Extensions of Dolev-Yao theory and the secrecy problem

A Baskar (CMI) R Ramanujam (IMSc) S P Suresh (CMI)

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Outline

1. Security protocols
2. Dolev-Yao model
3. Extensions of the basic model
4. An automaton construction

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Security protocols

Security protocols are three line programs that people still manage to get wrong.

Roger Needham

An example protocol

$$A \rightarrow B : \{n\}_B$$
$$B \rightarrow A : \{n\}_A$$

Attacks

- Another look at the same

Attacks

- Another look at the same

  $A!B : \{n\}_B$

  $A?B : \{n\}_A$

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$A!B : \{n\}_B \qquad B?A : \{n\}_B$

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$$A!B:\{n\}_B \qquad B?A:\{n\}_B$$
$$A?B:\{n\}_A \qquad B!A:\{n\}_A$$

- …and an attack!

Attacks

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  $A!B:\{p\}_B$

Attacks

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  $A!B:\{n\}_B$ $\quad$ $B?A:\{n\}_B$

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  $B?I:\{p\}_B$

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- Secrecy problem Is a secret leaked to the intruder by some run of the protocol?

Message construction rules

$\dfrac{}{t}\,Ax\ (t \in X)$	
$\dfrac{(t_0, t_1)}{t_i}\,split_i\ (i = 0, 1)$	$\dfrac{t_0 \quad t_1}{(t_0, t_1)}\,pair$
$\dfrac{\{t\}_k \quad inv(k)}{t}\,decrypt$	$\dfrac{t \quad k}{\{t\}_k}\,encrypt$
destruction rules	construction rules

Figure: Derivation rules (from X)

Decidability

- The passive intruder deduction problem: given X and t, check if there is proof of $X \vdash t$
- This problem is decidable.
  - A notion of normal proofs.
  - If $X \vdash t$ is provable, there is a normal proof of $X \vdash t$.
  - Every term r occurring in a normal proof of $X \vdash t$ is a subterm of $X \cup \{t\}$.
  - Derive bounds on the size of normal proofs from this.

Non-normal proofs

- An example:

$$\dfrac{\dfrac{\dfrac{}{t}\,Ax \quad \dfrac{}{t}\,Ax}{(t,t)}\,pair}{t}\,split_0$$

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$$\dfrac{\dfrac{}{t}\,Ax \quad \dfrac{}{t}\,Ax}{\dfrac{(t,t)}{t}\,split_0}\,pair$$

- Another one:

$$\dfrac{\dfrac{\dfrac{}{t}\,Ax \quad \dfrac{}{k}\,Ax}{\{t\}_k}\,encrypt \quad \dfrac{}{k}\,Ax}{t}\,decrypt$$

Normalization rules

$$\dfrac{\dfrac{\begin{matrix}\vdots\,\pi_1 \\ t\end{matrix} \qquad \begin{matrix}\vdots\,\pi_2 \\ t'\end{matrix}}{(t, t')}\,\mathit{pair}}{t}\,\mathit{split}_0 \quad \rightsquigarrow \quad \begin{matrix}\vdots\,\pi_1 \\ t\end{matrix}$$

$$\dfrac{\dfrac{\begin{matrix}\vdots\,\pi_1 \\ t\end{matrix} \qquad \begin{matrix}\vdots\,\pi_2 \\ k\end{matrix}}{\{t\}_k}\,\mathit{pair} \qquad \begin{matrix}\vdots\,\pi_3 \\ \mathit{inv}(k)\end{matrix}}{t}\,\mathit{decrypt} \quad \rightsquigarrow \quad \begin{matrix}\vdots\,\pi_1 \\ t\end{matrix}$$

Subterm property

Lemma

If π is a normal proof of $X \vdash t$ and r occurs in π:

- $r \in st(X \cup \{t\})$
- *if π ends in a destruction rule, then $r \in st(X)$.*

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$$\dfrac{\begin{array}{c}\vdots\,\pi_1\\ t\end{array} \qquad \begin{array}{c}\vdots\,\pi_2\\ k\end{array}}{\{t\}_k}\ \textit{encrypt}$$

- if r occurs in π_1,
  $r \in st(X \cup \{t\})$
- if r occurs in π_2,
  $r \in st(X \cup \{k\})$
- therefore, if r occurs in π,
  $r \in st(X \cup \{\{t\}_k\})$

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$$\dfrac{\begin{matrix} \vdots\, \pi_1 \\ \{t\}_k \end{matrix} \qquad \begin{matrix} \vdots\, \pi_2 \\ inv(k) \end{matrix}}{t}\ decrypt$$

- if r occurs in π_1 or π_2, $r \in st(X \cup \{\{t\}_k\})$
- since π is normal, π_1 does not end with the *encrypt* rule
- so it ends with a destruction rule, and $\{t\}_k \in st(X)$
- so any r occurring in π is in $st(X)$.

