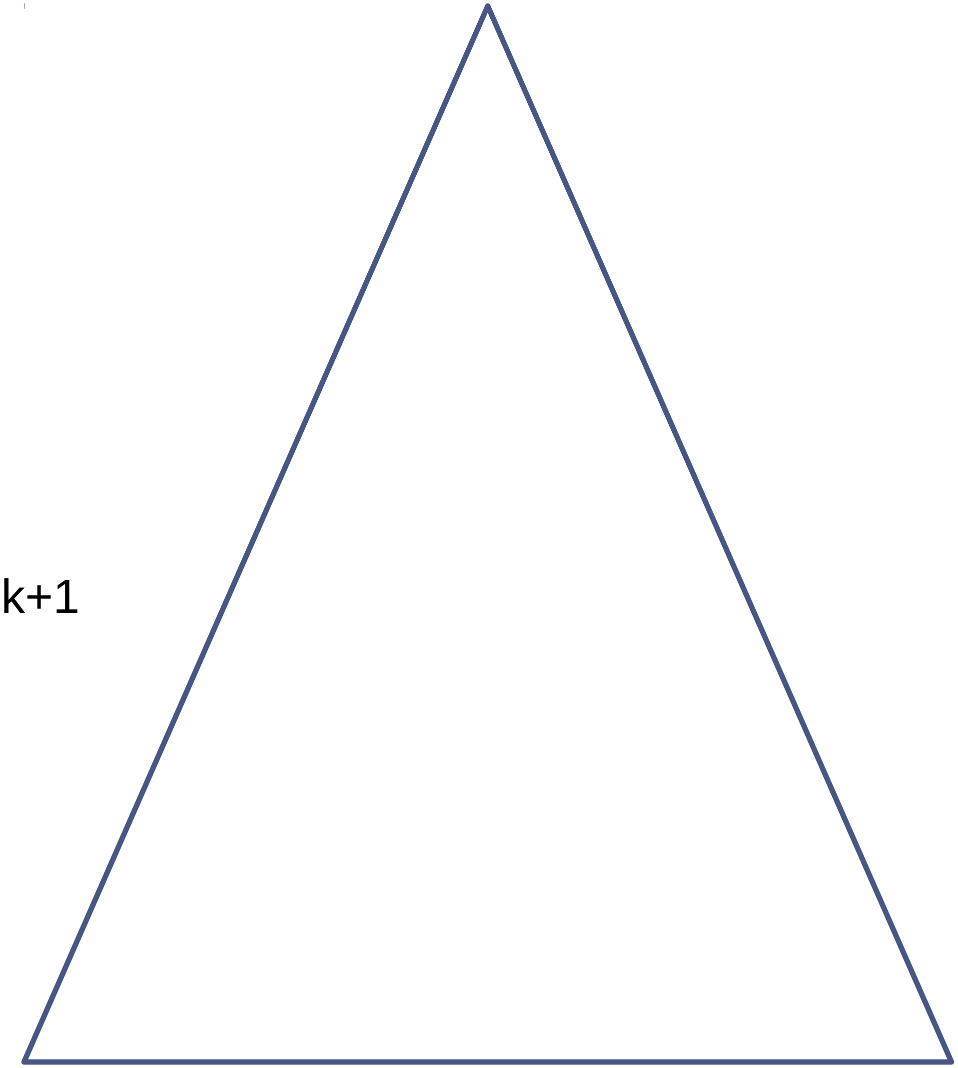
M = max Φ
Ck

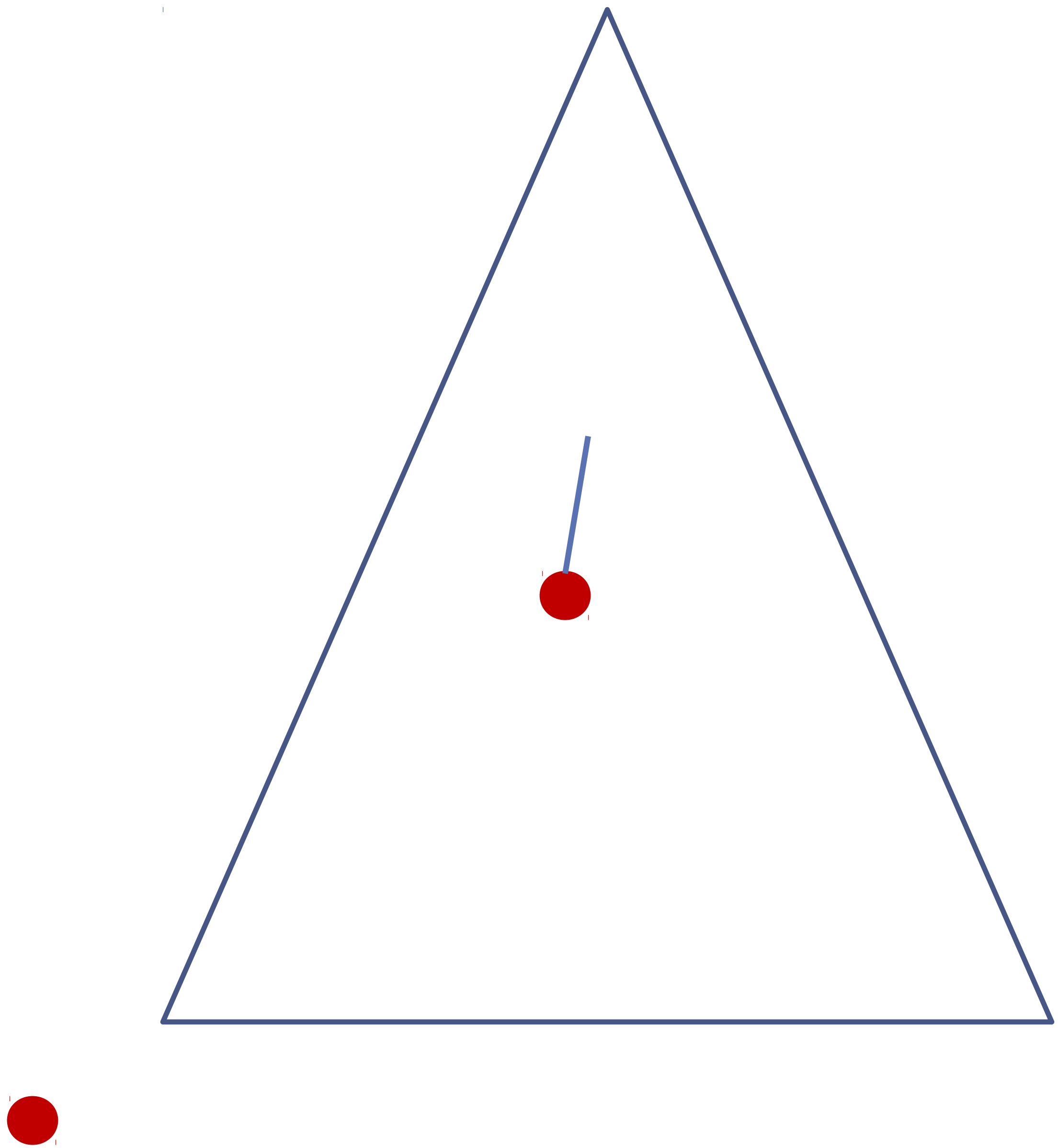

M = max Φ

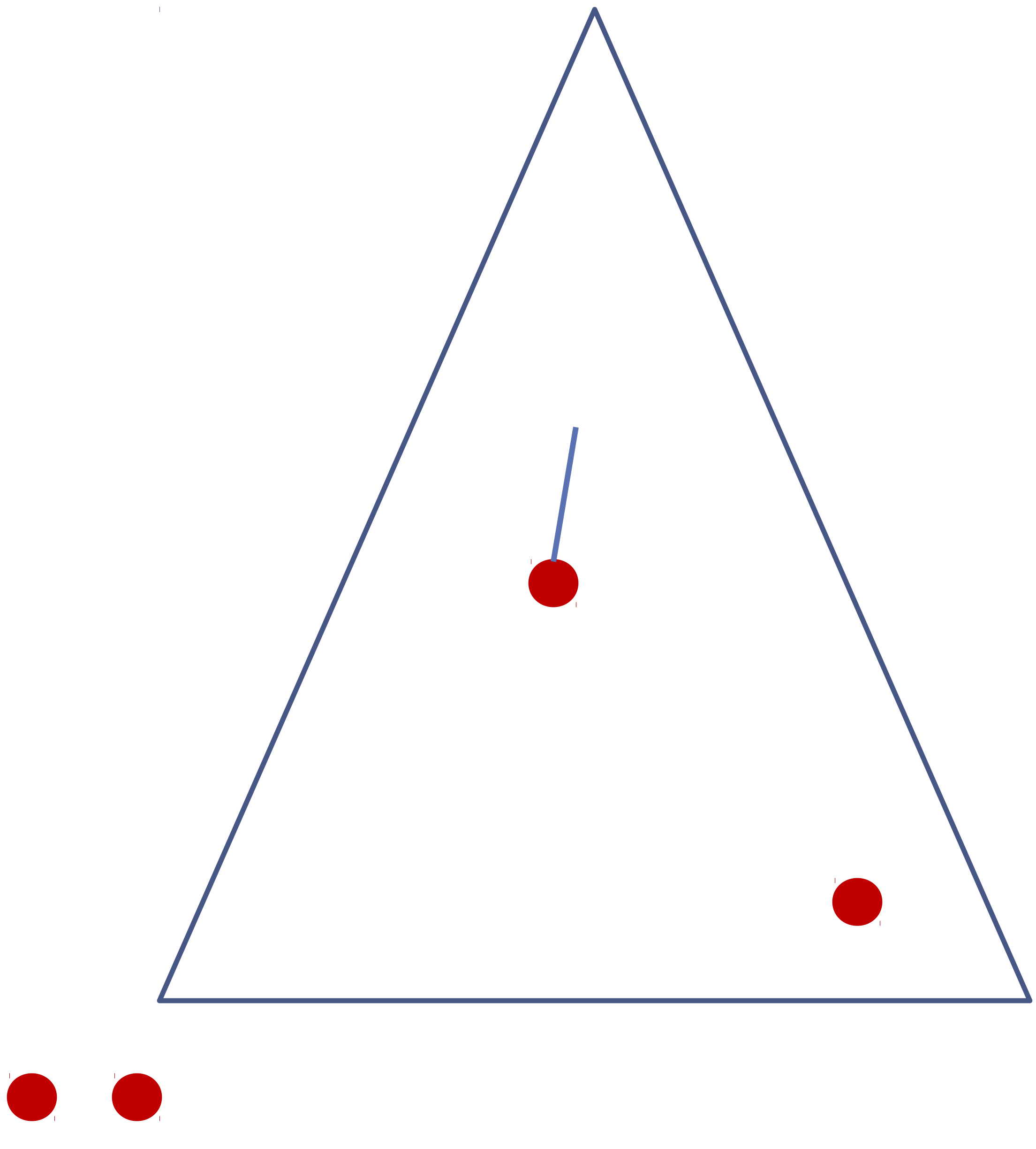

M = max Φ
M+1

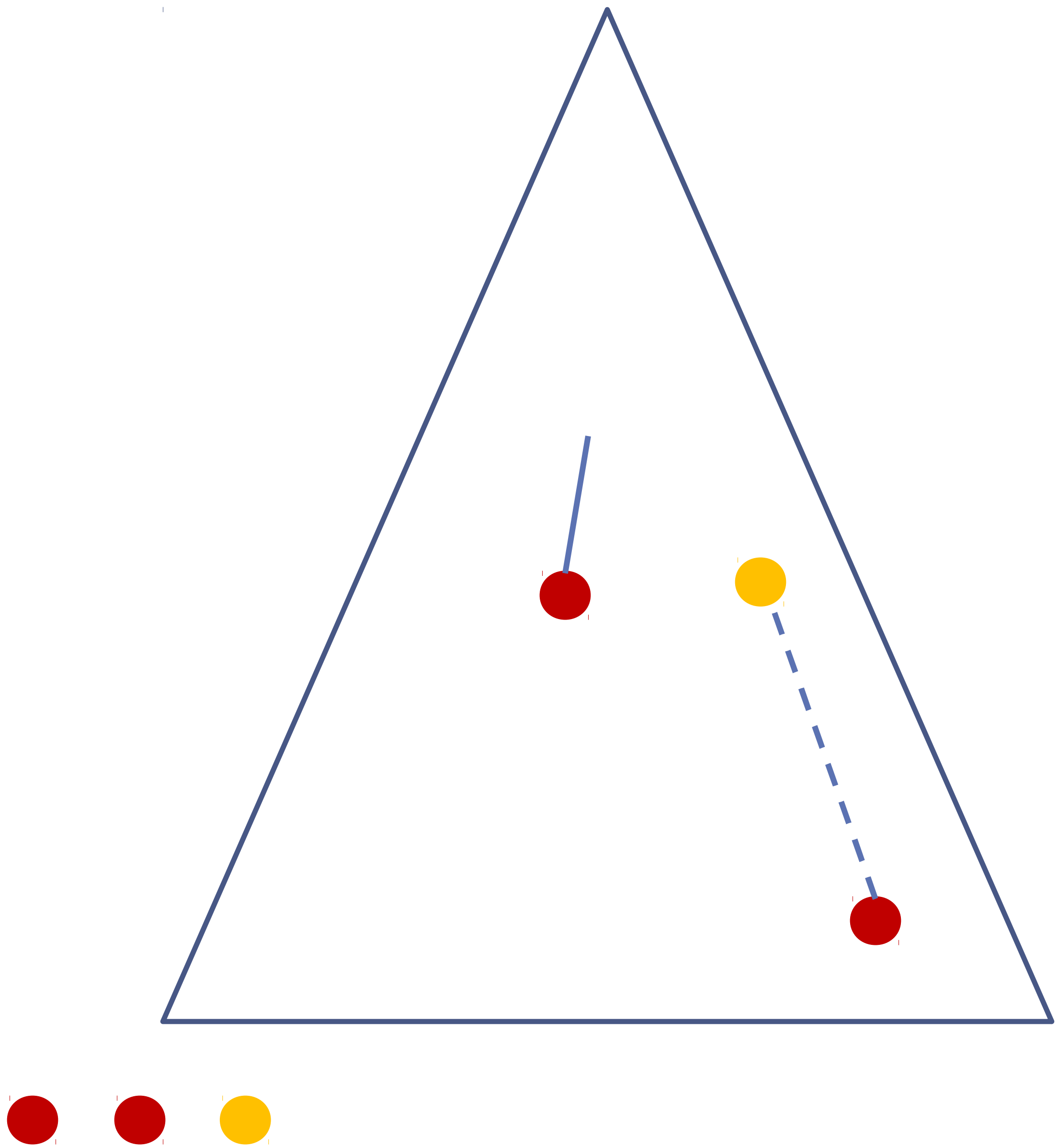

