Conflict-Tolerant Specifications for Hybrid Systems

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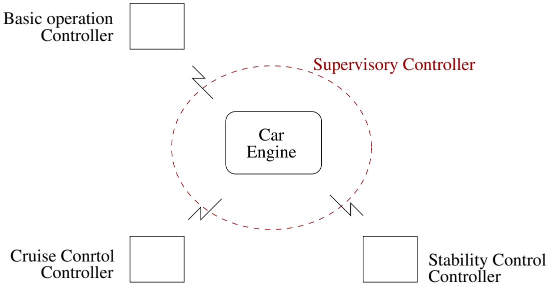
Outline

Overview

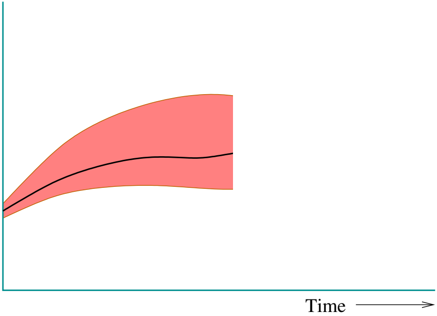
- We consider systems which are composed of a base system + multiple controllers + a supervisor.
- Supervisor chooses when to provide the control input of a controller to the base system (e.g. based on priority).



We propose a way of specifying the behaviour of individual controllers.

Why a classical safety specification is inadaquate

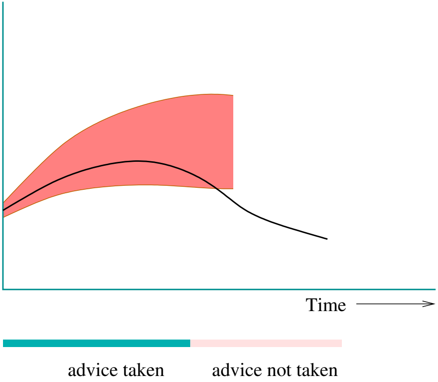
Specifies a safety "cone" (prefix-closed set of behaviours) within which the behaviour of the controlled system must lie.



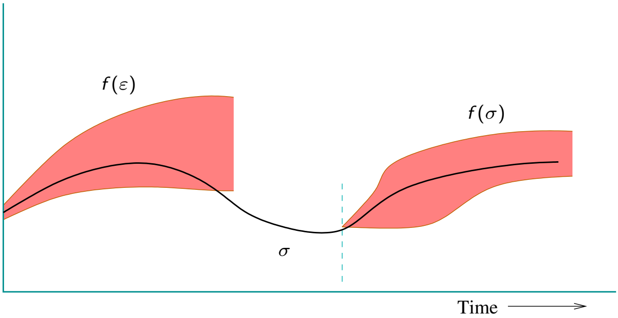
Why a classical safety specification is inadaquate

What happens if the controller's input is disregarded by the supervisor (due to a conflict)?

- The resumed controller has no specification to adhere to.



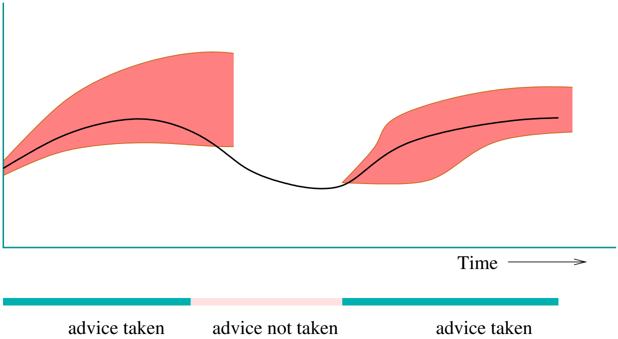
Conflict-tolerant specification



An advice function f which specifies a safety cone $f(\sigma)$ after each behaviour σ of the base system.