A polynomial-time algorithm

- The height of a normal proof of $X \vdash t$ is bounded by $n = |st(X \cup \{t\})|$.
- Let $X_0 = X$
- Compute $X_i = \text{one-step-derivable}(X_{i-1}) \cap st(X \cup \{t\})$, for $i \leq n$
- Check if $t \in X_n$!

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Extensions

- What about other cryptographic primitives?
- Diffie-Hellman encryption, exclusive or, homomorphic encryption, blind signatures, ...
- A large body of results: Rusinowitch & Turuani 2003, Millen & Shmatikov 2001, Comon & Shmatikov 2003, Chevalier, Küsters, Rusinowitch & Turuani 2005, Delaune & Jacquemard 2006, Bursuc, Comon & Delaune 2007, Lafourcade, Lugiez & Treinen 2007

Cancellations: the xor case

- One new construction rule:

$$\frac{t_1 \quad \cdots \quad t_n}{(t_1 \oplus \cdots \oplus t_n)\downarrow}$$

- Normalization rules: no more than one occurrence of any term as a premise of an *xor* rule
- Simplify

$$\dfrac{\dfrac{\begin{array}{c}\vdots\,\pi'_1\\ t'_1\end{array} \quad \cdots \quad \begin{array}{c}\vdots\,\pi'_m\\ t'_m\end{array}}{t_1}\,\mathit{xor} \qquad \begin{array}{c}\vdots\,\pi_2\\ t_2\end{array} \quad \cdots \quad \begin{array}{c}\vdots\,\pi_n\\ t_n\end{array}}{t}\,\mathit{xor}$$

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to

$$\dfrac{\dfrac{\vdots\ \pi'_1}{t'_1} \quad \cdots \quad \dfrac{\vdots\ \pi'_m}{t'_m} \quad \dfrac{\vdots\ \pi_2}{t_2} \quad \cdots \quad \dfrac{\vdots\ \pi_n}{t_n}}{t}\,\mathit{xor}$$

Subterm property

- The cases other than *xor* go through smoothly
- *xor* brings cancellations to the party!

$$\dfrac{\begin{array}{c}\vdots\ \pi_1 \\ t_1 \oplus t_2\end{array} \qquad \begin{array}{c}\vdots\ \pi_2 \\ t_2 \oplus t_3\end{array}}{t_1 \oplus t_3}$$

- t_2 is not a subterm of the conclusion. Is it a subterm of the premises?

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- t_2 is not a subterm of the conclusion. Is it a subterm of the premises? One can argue that it is!
- Moral: We cannot work with syntactic subterms any more, but there is still some way of bounding the set terms occurring in proofs.

Term syntax

$$\mathscr{T} ::= m \mid (t_1, t_2) \mid [t_1, t_2] \mid \{t\}_k$$

Normal terms: Terms that do not contain a subterm of the form $\{[t_1, t_2]\}_k$. For a term t, get its normal form $t\downarrow$ by pushing encryptions over blind pairs, all the way inside.

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$\dfrac{[t, t'] \quad k}{[\{t\}_k\downarrow, \{t'\}_k\downarrow]}\ \mathit{encrypt}$	$\dfrac{\{t\}_k\downarrow \quad \mathit{inv}(k)}{t}\ \mathit{decrypt}$	$\dfrac{(t_0, t_1)}{t_i}\ \mathit{split}_i$	$\dfrac{[t_0, t_1]\downarrow \quad t_i\downarrow}{t_{1-i}}\ \mathit{blindsplit}_i$
$\dfrac{}{t}\ \mathit{Ax}\ (t \in X)$	$\dfrac{t \quad k}{\{t\}_k\downarrow}\ \mathit{encrypt}\ (t \text{ not a blind pair})$	$\dfrac{t_1 \quad t_2}{(t_1, t_2)}\ \mathit{pair}$	$\dfrac{t_1 \quad t_2}{[t_1, t_2]}\ \mathit{bpair}$

