Determinization and Expressiveness of Integer Reset Timed Automata with Silent Transitions

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January, 2009

Workshop on Automata, Concurrency and Timed Systems

Outline

Timed Automata

- Timed Automata (TA): Finite State Automata + Clocks running at the rate of global time
- Extensions of timed automata with Periodic Guards [CG00] and Silent Transitions [BPGD98] have been introduced to model periodic behaviours
- Expressiveness: TA $<$ Per-TA $<$ ϵ-TA
- TA are closed under union and intersection but not closed under determinization and complementation [AD94]
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Integer Resets

- Integer Reset Timed Automata (IRTA): Clock resets happen at integral time points. Each resetting edge has an atomic constraint of the form $x = c$
  - Fractional values of all the clocks remain equal
  - Also equal to the fractional value of the global time
- Extensions of IRTA with periodic constraints and silent transitions can be defined. However clock resets happen at integral time points: In case of Per-IRTA, resetting edge consists an atomic constraint of the form $\exists k \in \mathbb{N} : x \in [a + kp, a + kp]$
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$\delta\checkmark$-Representation of Timed Words

- Each integer time stamp i (except 0) is marked by a $\checkmark$. All the events occurring at time point i appear immediately after this $\checkmark$
- Entering an interval $(i, i+1)$ is marked by a δ. All the events occurring in the time interval $(i, i+1)$ appear immediately after this δ
- **The mapping** f: example timed words and their mappings:
  $\rho_1 = \langle (a, 1.2), (b, 3.5), (c, 4), (d, 4.5), (e, 4.5), (f, 5.6), (g, 5.8)) \rangle$
  $f(\rho_1) = \delta\checkmark\delta a\checkmark\delta\checkmark\delta b\checkmark c\delta d\ e\checkmark\delta fg$
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- For every ϵ-TA $\mathcal{A}$, one can effectively construct a finite state automaton $\mathcal{B}$ such that $L(\mathcal{B}) = f(L(\mathcal{A}))$.
- Apply following.

- Then apply region construction

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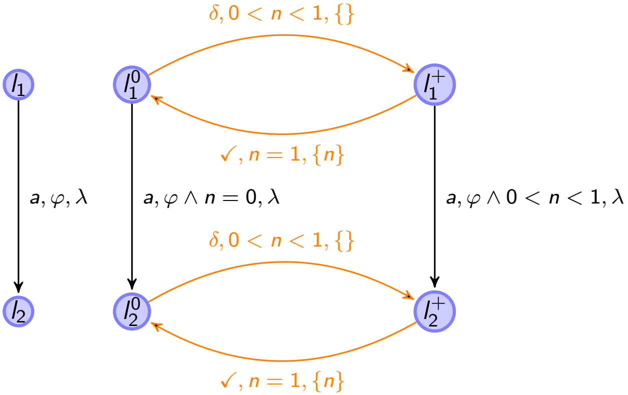

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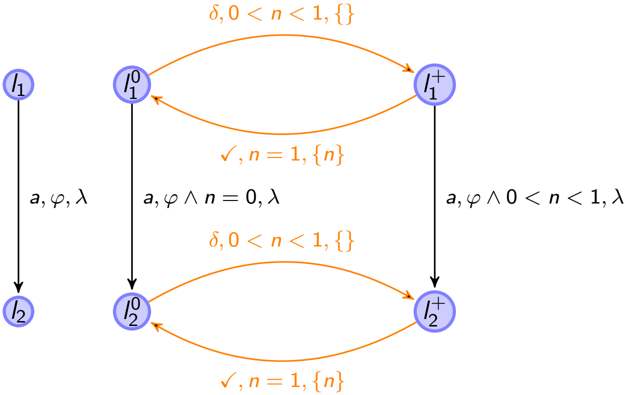

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Key Properties of f

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Language inclusion problem $L(\mathcal{A}) \subseteq L(\mathcal{B})$?, where $\mathcal{A}$ is an ϵ-TA and $\mathcal{B}$ is an ϵ-IRTA, can be decided (EXPSPACE) in the $\delta\checkmark$-world [SPKM08]

- If $\mathcal{A}$ is an ϵ-IRTA and $f(\rho) = f(\rho')$ then $\rho \in L(\mathcal{A})$ iff $\rho' \in L(\mathcal{A})$ [SPKM08]
- If $\mathcal{A}$ is an...
  - ϵ-TA: $f^{-1}(f(L(\mathcal{A}))) \supseteq L(\mathcal{A})$
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- $\delta\checkmark$-Regular Language: a regular subset of $(\Sigma^* \parallel ((\delta\checkmark)^* + \delta(\checkmark\delta)^*)).\Sigma$
- In a $\delta\checkmark$-word, when labels from Σ are ignored...
  - Pattern if nonempty starts with a δ
  - δ's and $\checkmark$'s strictly alternate
- $\delta\checkmark$-Automaton: Deterministic finite state automaton accepting a $\delta\checkmark$-regular language
- Questions: Where do we end up if we go back to the timed world? How about ϵ-freeness?