Conflict-tolerant Specification: Guarantee

Suppose a controller $\mathcal{C}$ satisfies its tolerant specification $\mathcal{S}$ wrt a base system $\mathcal{B}$.



- Then in every period in which it is control, $\mathcal{C}$ does "the right thing" (according to $\mathcal{S}$).
- This guarantee is regardless of other controllers/supervisors it is composed with.

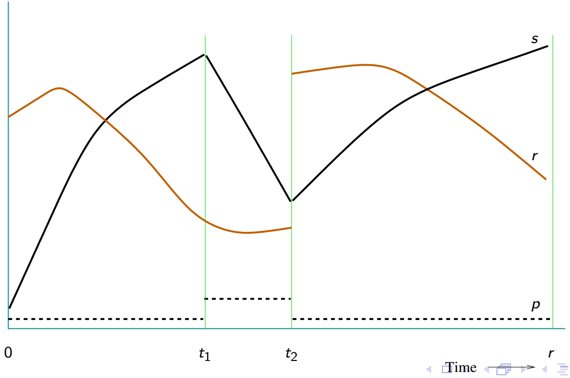
In this talk

- We give a mechanism to describe conflict-tolerant specifications for hybrid systems.
- Solve the verification problem when components are given as initialized rectangular hybrid automata.

Behaviour of a hybrid system

A signal over a set of variables W is a function $\sigma : [0, r) \longrightarrow \mathbf{W}$ which has only finitely many points of discontinuity.

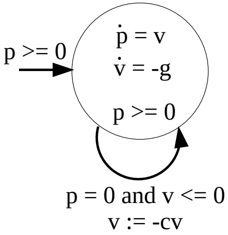
- There is a strictly increasing sequence of time points $t_0 = 0 < t_1 < t_2 < \cdots < t_n = r$ such that σ is continuous in the interval $[t_k, t_{k+1})$.



Example hybrid system: Bouncing ball

System variables:

- p: vertical position (height) of the ball.
- v: velocity of ball.



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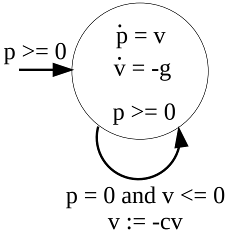
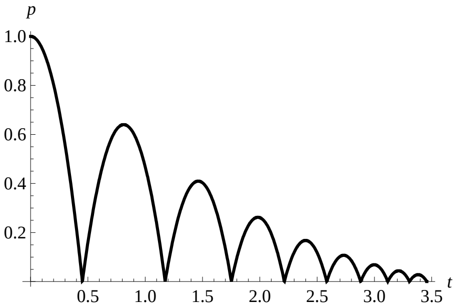
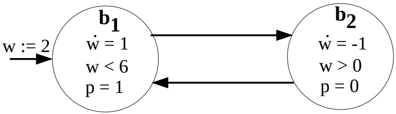




Example hybrid system: Water tank with pump

System variables:

- w: level of water in tank.
- p: On/Off status of pump (1="on").



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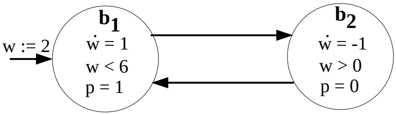
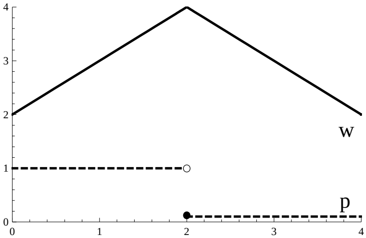
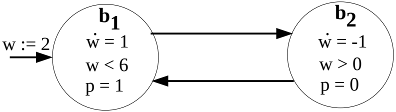


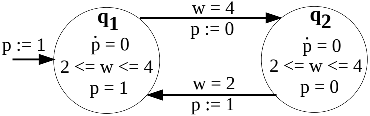


Controller for water tank

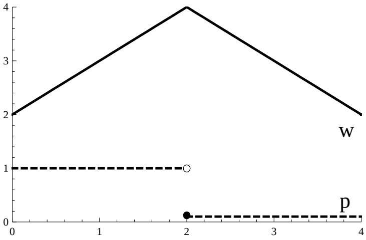
- System variables: $X = \{w\}$
- Control variables: $U = \{p\}$.



