Small Vertex Cover makes Petri Net Coverability and Boundedness Easier

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Petri nets — example

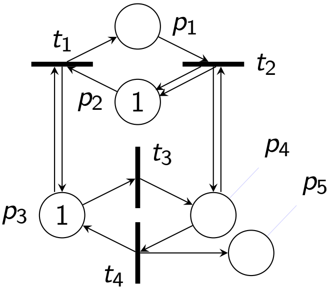

Figure: Hopcroft and Pansiot's example Petri net

Initial marking: $(0, 1, 1, 0, 0)$

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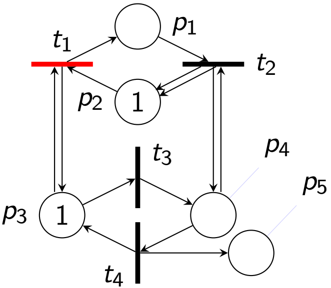

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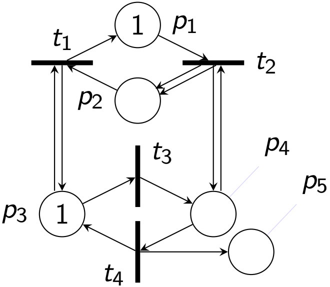

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Current marking: $(1, 0, 1, 0, 0)$

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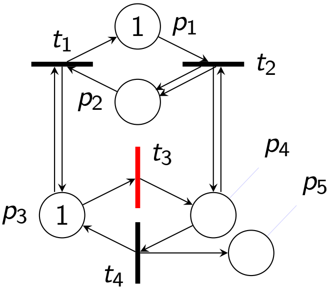

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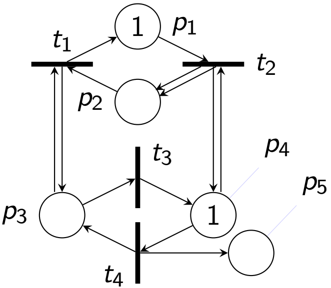

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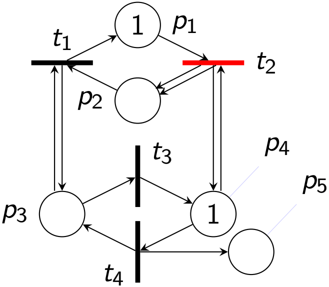

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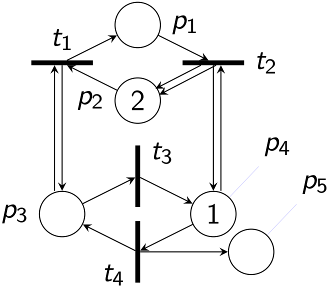

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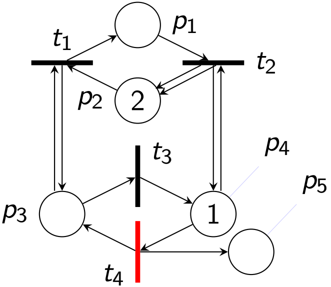
Current marking: $(0, 2, 0, 1, 0)$

Petri nets — example



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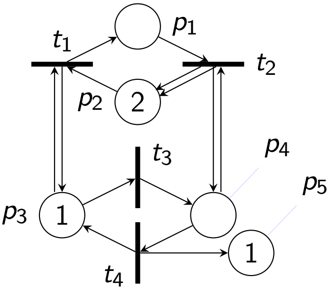

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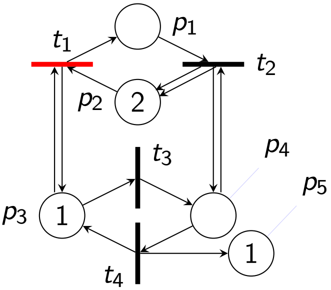

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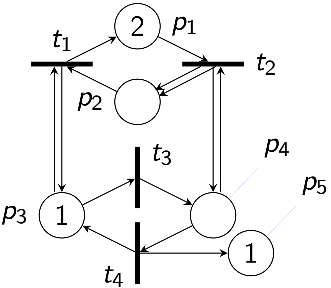

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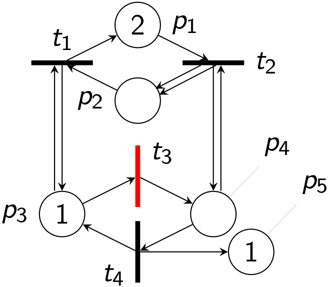
Current marking: $(2, 0, 1, 0, 1)$

Petri nets — example



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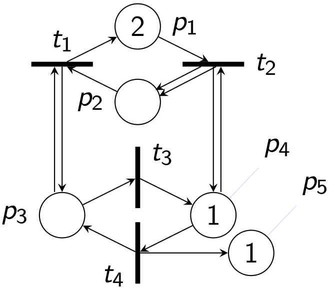

