Church Synthesis Problem for Noisy Input

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We define an extension of the Church's synthesis problem for the cases where some signals from the environment are noisy. We study several types of games in this framework, with the following parameters:

Type of errors: In *Games with detected errors* the environment (i.e., Player 1) may send a special signal outside the original alphabet. Player 2 wins such a play if however Player 1 replaces the special signals with real signals, the word obtained is in the required language. In *Games with undetected errors* the system (i.e., Player 2) is not able to detect the errors. Technically, it means that, at the end of the play, the opponent may chose to change the value of some of the letters it has played. Player 2 wins such a game if all the possible modified words belong to the language defining the winning condition.

Since it would be meaningless unless we restrict the errors that can occur, the second parameter is **Amount of errors tolerated:**

- Fixed number of errors: we assume that at most $n \in \mathbb{N}$ errors will occur.
- Finite number of errors: we assume that only a finite number of errors will occur.
- Threshold on the error rate: we assume that the error rate, which is defined as $\limsup_{n\to\infty} \frac{\text{number of errors until round } n}{n}$, will be under a given threshold $\delta \in \mathbb{Q}$.

The last parameter is the **Type of language**, which is described either by a Parity automaton or by a Mean-payoff automaton.

We are interested in deciding the winner of such games, and computing winning strategies for all these types of games. To solve mean payoff games with errors, we make a detour through multidimensional mean payoff games without errors (games played on graphs with edges labeled by k-dimensional weights). We extend previous results on such games - that were limited to players with finite-state strategies - by proving decidability and giving precise complexity bounds for the problem of deciding the winner of such games.

Table 1 summarizes our decidability results for games with detected and undetected errors with regular and mean-payoff winning condition.

	Fixed		Finite		Rate $\delta = 0$		Rate $\delta \in (0,1)$		Rate $\delta = 1$	
	Regular	MP	Regular	MP	Regular	MP	Regular	MP	Regular	MP
Detected	✓	✓	✓	✓	✓	✓	✓	×	✓	×
Undetected	?	?	✓	?	?	?	×	×	✓	×

Table 1. ✓ - Decidable. × - Undecidable. ? - Open.