Water Tank

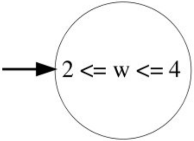


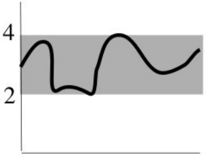
Controller



A classical specification for water level controller

Classical safety specification = prefix-closed set of signals.



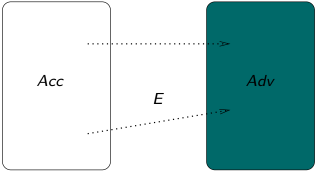


Conflict-tolerant specification

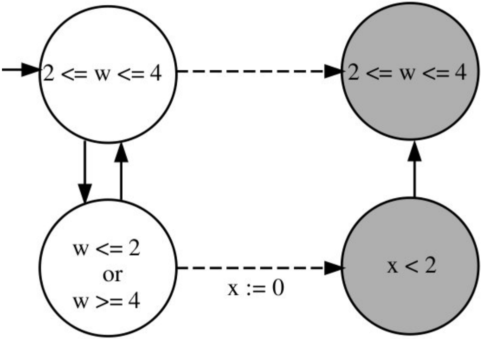
An advice function over a set of variables W is function $f : Signals \rightarrow 2^{Signals}$ such that each $f(\sigma)$ is prefix-closed.

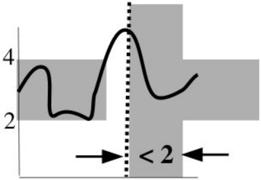
Mechanism to specify such advice functions: $\mathcal{S} = (Acc, Adv, E)$ where

- Acc and Adv are hybrid automata over W
- E is a set of edges between Acc and Adv called an advice relation.

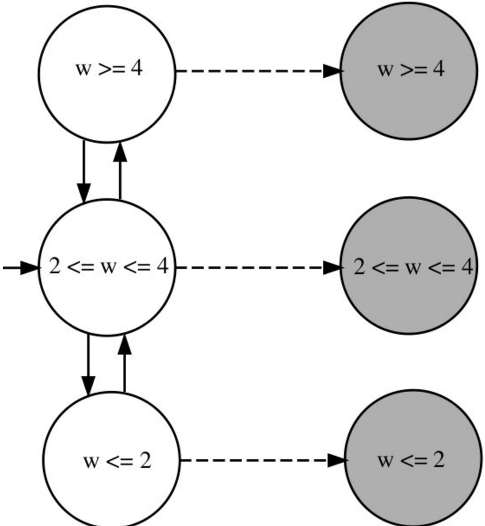


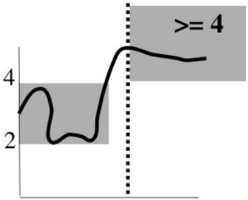
Example tolerant specifications for water level controller I





Example tolerant specifications for water level controller II





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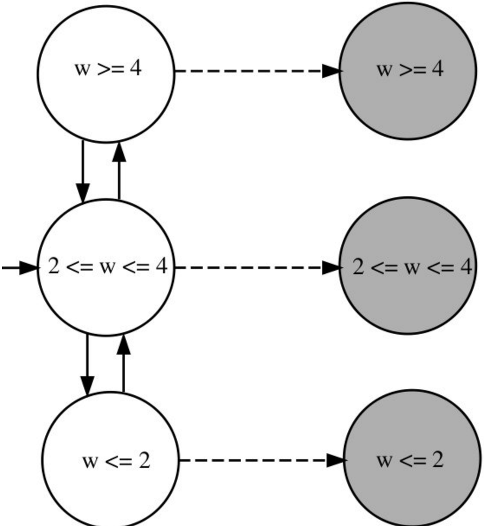
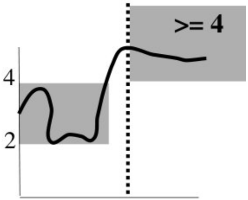
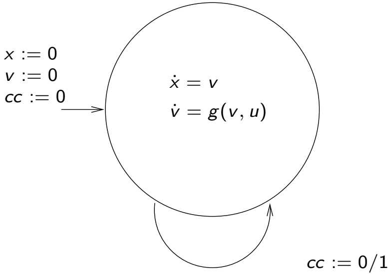