M = max Φ
M+1

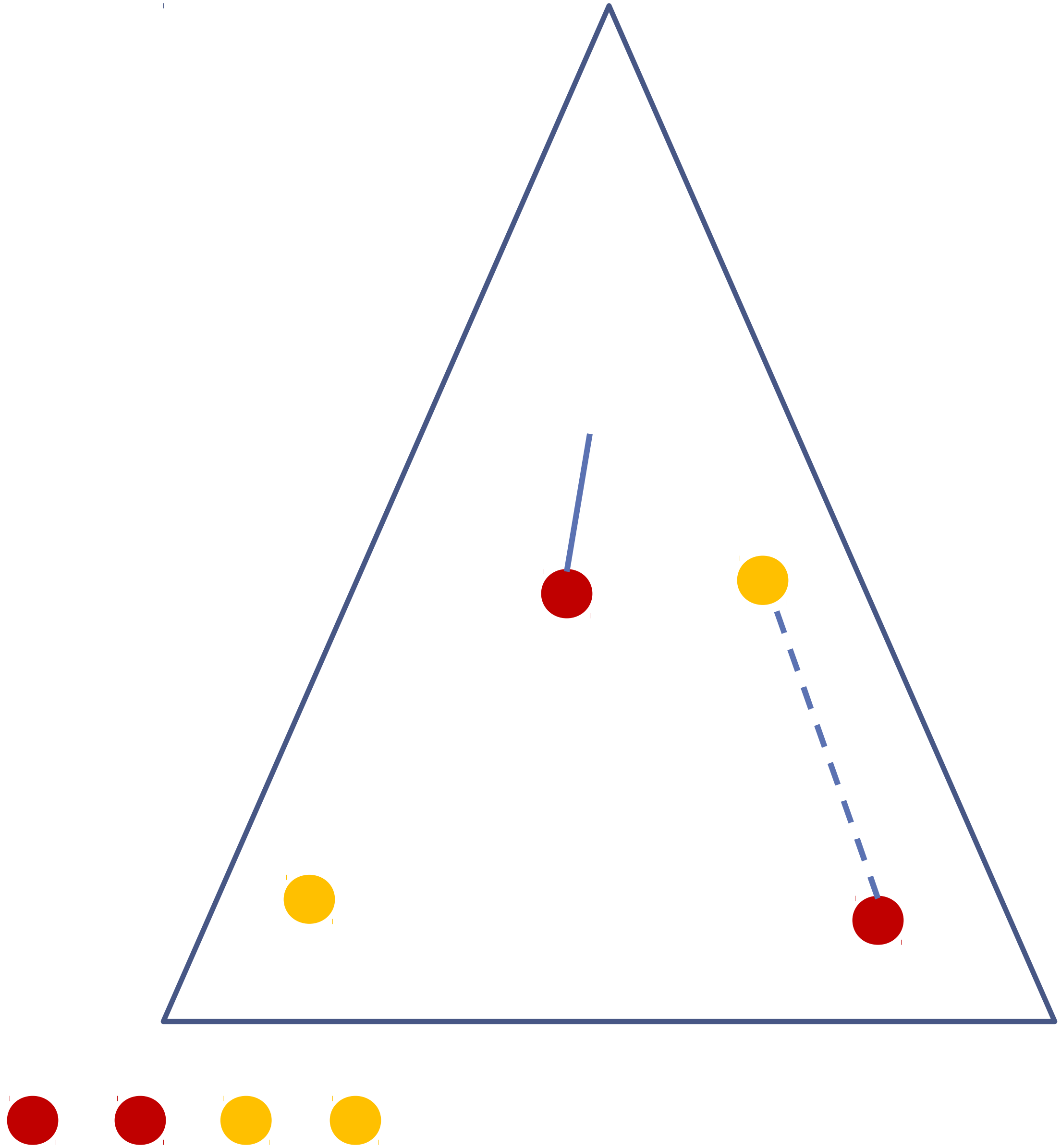
Ck+1

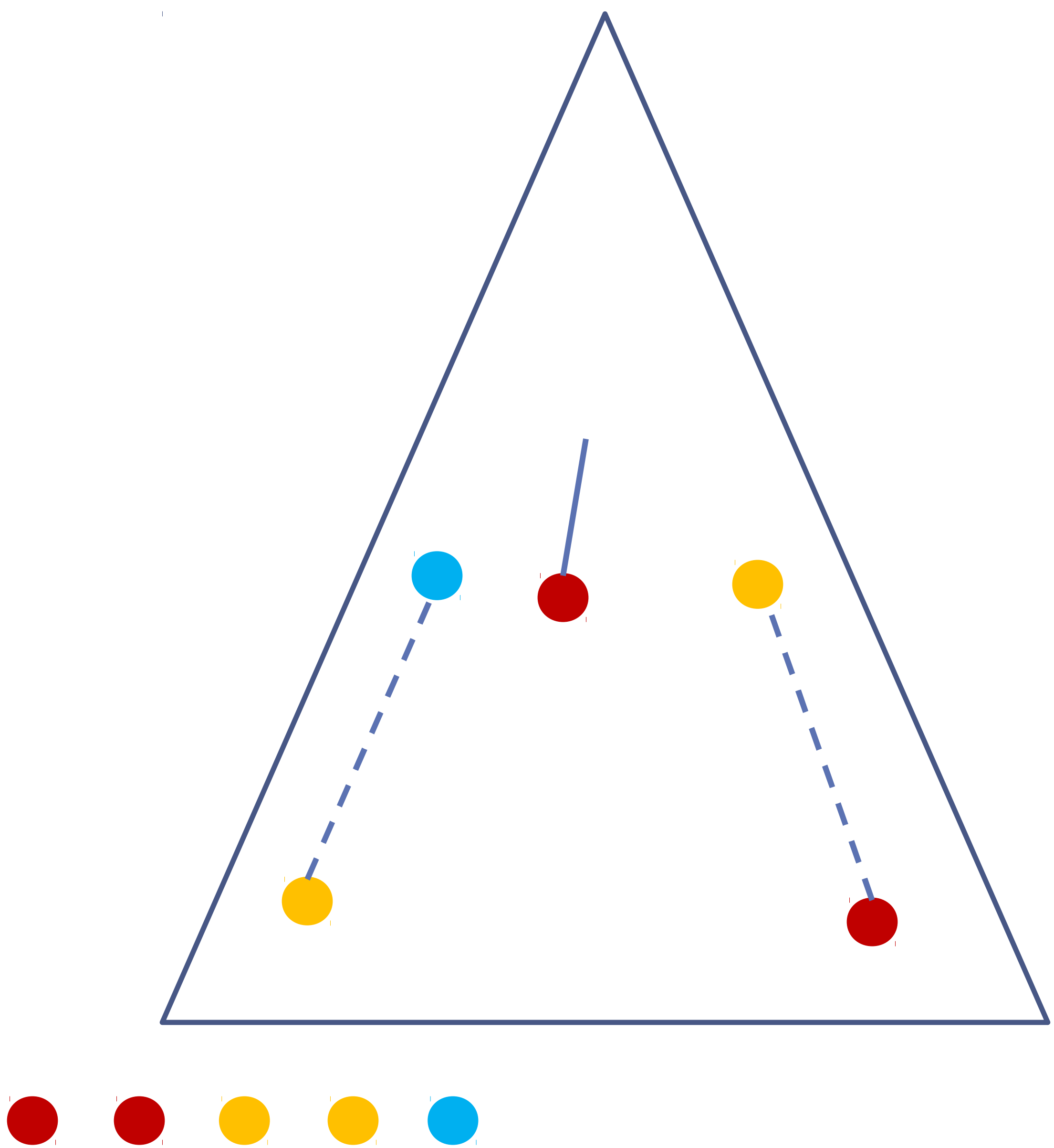


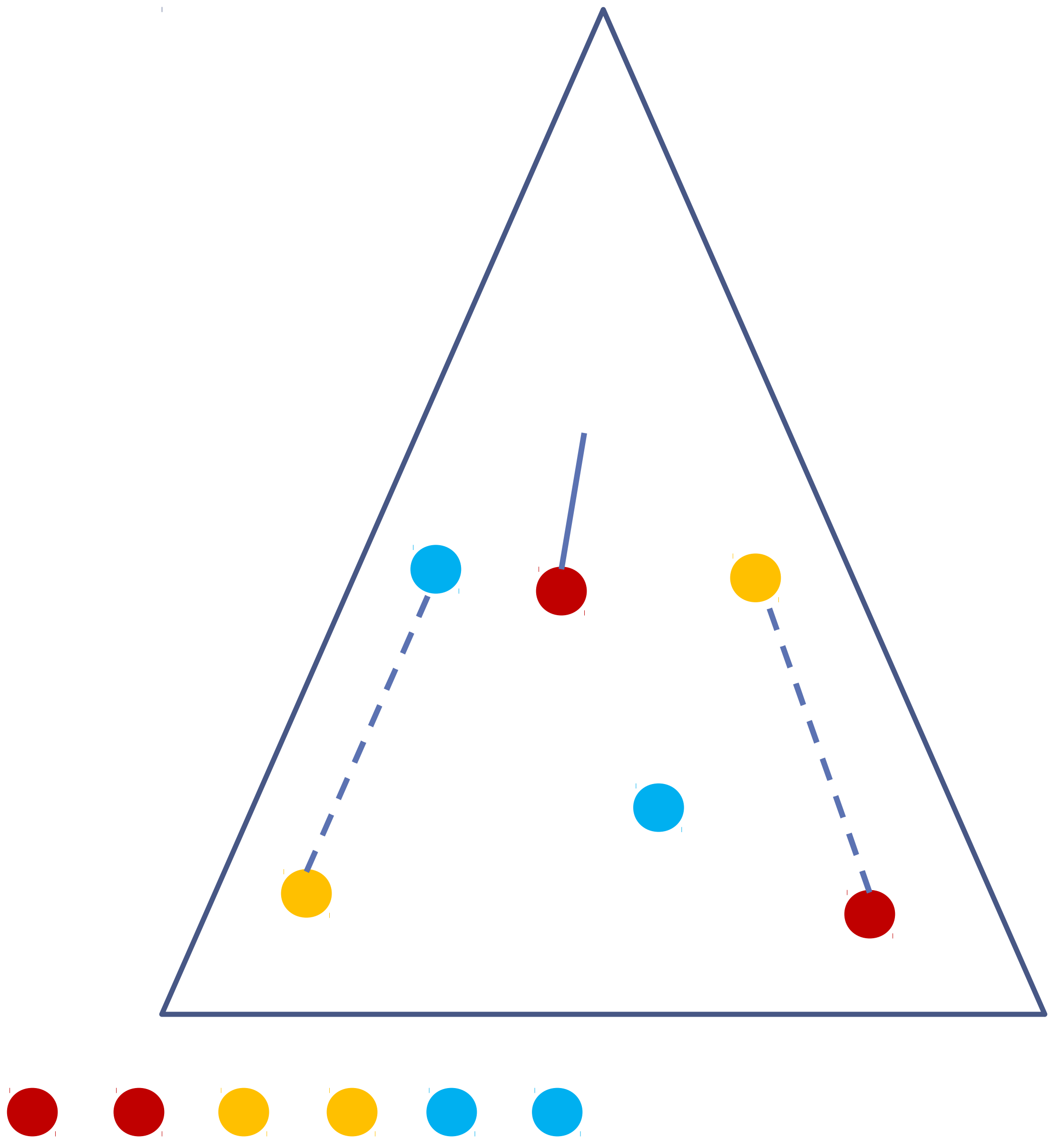


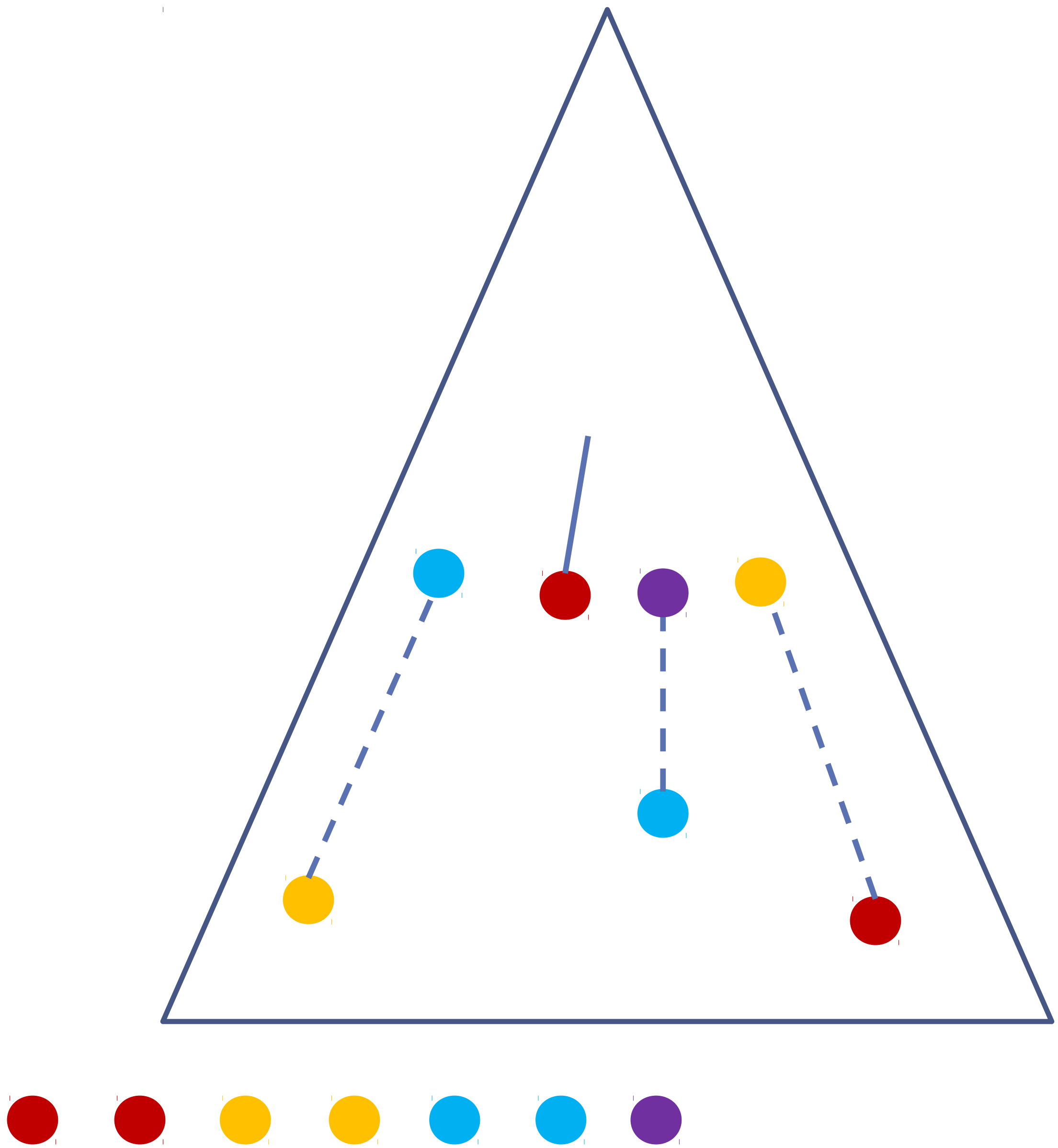