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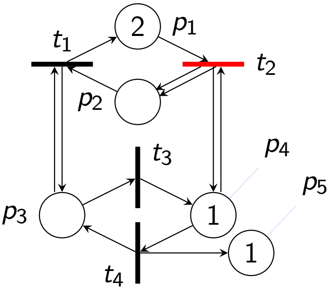

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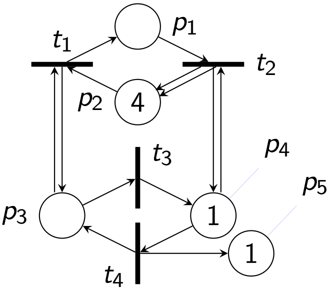

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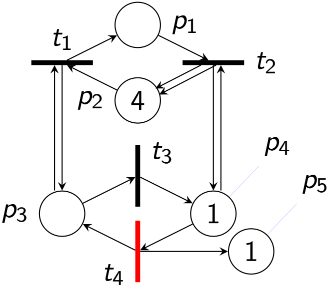
Current marking: $(0, 4, 0, 1, 1)$

Petri nets — example



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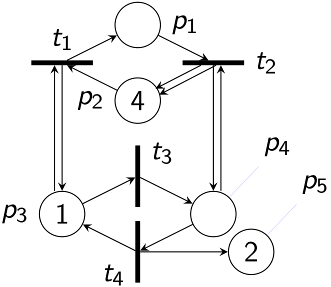

Figure: Hopcroft and Pansiot's example Petri net

Current marking: $(0, 4, 1, 0, 2)$

Problem definition

- Coverability: given a target marking M_{cov}, can we reach a marking M such that $M(p) \geq M_{cov}(p)$ for each place p?
- Boundedness: is there a bound on the number of tokens in any place in any reachable marking?
- [R. J. Lipton 75]: Exponential space lower bound.
- [C. Rackoff 78]: Almost matching upper bound.

Logic [Lodaya, P. 2009]

$$\tau ::= p,\ p \in P \mid \tau_1 + \tau_2 \mid c\tau,\ c \in \mathbb{N}$$

$$\kappa ::= \tau \geq c,\ c \in \mathbb{N} \mid \kappa_1 \wedge \kappa_2 \mid \kappa_1 \vee \kappa_2 \mid \mathbf{EF}\kappa$$

$$\beta ::= \{\tau_1, \ldots, \tau_r\} < \omega \mid \neg\beta \mid \beta_1 \vee \beta_2$$

$$\phi ::= \beta \mid \kappa \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2$$

Petri net analysis problems — related work

- [L.E. Rosier and H.-C. Yen 1986]: A multi-parameter analysis of the coverability and boundedness problems. They showed that space requirement is exponential in the number of places and logarithmic in the number of transitions and maximum arc weight. They showed corresponding lower bounds too.
- [P. Habermehl 1997]: Model checking linear time μ-calculus needs space polynomial in the size of the formula and exponential in the size of the net. Corresponding lower bounds are shown too.
- [S. Demri, F. Laroussinie and Ph. Schnoebelen 2002]: Parameterized analysis of reachability and other problems in synchronized transition systems, which are 1-safe nets.
- [Flum and Grohe 2003]: Describing parameterized classes. Fundamental results about parameterizing many standard complexity classes.

Our results

Let k be the vertex cover number of the graph underlying the Petri net.

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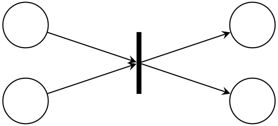

Figure: A synchronizing transition

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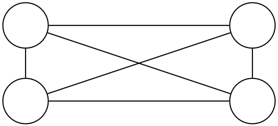

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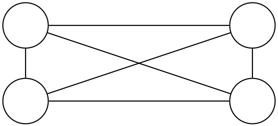

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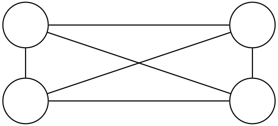

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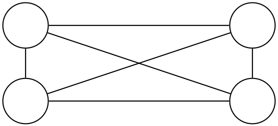

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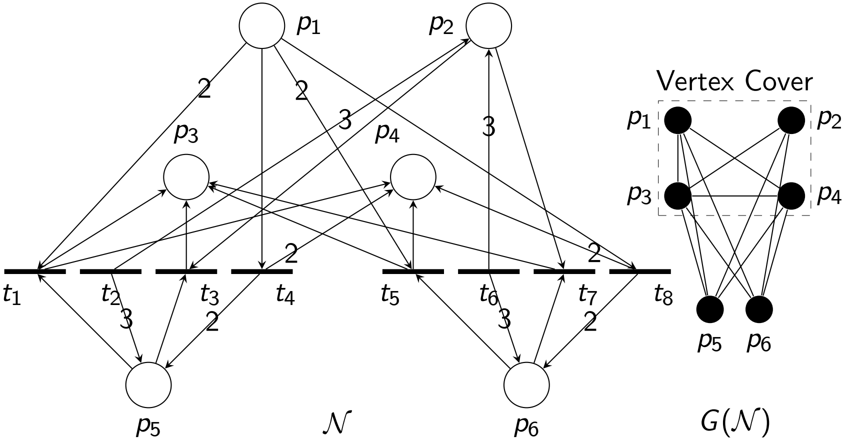
Similar result for model checking the logic mentioned earlier, assuming nesting depth of **EF** to be a constant.