Figure: analz and synth rules for normal terms (with assumptions from $X \subseteq \mathscr{T}$)

Modelling blind signatures

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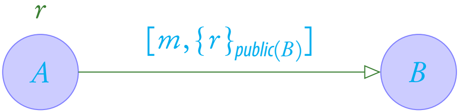

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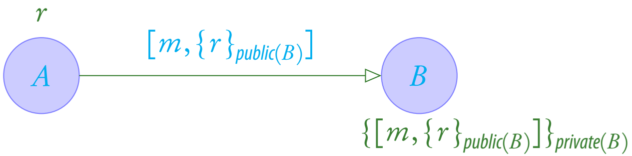

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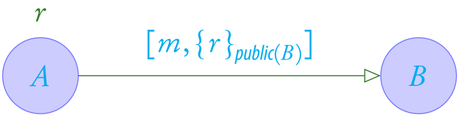

$$\{[m, \{r\}_{public(B)}]\}_{private(B)}$$

$$[\{m\}_{private(B)}, r]$$

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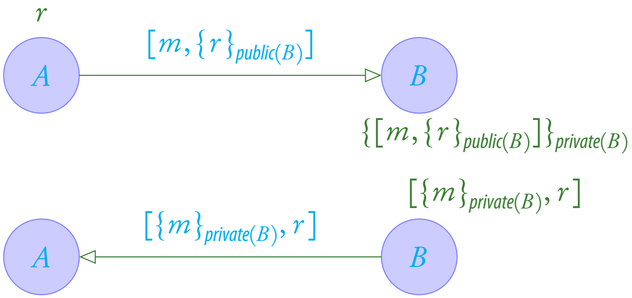

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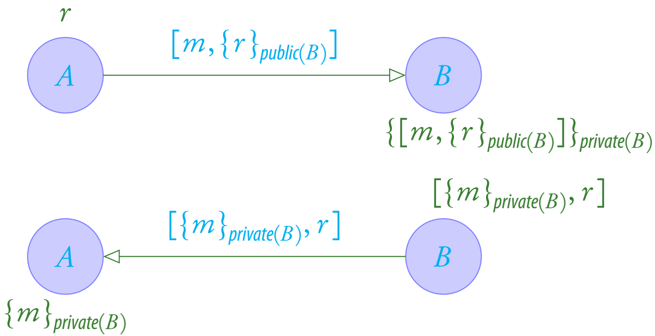

Alternative theories

- A simpler system. Delaune, Kremer, Ryan 2009, Baskar, Ramanujam, Suresh 2007.

$$\frac{[t, \{m\}_k] \quad inv(k)}{[\{t\}_{inv(k)}, m]}$$

  Passive intruder deduction is PTIME decidable.

- A much harder system. Lafourcade, Lugiez, Treinen 2007.

$$\frac{t_1 + \cdots + t_\ell \quad k}{\{t_1\}_k + \cdots + \{t_\ell\}_k}$$

$$\frac{t_1 + \cdots + t_\ell + \cdots + t_m \quad t_\ell + \cdots + t_m + \cdots + t_n}{t_1 + \cdots + t_{\ell-1} - t_{m+1} - \cdots - t_n}$$

  Decidable but non-elementary upper bound.

- Our system: Decidable with a DEXPTIME upper bound.

Some difficult proofs

$$\dfrac{\dfrac{}{[a,\{b\}_k]}\,\textit{Ax} \qquad \dfrac{\dfrac{}{b}\,\textit{Ax} \quad \dfrac{}{k}\,\textit{Ax}}{\{b\}_k}\,\textit{encrypt}}{a}\,\textit{blindsplit}_1$$

Some difficult proofs . . .

$$
\dfrac{\dfrac{\dfrac{}{[a, b]}\,\mathit{Ax} \qquad \dfrac{}{k}\,\mathit{Ax}}{[\{a\}_k, \{b\}_k]}\,\mathit{encrypt} \qquad \dfrac{}{\{b\}_k}\,\mathit{Ax}}{\{a\}_k}\,\mathit{blindsplit}_1
$$