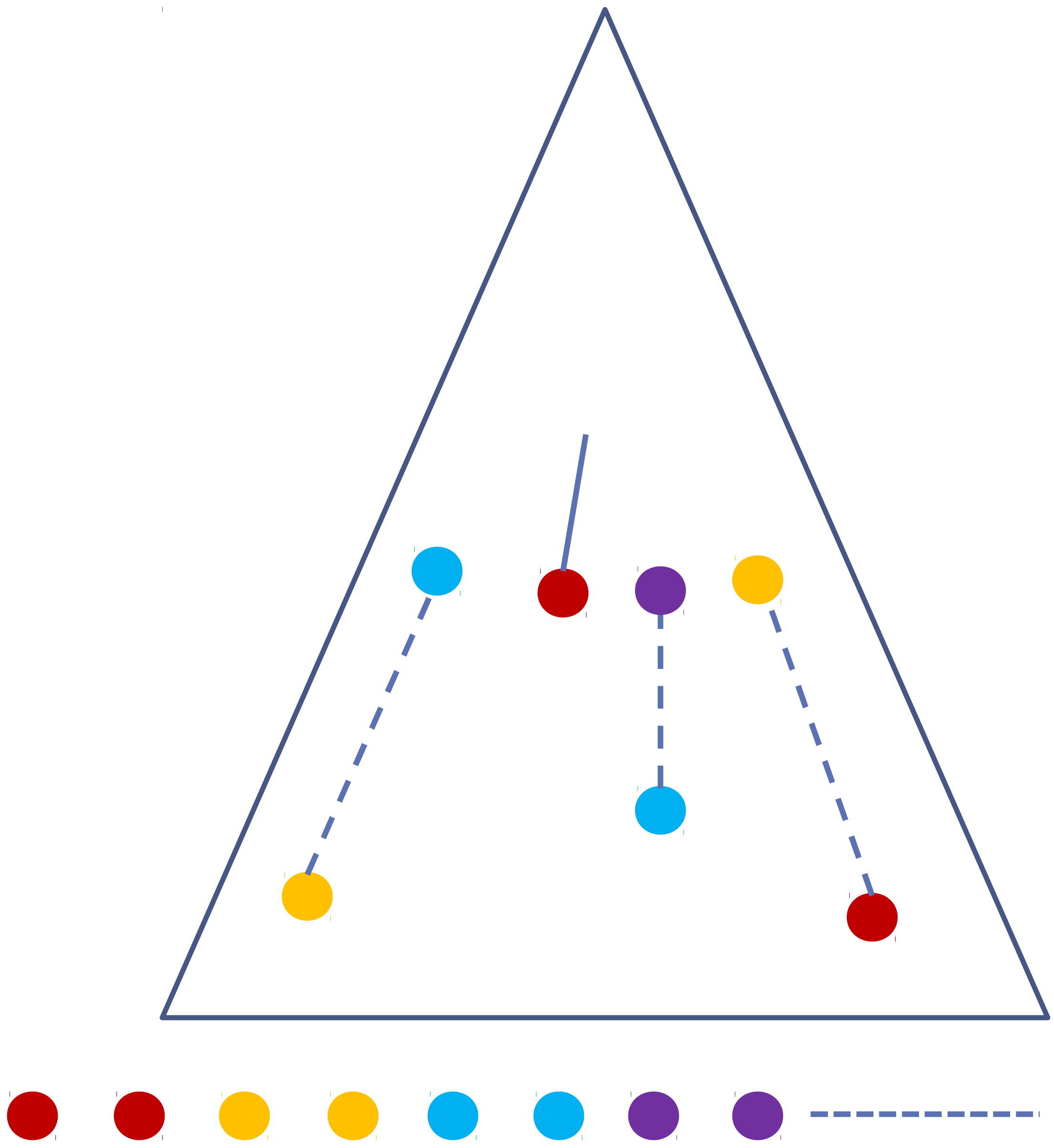


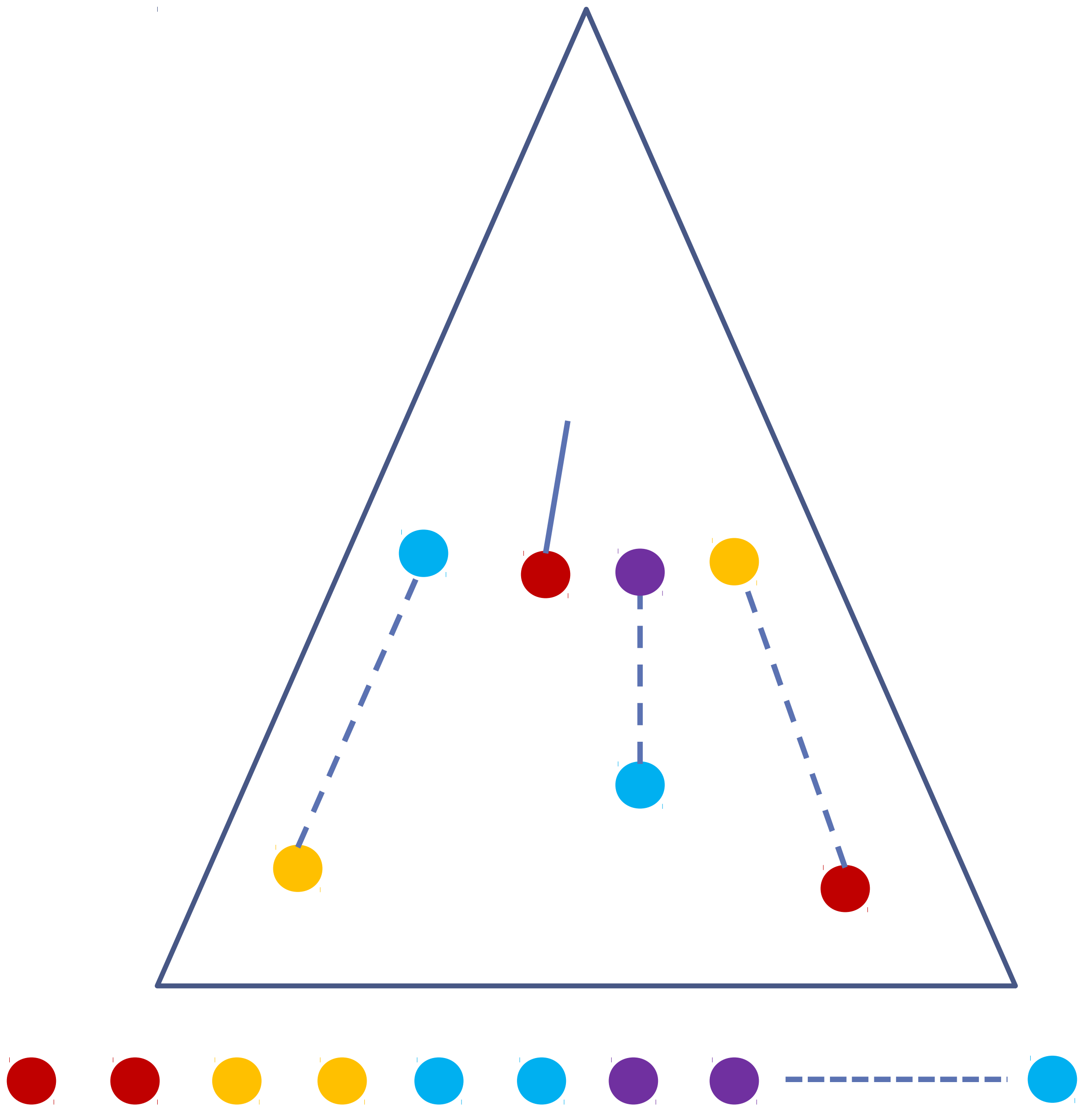


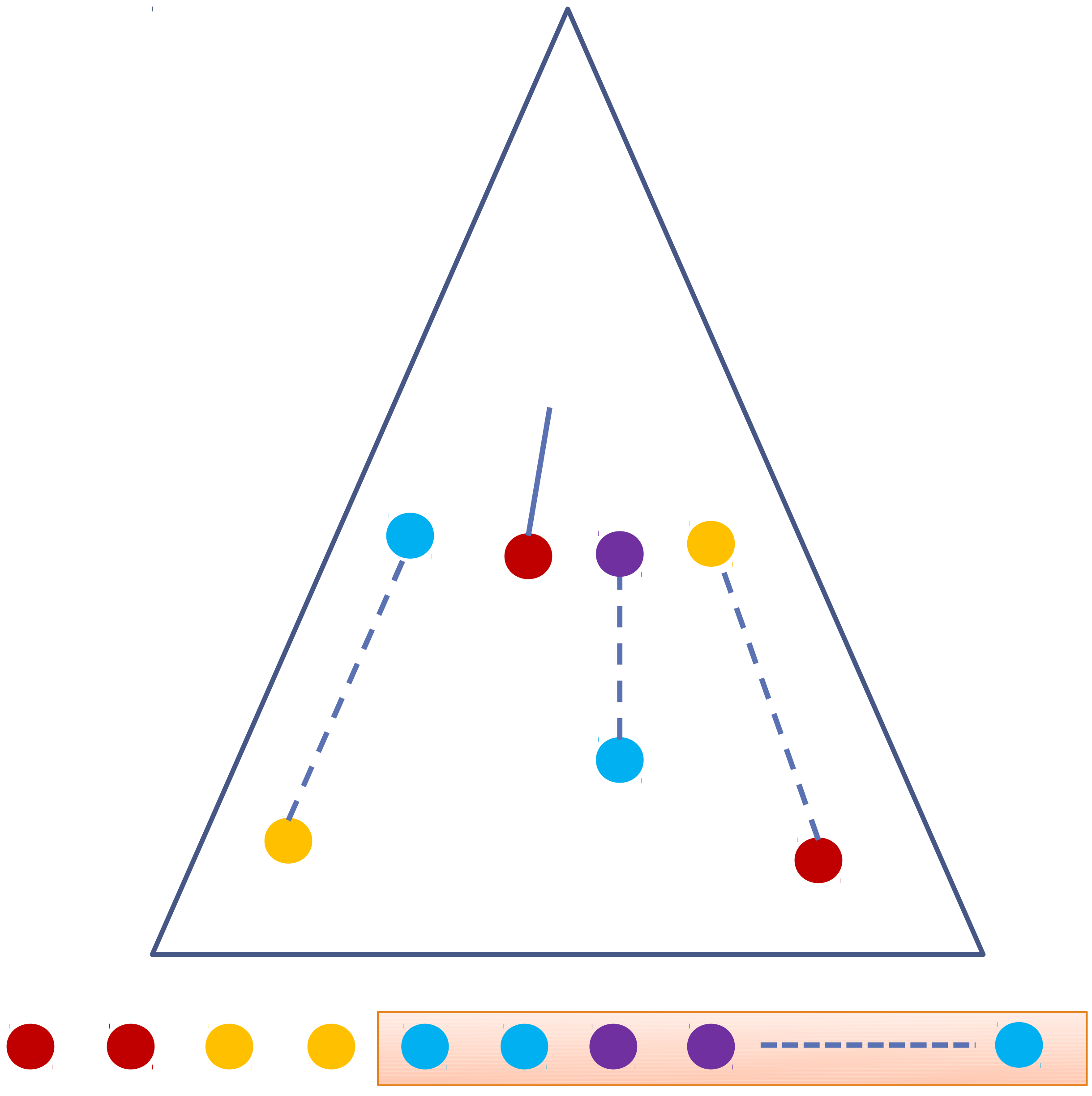


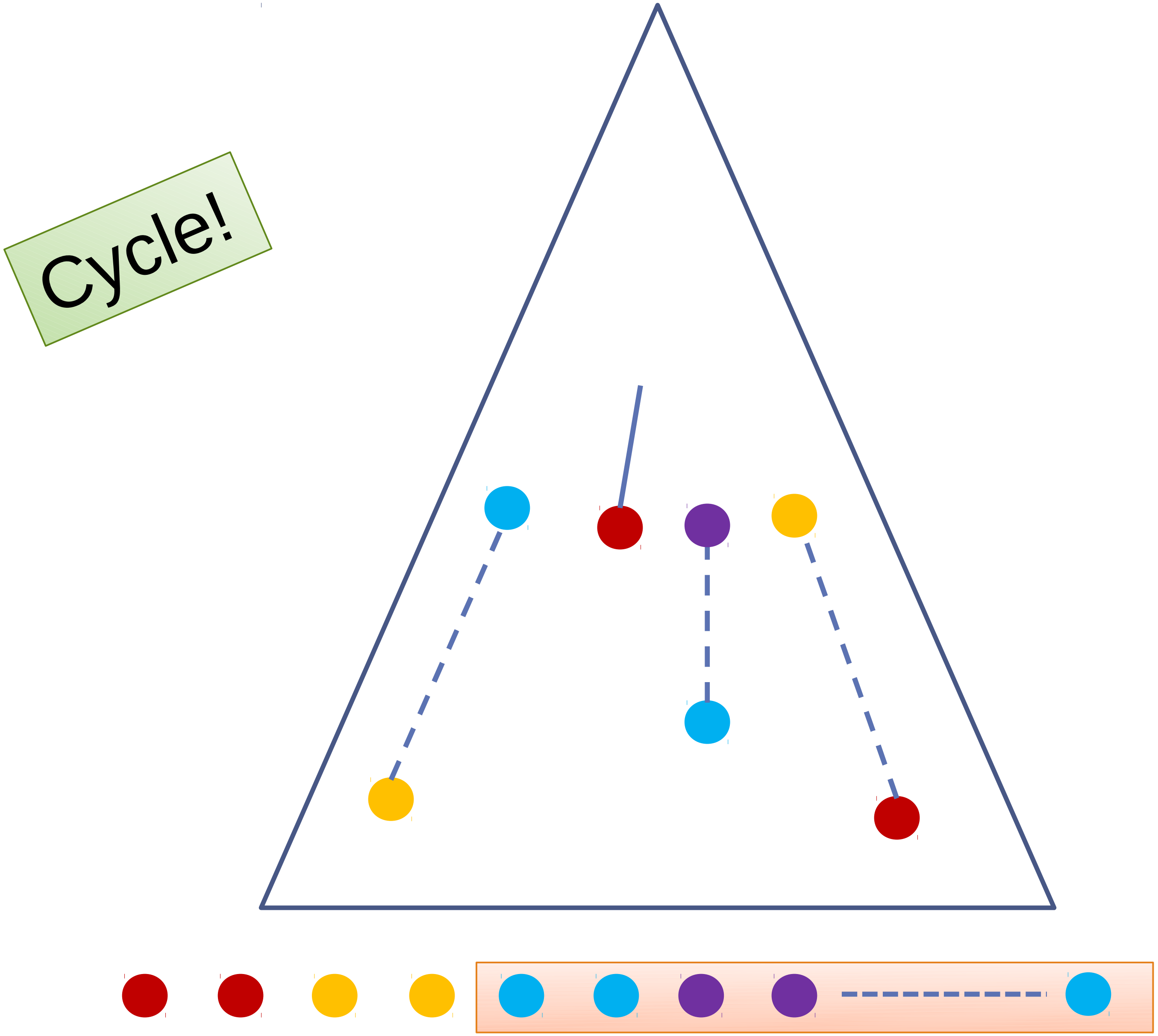