Vertex Cover Example



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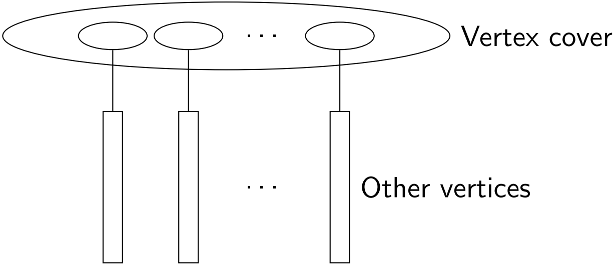
Motivation

- Classifying Petri nets according to hardness.
- Some insight into what makes them so hard.

Overview of the technique

Vertex cover of size k implies that remaining vertices can be classified into 2^k groups, each having "similar" vertices.



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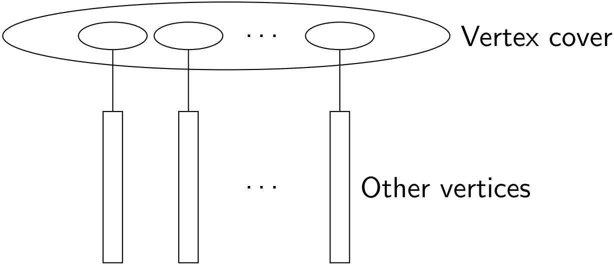
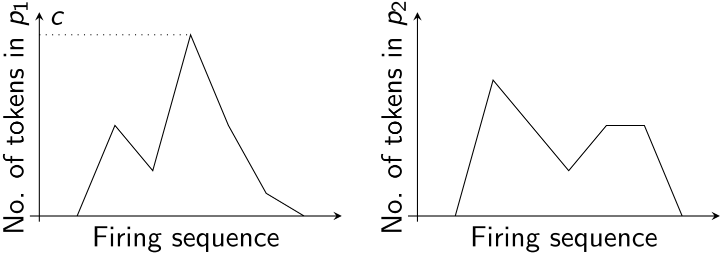


Similar classification can be done in Petri nets. Places in the same group will have same type of transitions incident on them.

Technique continued (Truncation lemma)



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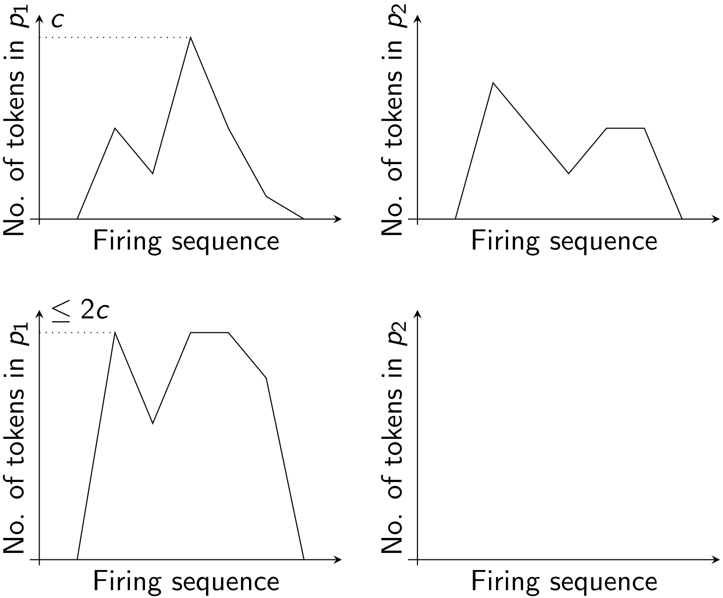

Using truncation lemma for better length bounds

- *Pigeon hole principle for bounding the length of firing sequences:* In a Petri net with two places, suppose there is a firing sequence with at most c tokens in all intermediate markings. Then we can obtain a sequence of length at most $(c+1)^2$ reaching the same final marking.

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- If there is a firing sequence covering the given target marking, Rackoff gives a recurrence relation for the length of a shortest such sequence.
- Using truncation lemma, we can modify the recurrence relation to get better bounds.

Conclusion

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- Future work:
  - Time complexity.
  - Size of smallest feedback vertex set instead of vertex cover.

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Thank you. Questions?