Some difficult proofs ...

$$
\dfrac{\dfrac{}{[\{a\}_{k_1k_2}, b]}\,\mathit{Ax} \qquad \dfrac{\dfrac{}{[b, \{a\}_{k_1k_2k_1}]}\,\mathit{Ax} \qquad \dfrac{\dfrac{\dfrac{}{[\{a\}_{k_1k_2}, b]}\,\mathit{Ax} \quad \dfrac{}{k_1}\,\mathit{Ax}}{[\{a\}_{k_1k_2k_1}, \{b\}_{k_1}]}\,\mathit{encrypt} \qquad \dfrac{}{\{b\}_{k_1}}\,\mathit{Ax}}{\{a\}_{k_1k_2k_1}}\,\mathit{blindsplit}_1}{b}\,\mathit{blindsplit}_1}{\{a\}_{k_1k_2}}\,\mathit{blindsplit}_1
$$

Some difficult proofs ...

$$
\dfrac{\dfrac{}{[a,\{a\}_{k_1}]}\,Ax \qquad \dfrac{\dfrac{\dfrac{}{[a,\{a\}_{k_1}]}\,Ax \quad \dfrac{}{k_1}\,Ax}{[\{a\}_{k_1},\{a\}_{k_1 k_1}]}\,encrypt \qquad \dfrac{}{\{a\}_{k_1 k_1}}\,Ax}{\{a\}_{k_1}}\,blindsplit_1}{a}\,blindsplit_1
$$

Decidability: the proof idea

- The examples suggest that it is not easy to come up with a bound on the terms occurring in the proof.
- Instead of trying to prove that it is finite, we prove that it is regular.
  - Show that every term in a normal proof of $X \vdash t$ is of the form $\{p\}_x$ where $p \in st(X \cup \{t\})$ and x is a sequence of keys from $st(X \cup \{t\})$.
  - Show that for each $p \in st(X \cup \{t\})$, $\mathscr{L}_p = \{x \in \mathscr{K}^* | X \vdash \{p\}_x\}$ is a regular set.
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  - To check whether $X \vdash t$, check whether $\varepsilon \in \mathscr{L}_t$.
  - Properties of the $\mathscr{L}_p$:
    - $kx \in \mathscr{L}_p$ iff $x \in \mathscr{L}_{\{p\}_k}$
    - if $x \in \mathscr{L}_p$ and $x \in \mathscr{L}_{[p,p']}$, then $x \in \mathscr{L}_{p'}$
    - if $x \in \mathscr{L}_p$ and $\varepsilon \in \mathscr{L}_k$, then $xk \in \mathscr{L}_p$
    - if $\varepsilon \in \{t\}_k$ and $\varepsilon \in inv(k)$ then $\varepsilon \in t$.

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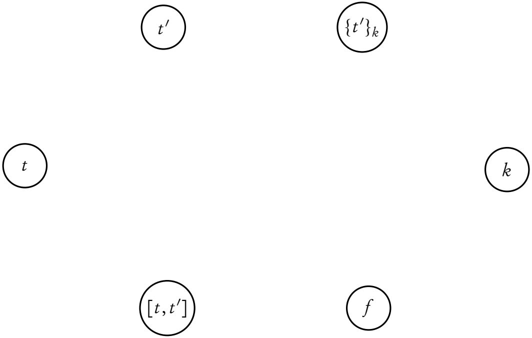
An example

$$\{[t,t'],\{t'\}_k,k\} \vdash \{t\}_k$$



the set of subterms

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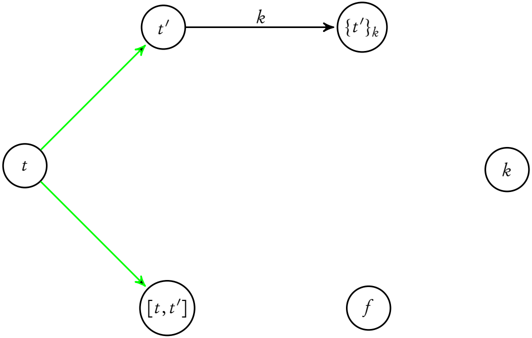
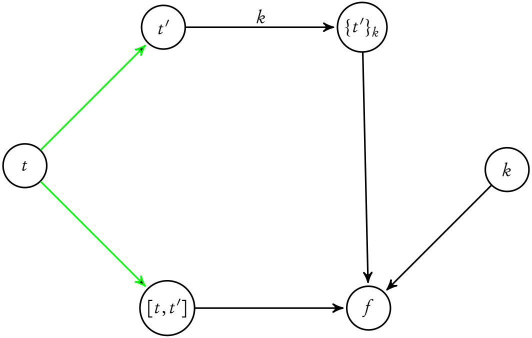