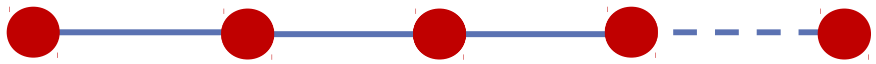


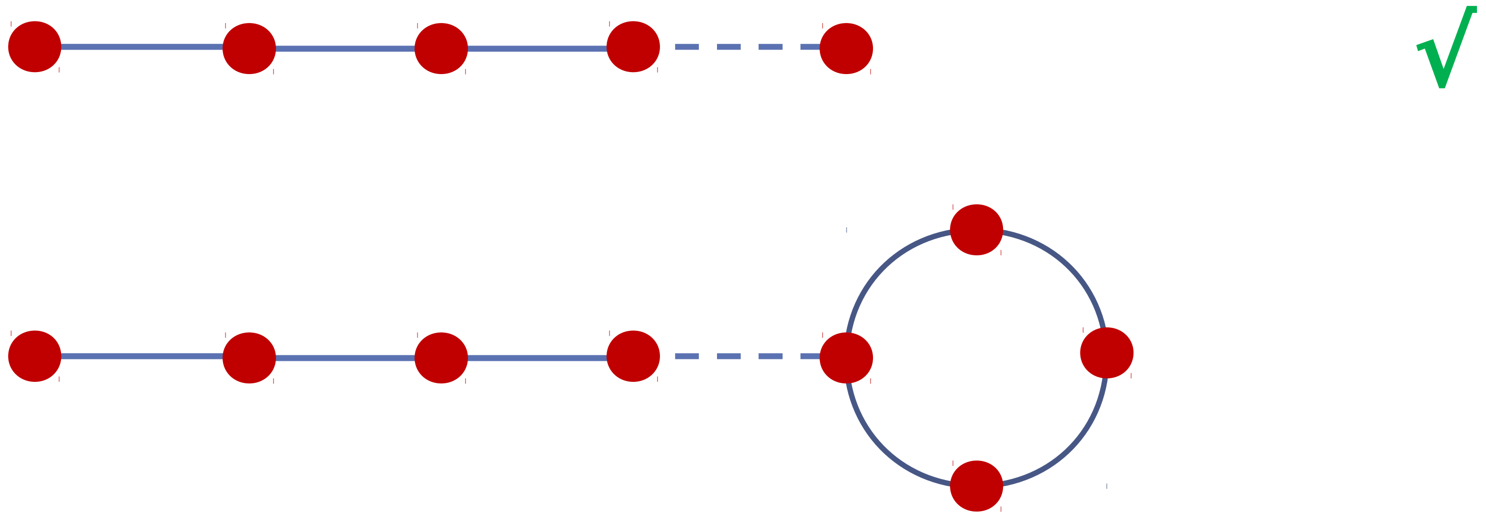


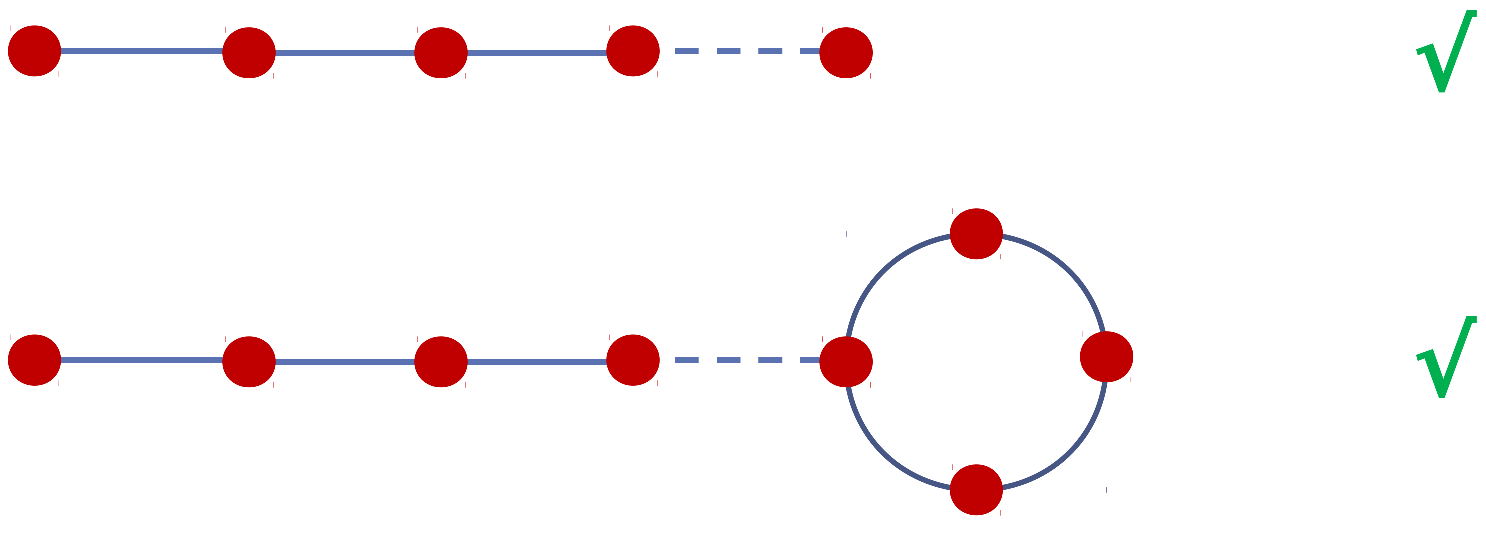


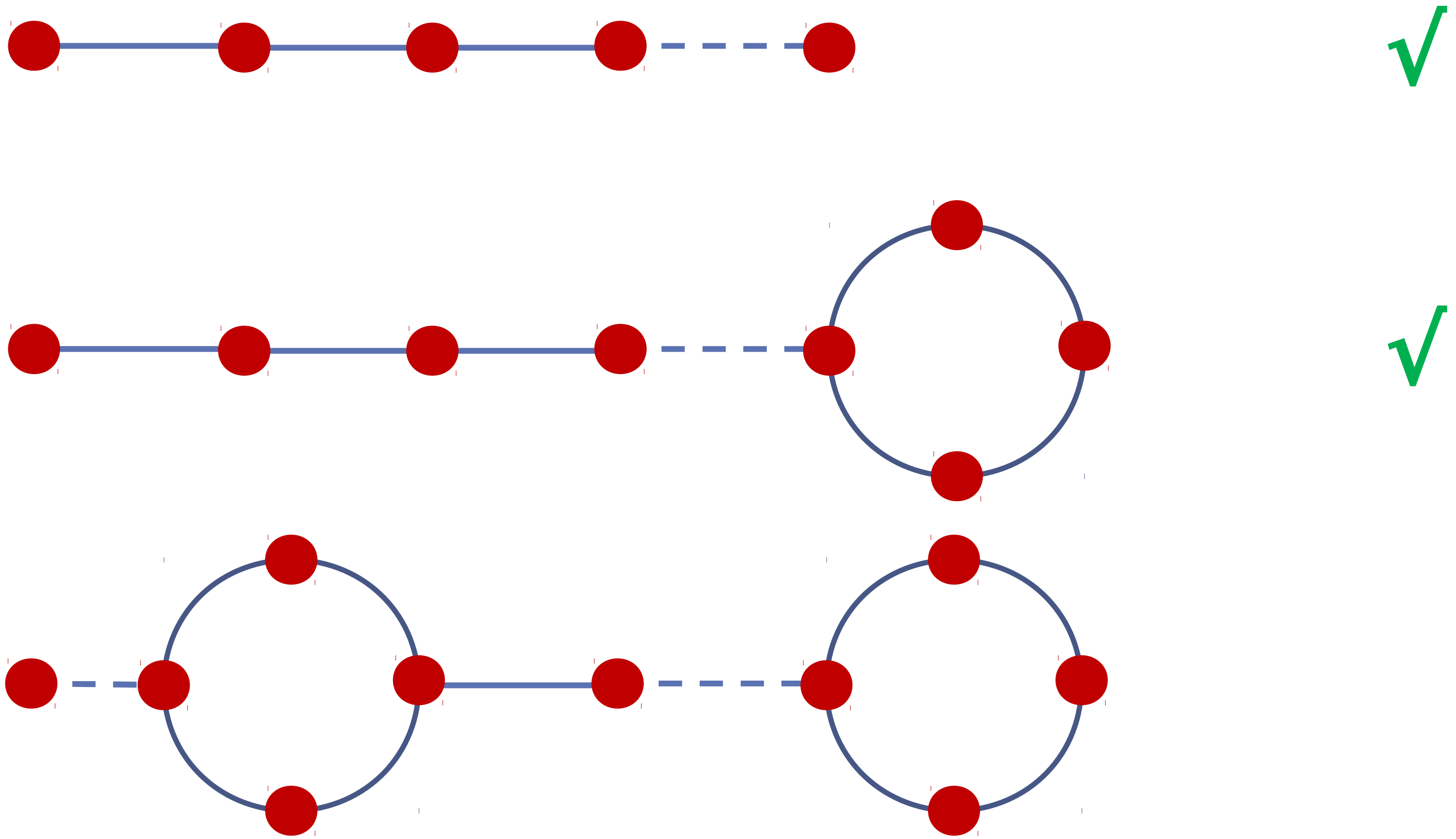

Cycle!

Generalising Stability