Note: Both tolerant specs induce the same classical spec but are quite different as tolerant specs.

Car motor base system

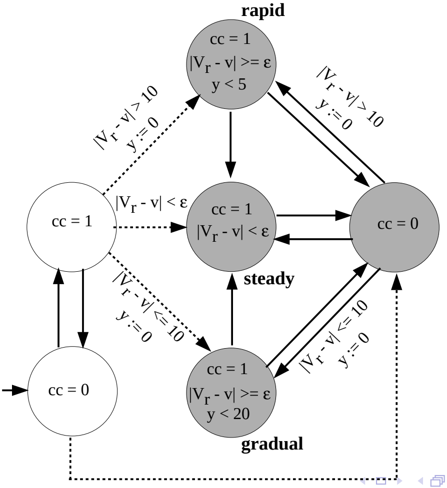
System variables: $\{x, v, cc\}$, Input variables: $\{u\}$.



Consider two controllers: for "cruise-control" and "stability-control".

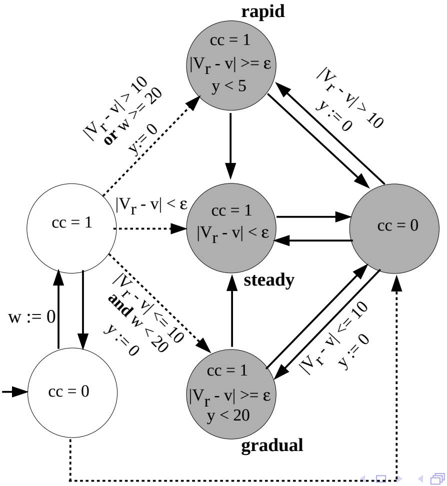
Cruise control tolerant specification I

"Reach set point within 20 sec if already close to it, else reach within 5 sec."



Cruise control tolerant specification II

"Reach set point within 5 sec if far from it or if *cc* has been on for more than 20 sec; else reach within 20 sec."



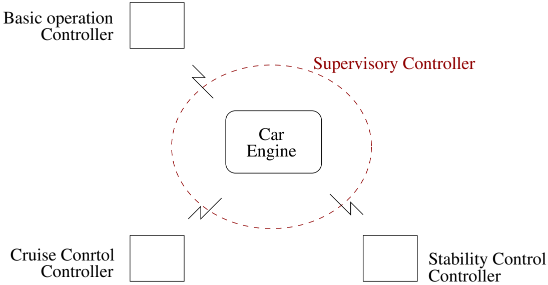
Verification for rectangular hybrid automata

We can solve the verification problem for such specs: Given

- a base system $\mathcal{B}$ modelled as an initialized rectangular automaton over (X, Y),
- a controller $\mathcal{C}$ modelled as an initialized rectangular automaton over (X, Y),
- a tolerant spec $\mathcal{S} = (Acc, Adv, E)$ whose components are IRHA:

we can check whether $\mathcal{C}$ satisfies $\mathcal{S}$ wrt $\mathcal{B}$.

Conclusion



- Conflict-tolerant specifications more richly capture a controller's specification.
- A modular or "compositional" way of developing and reasoning about systems with multiple controllers.
- A mechanism to specify them via hybrid automata.
- Decision procedure for the verification problem when components are given as initialised rectangular hybrid