$t',[t,t'] \vdash t$ and t' encrypted with k is $\{t'\}_k$

An example

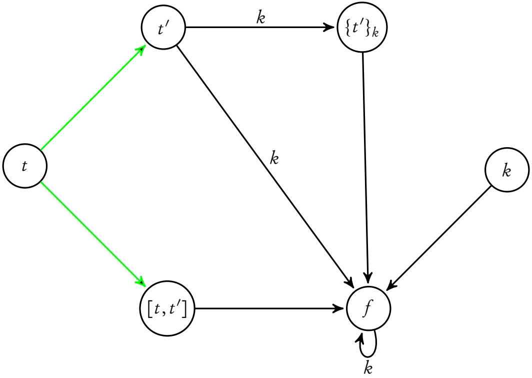
$$\{[t,t'],\{t'\}_k,k\} \vdash \{t\}_k$$



the initial set of terms X

An example

$$\{[t,t'],\{t'\}_k,k\}\vdash\{t\}_k$$



$k \in X$ and $t' \overset{k}{\Rightarrow} f$

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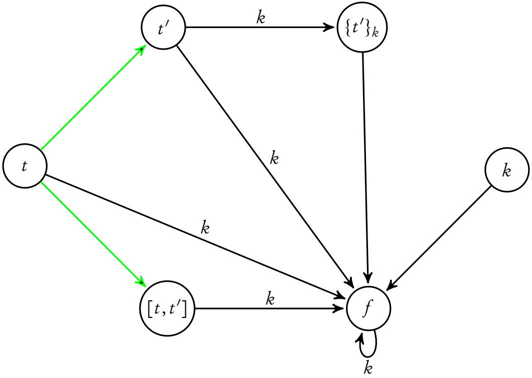


$[t,t'] \overset{k}{\Rightarrow} f$ and $t \overset{k}{\Rightarrow} f$

Another example

$$\{[t, \{t'\}_k], t', k\} \vdash t$$

$\{t'\}_k$

t'

t

k

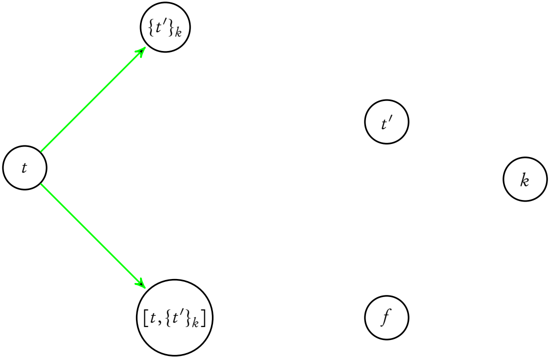
$[t, \{t'\}_k]$

f

the set of subterms

Another example

$$\{[t,\{t'\}_k],t',k\} \vdash t$$



$$\{t'\}_k,[t,\{t'\}_k] \vdash t$$

Another example

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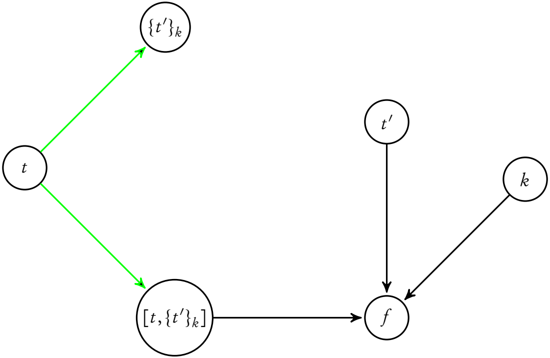