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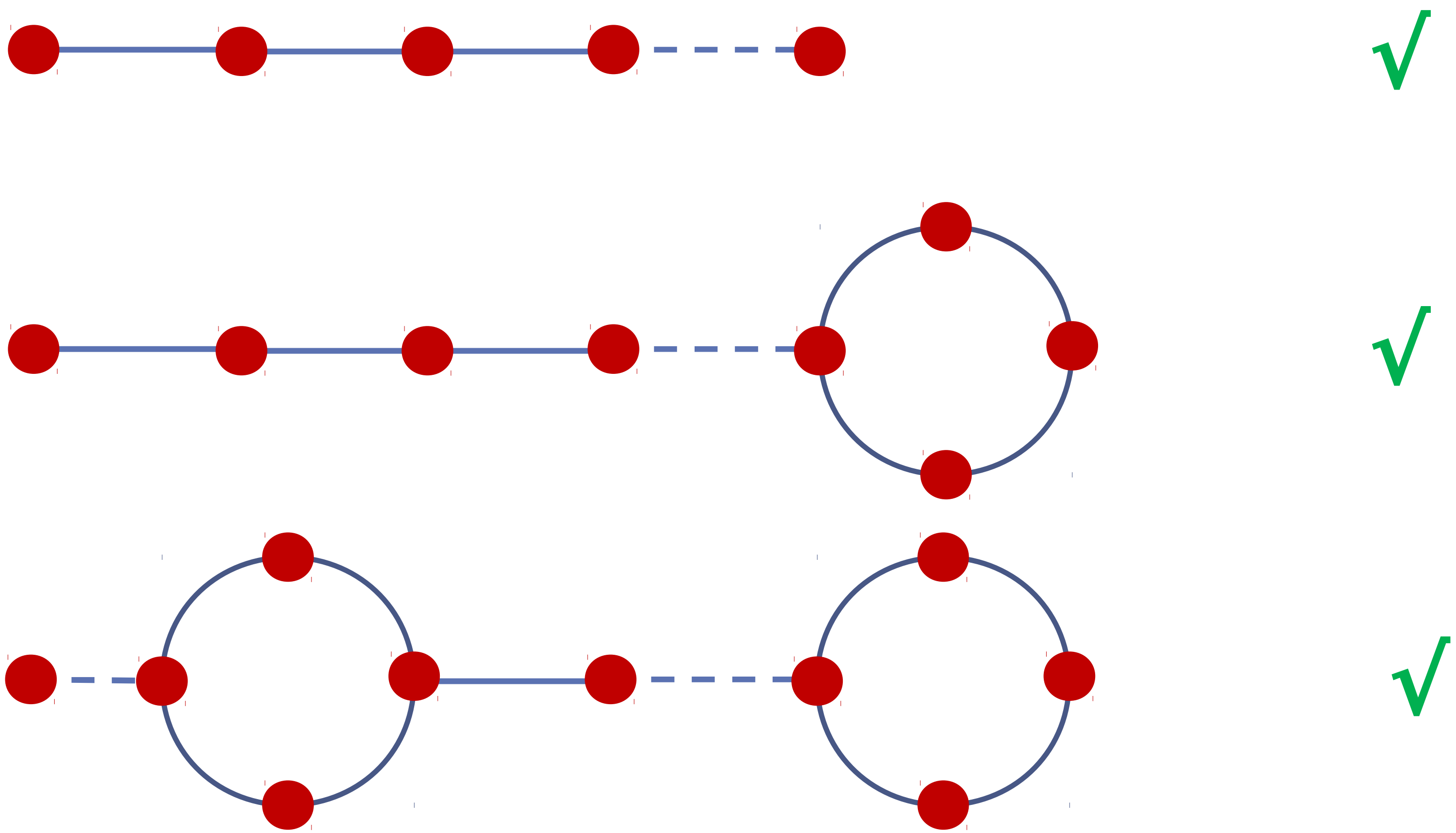

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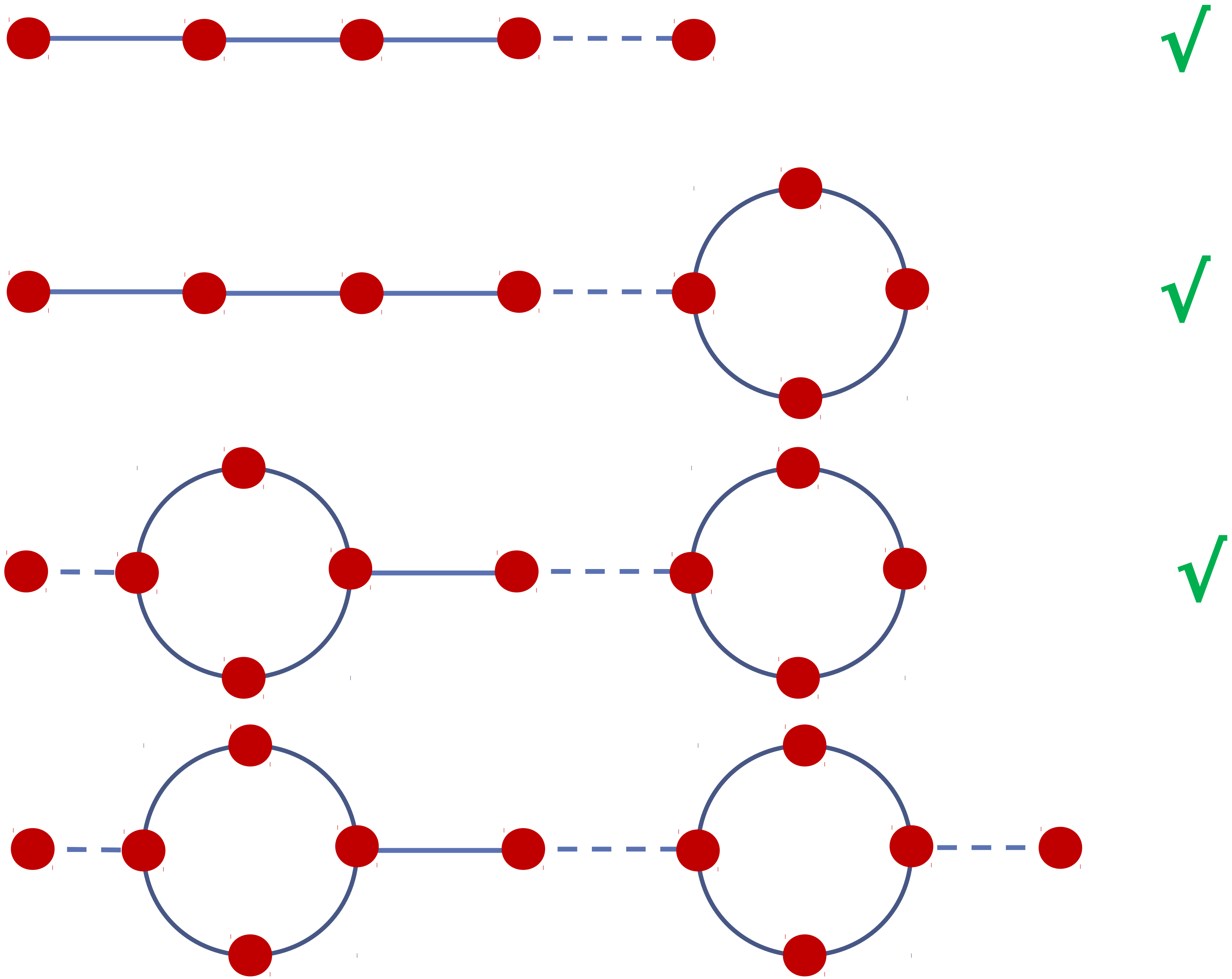