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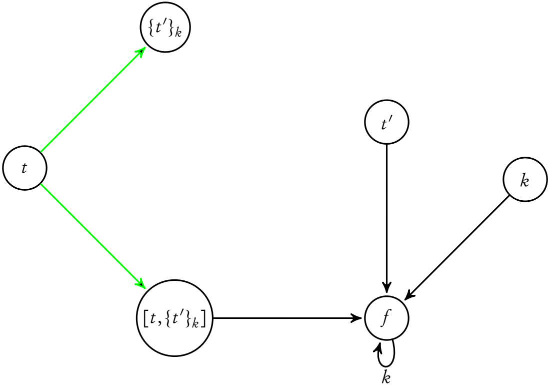

$k \in X$

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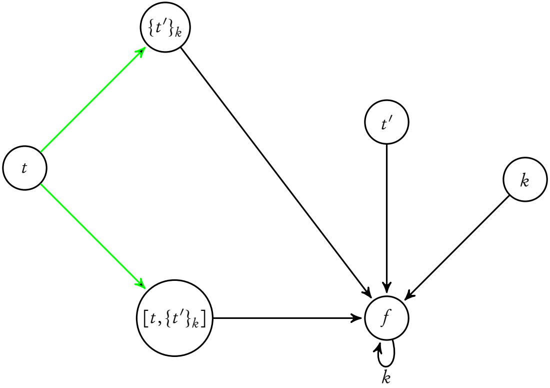

$t' \overset{k}{\Rightarrow} f$

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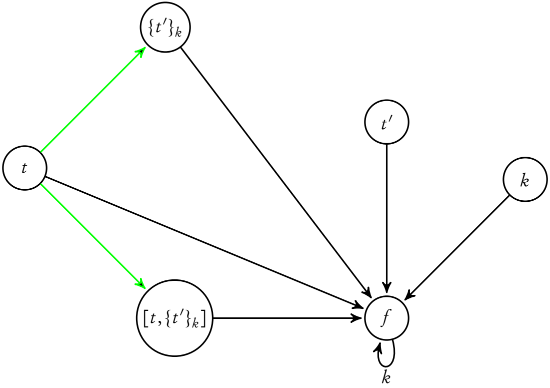

$t \Rightarrow f$

Proof normalization

$$\dfrac{\dfrac{\begin{matrix}\vdots\,\pi' \\ t'\end{matrix} \qquad \begin{matrix}\vdots\,\pi'' \\ t''\end{matrix}}{[t', t'']}\,\textit{bpair} \qquad \begin{matrix}\vdots\,\delta \\ k\end{matrix}}{[\{t'\}_k\!\downarrow, \{t''\}_k\!\downarrow]}\,\textit{encrypt}$$

$$\dfrac{\dfrac{\begin{matrix}\vdots\,\pi' \\ t'\end{matrix} \qquad \begin{matrix}\vdots\,\delta \\ k\end{matrix}}{\{t'\}_k\!\downarrow}\,\textit{encrypt} \qquad \dfrac{\begin{matrix}\vdots\,\pi'' \\ t''\end{matrix} \qquad \begin{matrix}\vdots\,\delta \\ k\end{matrix}}{\{t''\}_k\!\downarrow}\,\textit{encrypt}}{[\{t'\}_k\!\downarrow, \{t''\}_k\!\downarrow]}\,\textit{bpair}$$

$$\dfrac{\dfrac{\begin{matrix}\vdots\,\pi' \\ \{t'\}_k\!\downarrow\end{matrix} \qquad \begin{matrix}\vdots\,\pi'' \\ \{t''\}_k\!\downarrow\end{matrix}}{[\{t'\}_k\!\downarrow, \{t''\}_k\!\downarrow]}\,\textit{bpair} \qquad \begin{matrix}\vdots\,\delta \\ \textit{inv}(k)\end{matrix}}{[t', t'']}\,\textit{decrypt}$$

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Figure: The normalization rules I

Proof normalization ...