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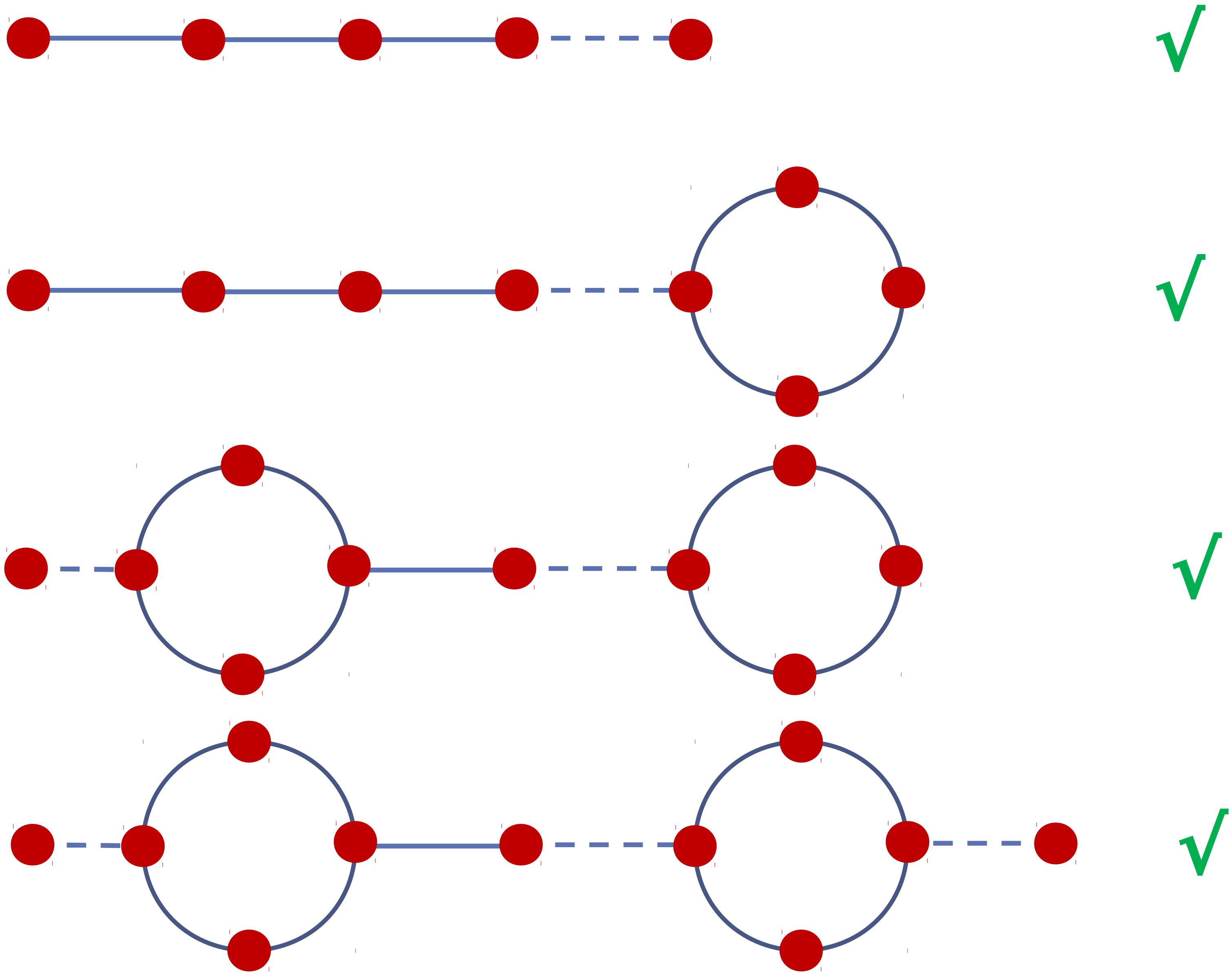

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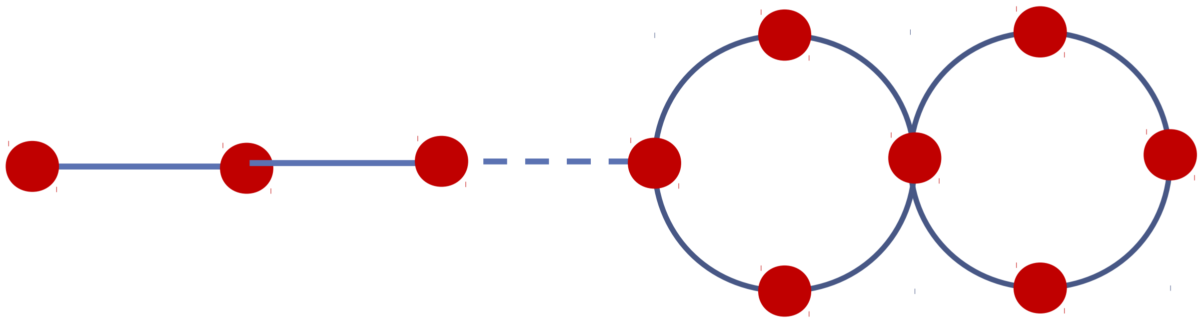

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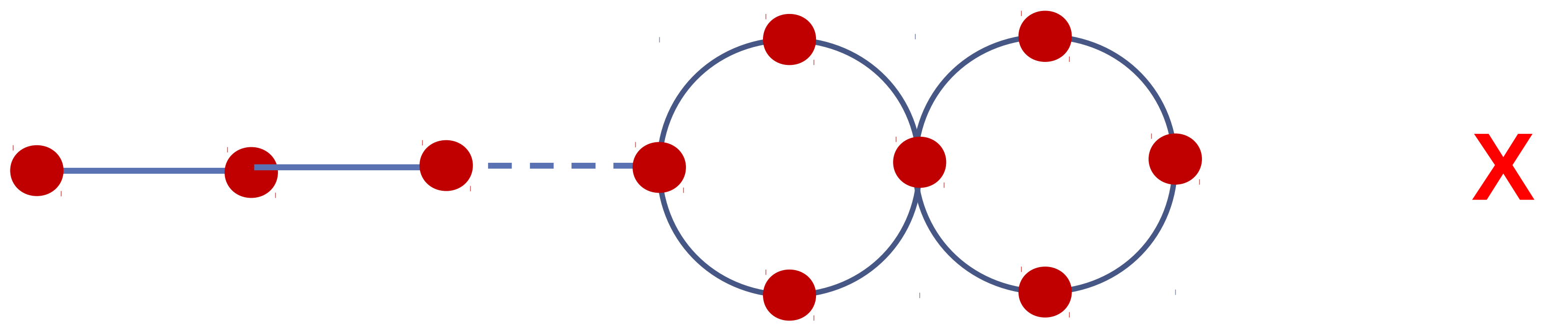
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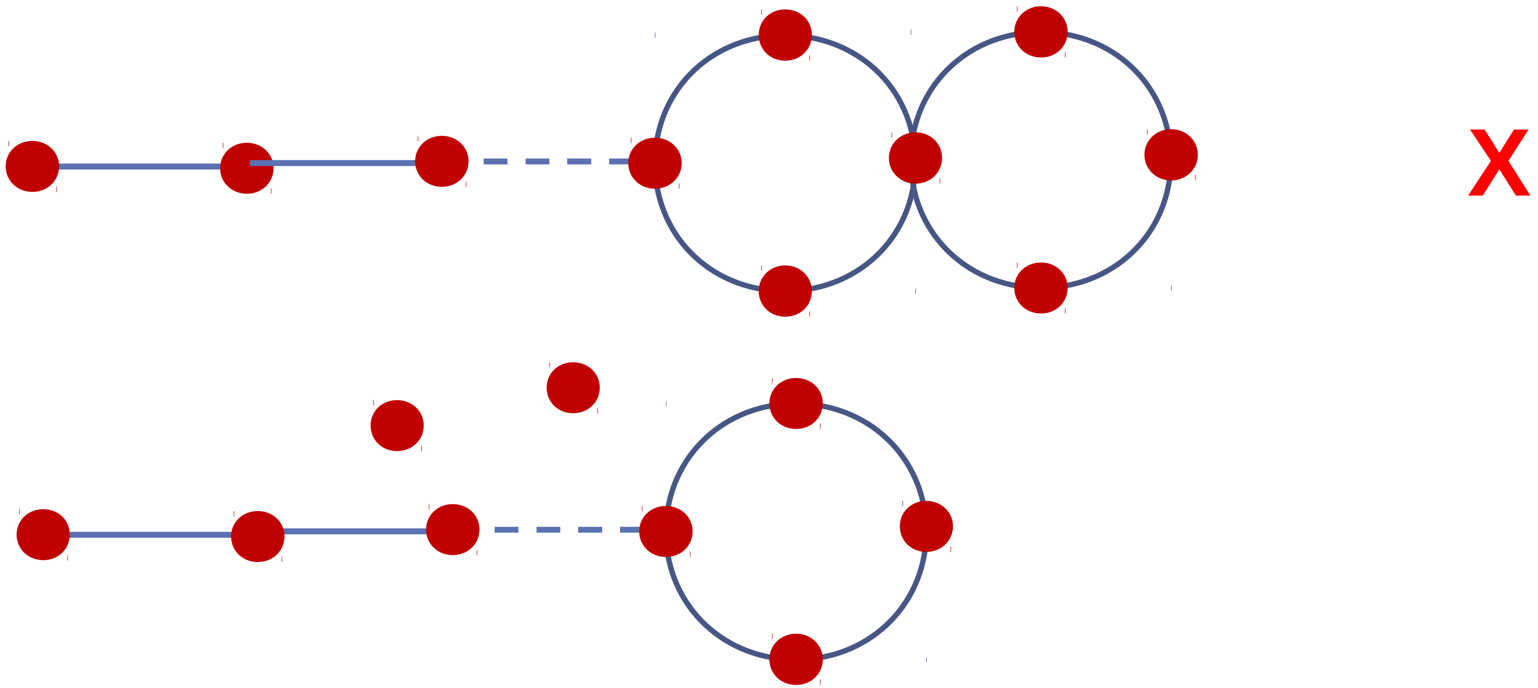