$$\cfrac{\cfrac{\overset{\vdots\,\pi'}{[t,t']} \quad \overset{\vdots\,\pi''}{t'}}{t}\,\textit{blindsplit} \qquad \overset{\vdots\,\delta}{k}}{\{t\}_k\downarrow}\,\textit{encrypt}$$

$$\cfrac{\cfrac{\overset{\vdots\,\pi'}{[t,t']} \quad \overset{\vdots\,\delta}{k}}{[\{t'\}_k\downarrow,\{t'\}_k\downarrow]}\,\textit{encrypt} \qquad \cfrac{\overset{\vdots\,\pi''}{t'} \quad \overset{\vdots\,\delta}{k}}{\{t'\}_k\downarrow}\,\textit{encrypt}}{\{t\}_k\downarrow}\,\textit{blindsplit}$$

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Figure: The normalization rules II

Proof normalization …

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Let π be a normal proof of t from X, and let δ be a sub-proof of π with root labelled r. Then the following hold:

1. *If δ ends with an analz rule, then for every u occurring in δ there is $p \in st(X)$ and keyword x such that $u = \{p\}_x\downarrow$.*
2. *If δ ends with a synth rule, then for every u occurring in δ, either $u \in st(X \cup \{r\})$ or there is $p \in st(X)$ and keyword x such that $u = \{p\}_x\downarrow$.*
3. *If the last rule of δ is decrypt or split with major premise r_1, then $r_1 \in st(X)$.*

The automaton construction

Similar to the construction in [Bouajjani, Esparza, Maler 1997]

$$\mathscr{A}_i = (Q, \Sigma, \hookrightarrow_i, F), Q = Y_0 \cup \{f\}, \Sigma = K_0, \text{ and } F = \{f\}.$$

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- 1. if $t \in Y_0, k \in K_0$ such that $\{t\}_k\!\downarrow \in Y_0$, then $t \overset{k}{\hookrightarrow}_0 \{\{t\}_k\!\downarrow\}$.
  2. if $t, t', t'' \in Y_0$ such that t is the conclusion of an instance of the *bpair* or *blindsplit*$_i$ rules with premises t' and t'', then $t \overset{\varepsilon}{\hookrightarrow}_0 \{t', t''\}$.

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- 1. if $q \overset{a}{\Rightarrow}_i C$, then $q \overset{a}{\hookrightarrow}_{i+1} C$.
  2. if $\{t\}_k\!\downarrow \in Y_0$ and $t \overset{k}{\Rightarrow}_i C$, then $\{t\}_k\!\downarrow \overset{\varepsilon}{\hookrightarrow}_{i+1} C$.
  3. if $k \in K_0$ and $k \overset{\varepsilon}{\Rightarrow}_i \{f\}$, then $f \overset{k}{\hookrightarrow}_{i+1} \{f\}$.
  4. if $\Gamma \subseteq Y_0$, $t \in Y_0$, and if there is an instance r of one of the rules whose set of premises is (exactly) Γ and conclusion is t the following holds:

     if $u \overset{\varepsilon}{\Rightarrow}_i \{f\}$ for every $u \in \Gamma$, then $t \overset{\varepsilon}{\hookrightarrow}_{i+1} \{f\}$.

Correctness of the construction

Theorem

*(**Completeness**) For any* $t \in Y_0$ *and any keyword* x*, if* $X_0 \vdash \{t\}_x\downarrow$*, then there exists* $i \geq 0$ *such that* $t \stackrel{x}{\Rightarrow}_i \{f\}$.

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Suppose $i, d \geq 0$*,* $t \in Y_0, x, y \in K_0^*$*, and* $C \subseteq Q$ *(with* $D = C \cap Y_0$*). Suppose the following also hold: 1)* $t \overset{x}{\Rightarrow}_{i,d} C$*, and 2)* $C \subseteq Y_0$ *or* $X_0 \vdash y$*. Then* $X_0 \cup \{D\}_y \vdash \{t\}_{xy}$.

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Theorem

(**Soundness**) *For any* i*, any* $t \in Y_0$*, and any keyword* x*, if* $t \overset{x}{\Rightarrow}_i \{f\}$*, then* $X_0 \vdash \{t\}_x\downarrow$.

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- Interesting extension of the Dolev-Yao theory
- Related work Tree automata used extensively in security protocol literature. Typically the accepted language is an over-approximation of the set of derivable terms. But there is a lot of potential for automata to be used in the analysis of derivations.
- Future work Lots of unresolved questions: Lower bounds or tighter upper bounds, complexity of the active intruder theory, better upper bounds for a general abelian group operator with encryption (the [LLT2007] result) etc.

Thank you!