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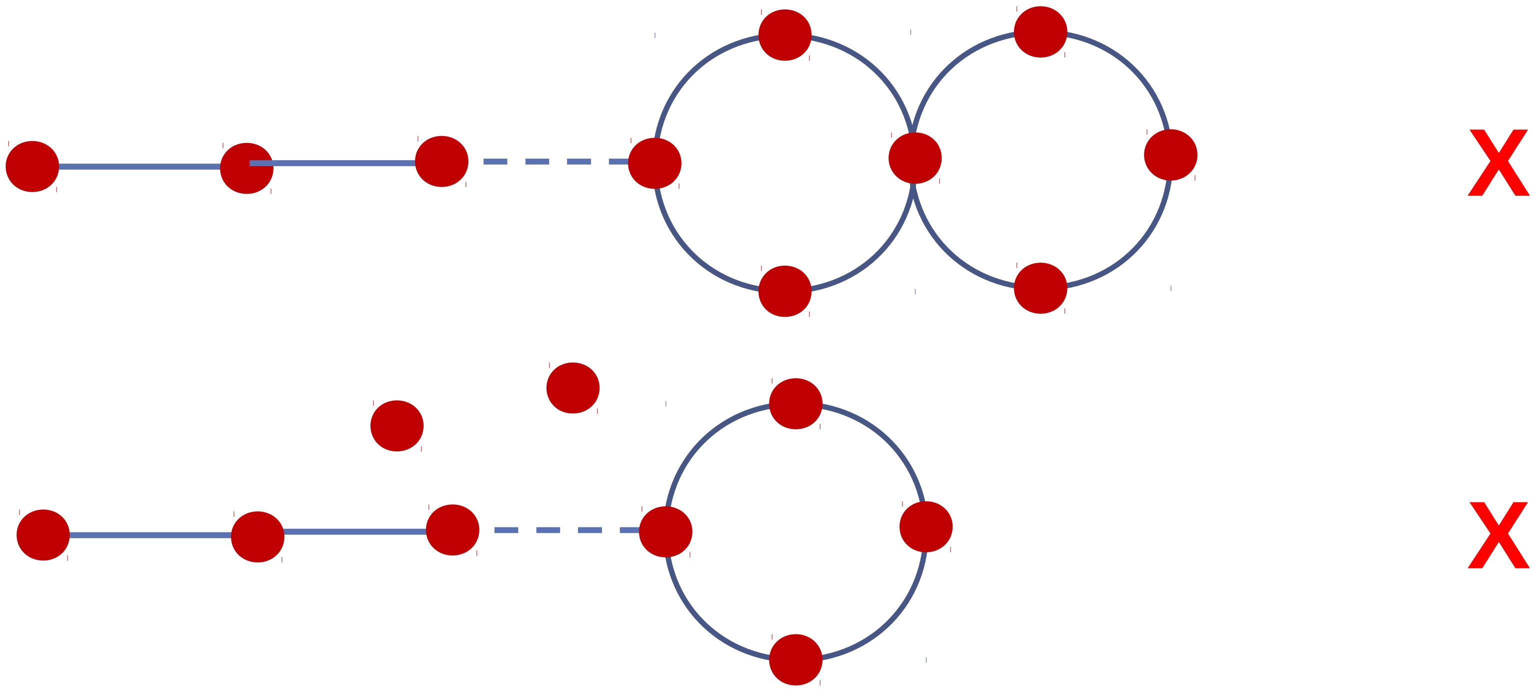
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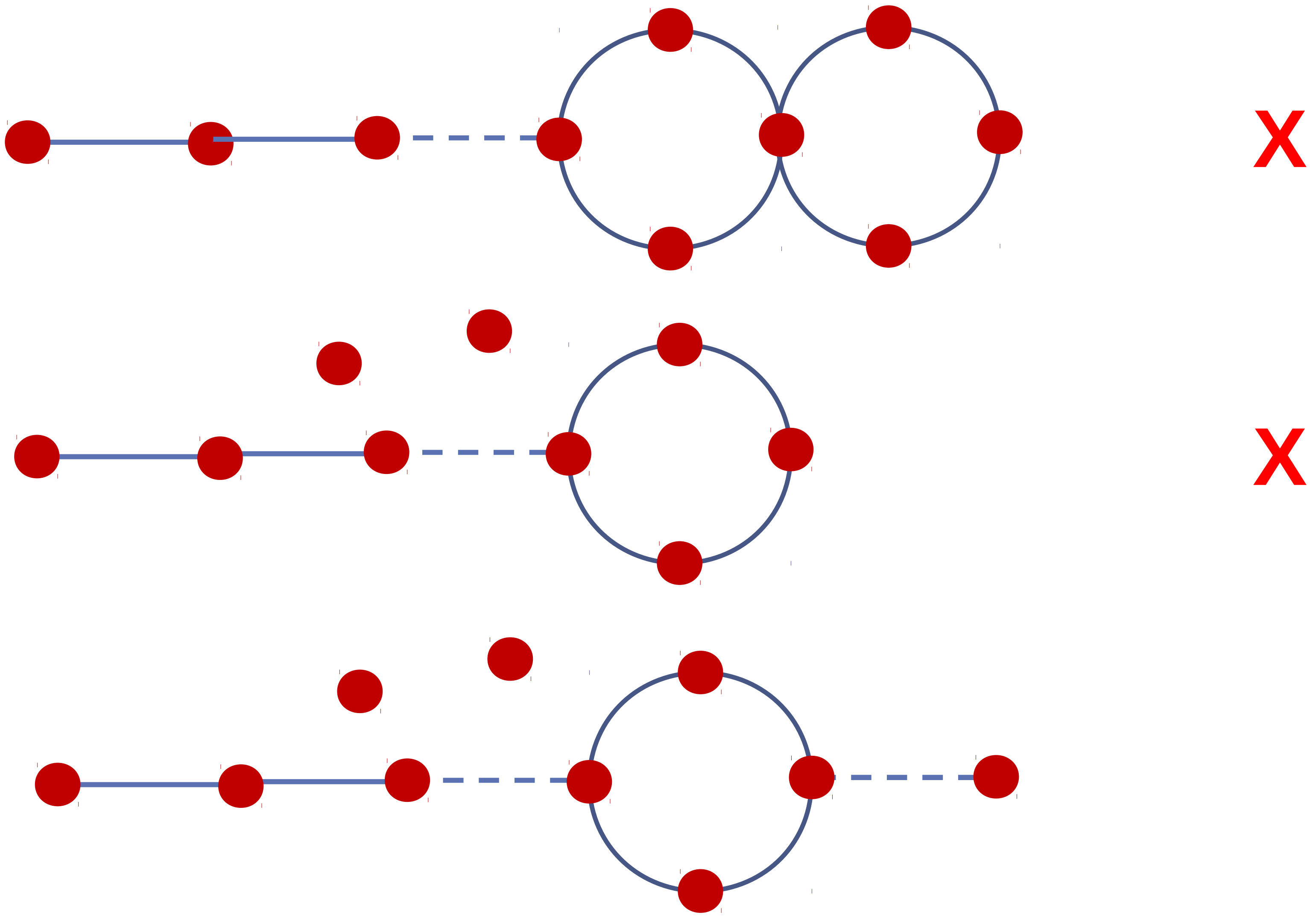

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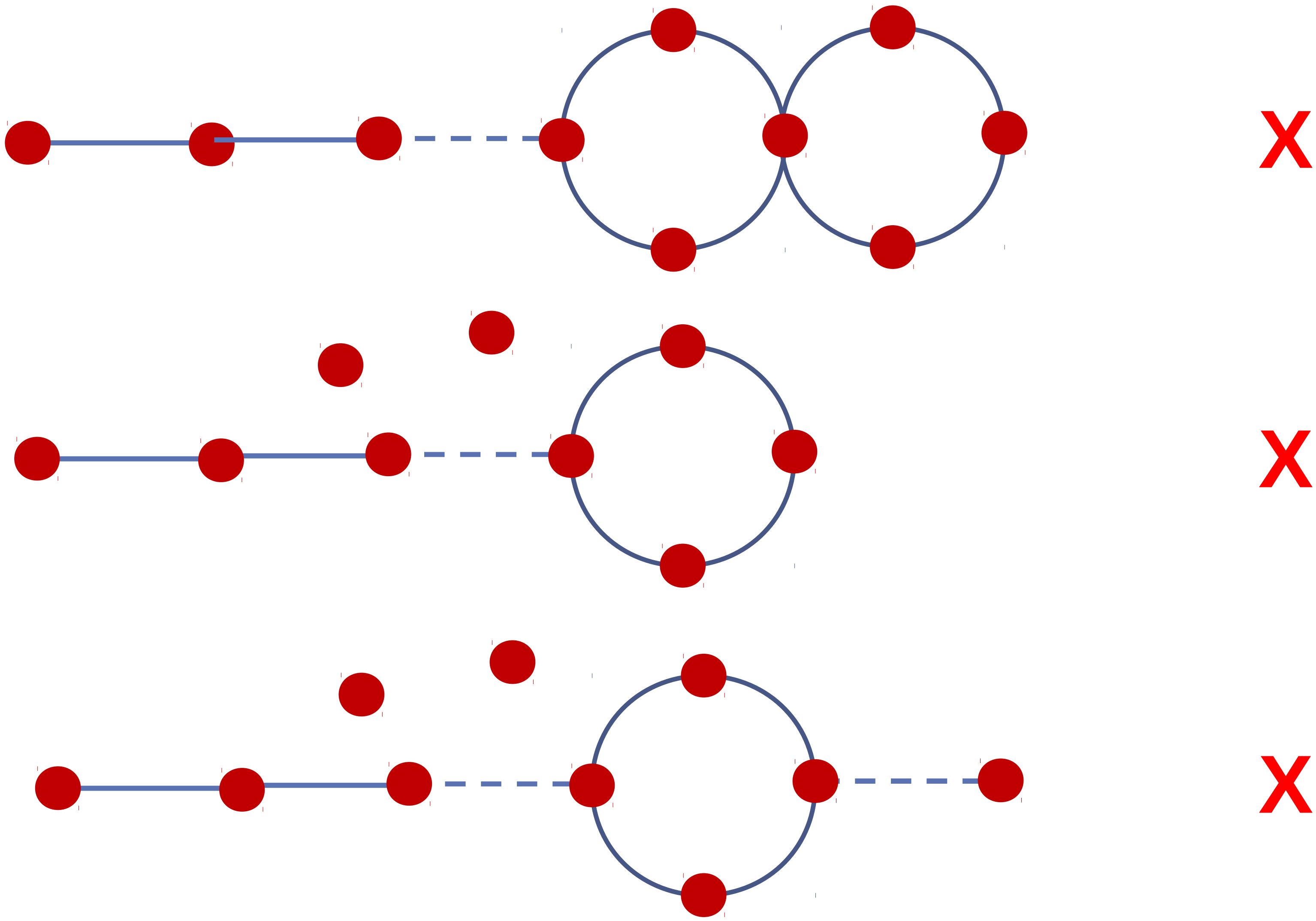


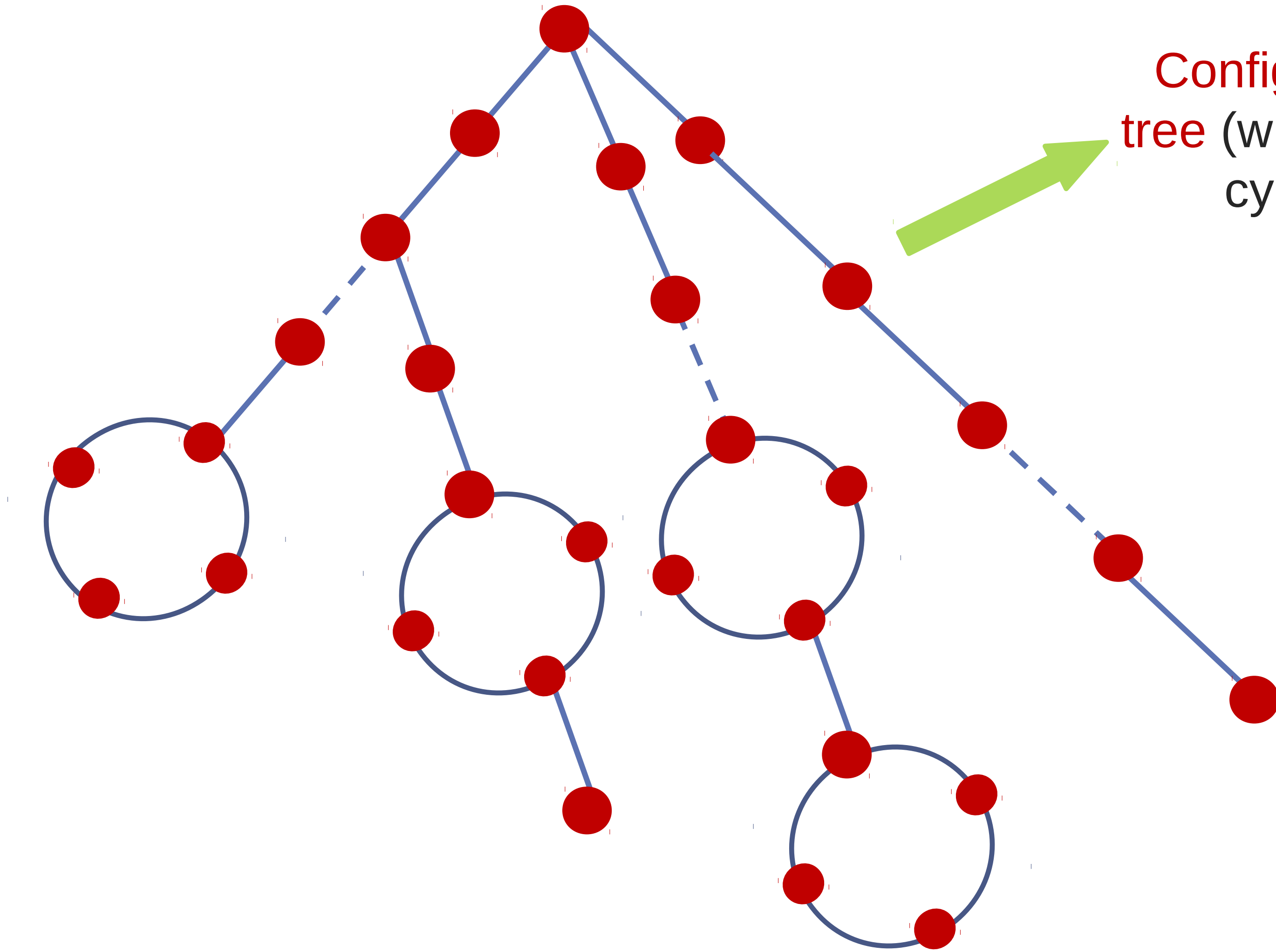
Neighbourhood Sturcture in Games

Theorem:

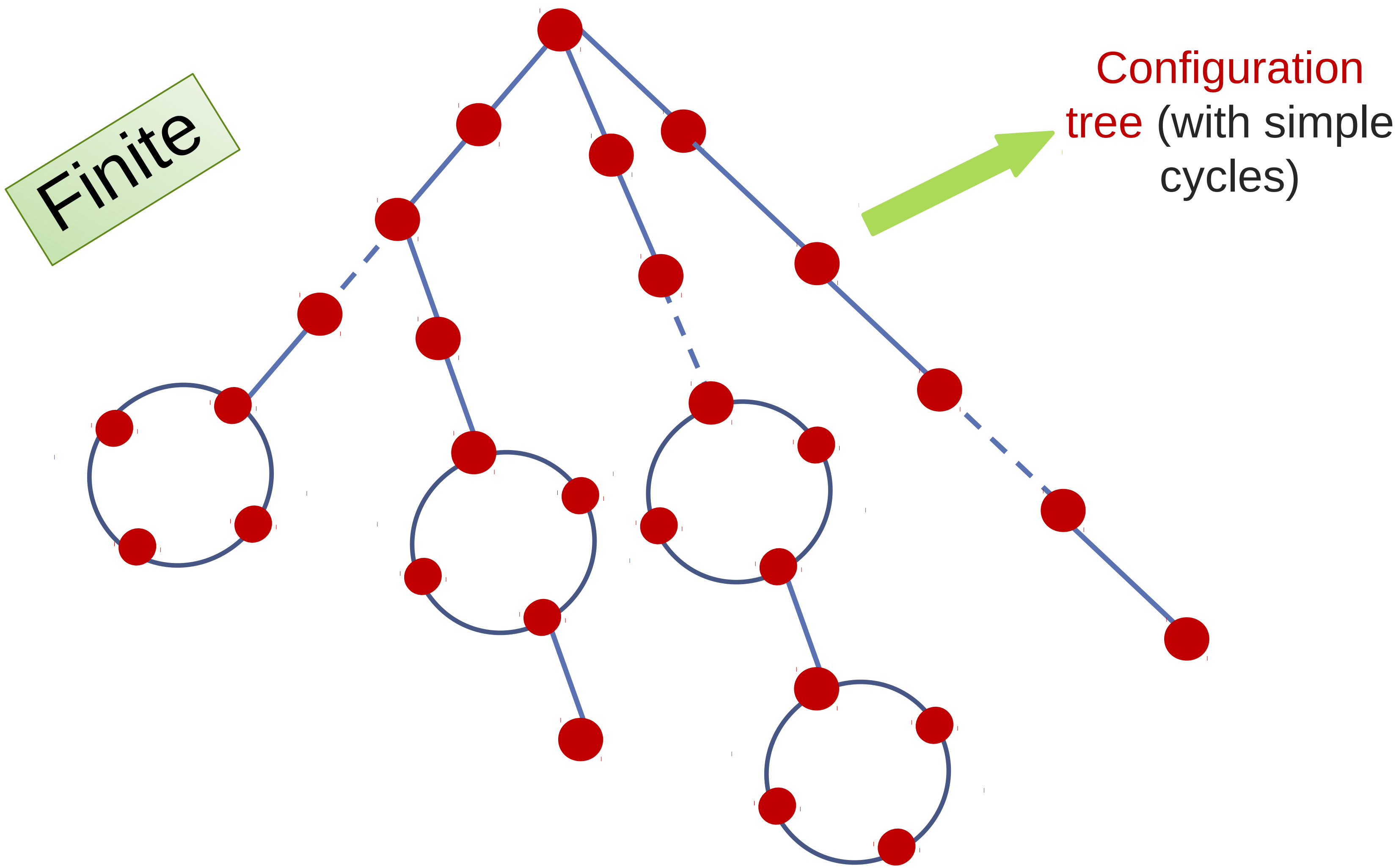
A general game with n players and with either a static or a dynamic neighbourhood structure eventually stabilises if and only if we can associate a potential Φk with every round k such that the following holds:

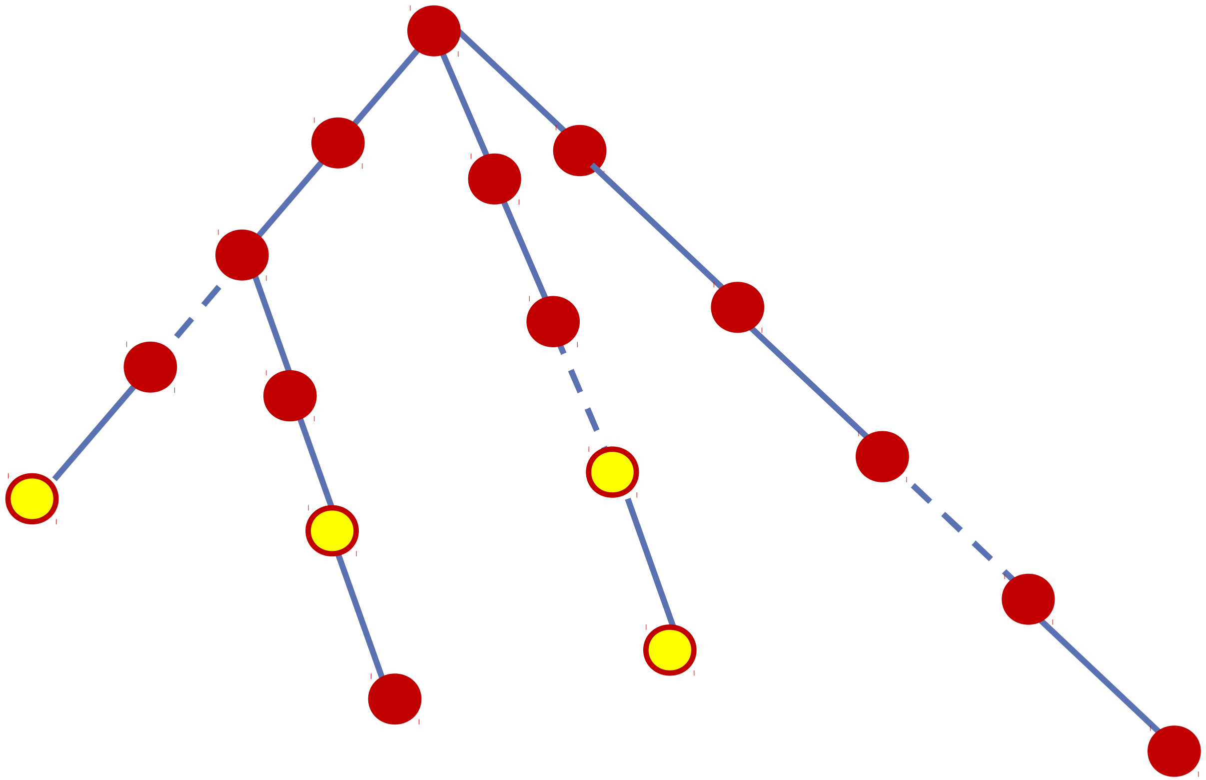
1. If the game has not yet stabilised in round k then there exists a round $k0 > k$ such that $\Phi k0 > k$
2. There exists $k0 \geq 0$ such that for all $k, k' > k0$, $\Phi k = \Phi k'$. That is, the potential of the game becomes constant eventually
3. The maximum potential of the game is bounded

Proof



Configuration tree (with simple cycles)


Finite
Configuration tree (with simple cycles)



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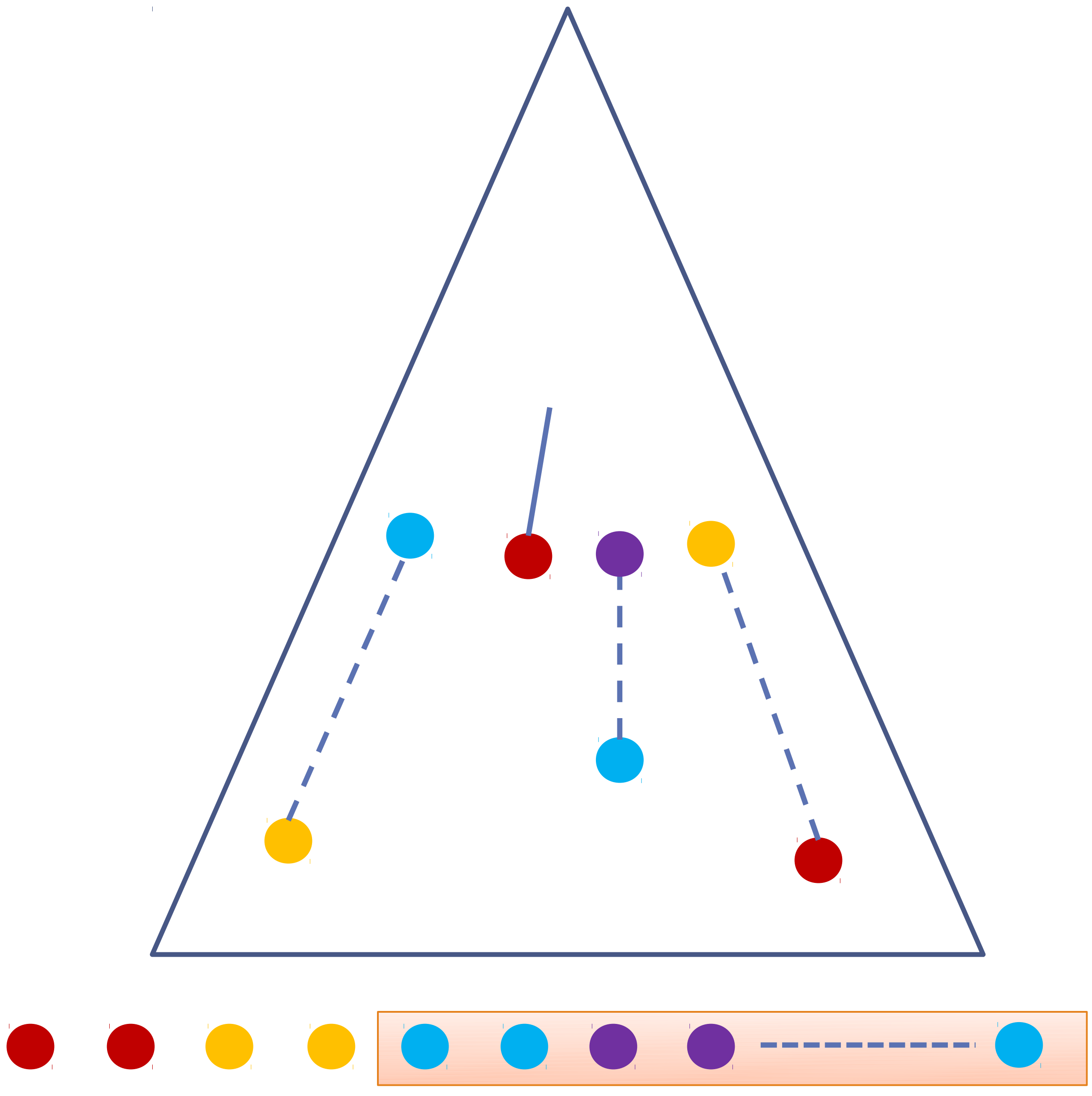
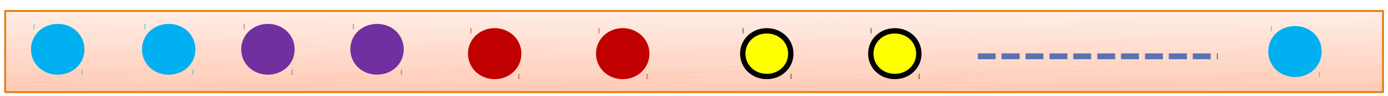


Neighbourhood Sturcture in Games

No cyclic configuration
implies
simple cycle
implies
unfolding was not correct



No cyclic configuration
implies
simple cycle
implies
unfolding was not correct

Cyclic configuration
implies
complex cycle present
contradicts
definition of stability

Questions?