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- Cannot go from $\delta\checkmark$-automata to Per-IRTA: Per-IRTA $\subsetneq$ ϵ-IRTA
- The following is an ϵ-IRTA

  Language accepted $L = \{\langle(a, \tau_1), (b, \tau_2)\rangle : \tau_1$ and τ_2 are non-integral, and are separated by at least one integer$\}$
- There exists no Per-IRTA accepting L:
  - Say there is a Per-IRTA $\mathcal{A}$ for L and k is the max. constant
  - Let $k < k_1 < k_2$. Therefore $\langle(a, k_1 + .5), (b, k_2 + .5)\rangle$ has an accepting run over $\mathcal{A}$
  - Since no clock was reset on the a-transition, k_1 and k_2 can be chose such that $\langle(a, k_2 + .5), (b, k_2 + .5)\rangle$ also has an accepting run via the same edges in $\mathcal{A}$

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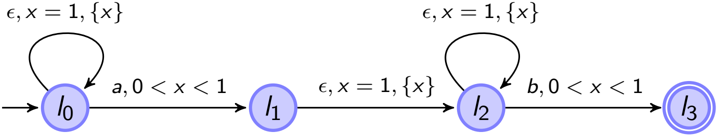

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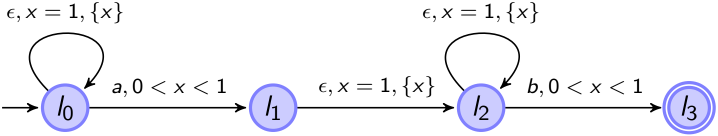

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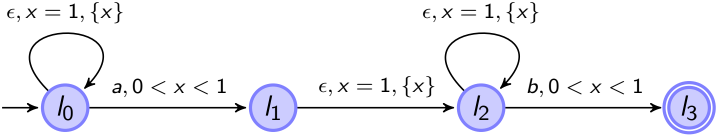

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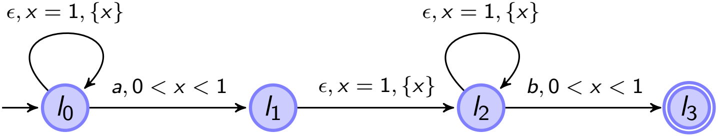

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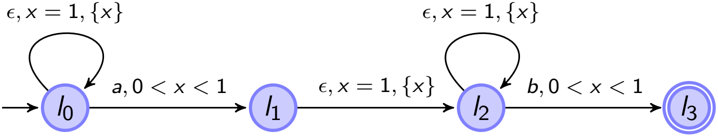

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- Per-GRTA: GRTA wherein the atomic constraints can be periodic. We will translate $\delta\checkmark$-automata to Per-GRTA.

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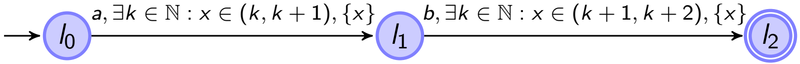
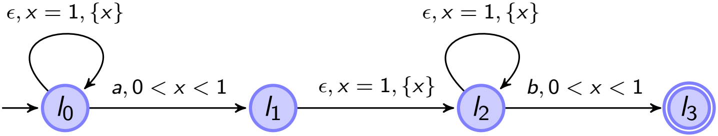


Construction by Example

- ϵ-IRTA



- $\delta\checkmark$-Automaton (equivalent to the one obtained by our construction)

- The idea is to encode the time distances represented by $\delta\checkmark$-paths into clock constraints. Deterministic, 1-Clock Per-GRTA

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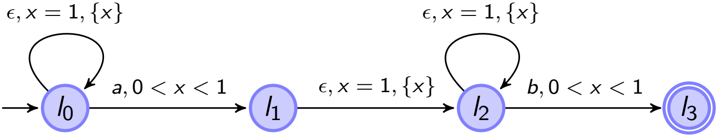

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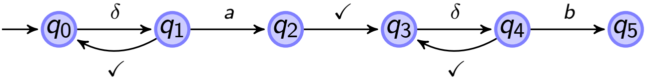

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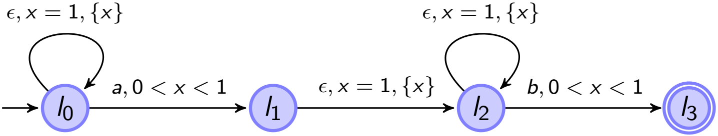

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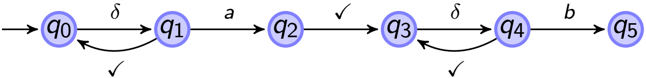

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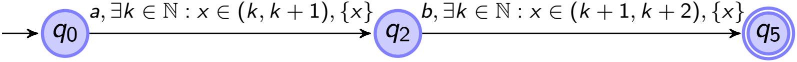

Hence. . .

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For every ϵ-IRTA $\mathcal{A}$ we can effectively construct a language equivalent 1-clock, deterministic Per-GRTA $\mathcal{B}$ with at most double exponential blowup in number of locations

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  - Replace each δ-transition by an ϵ-transition with constraint $0 < x < 1$
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Outline

Relative Expressive Power

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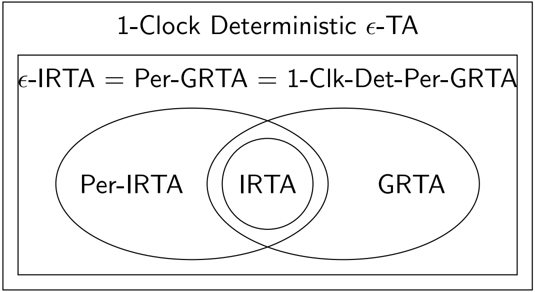

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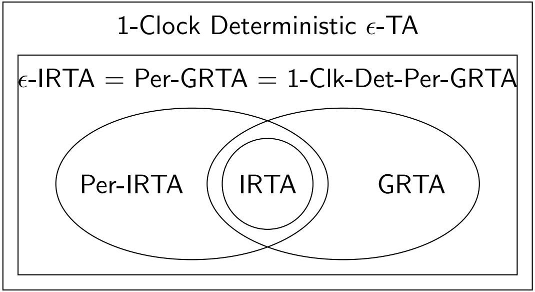

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  - $L(\mathcal{A}) \subseteq L(\mathcal{B})$ is decidable if $\mathcal{B}$ is 1C-TA [OW04]
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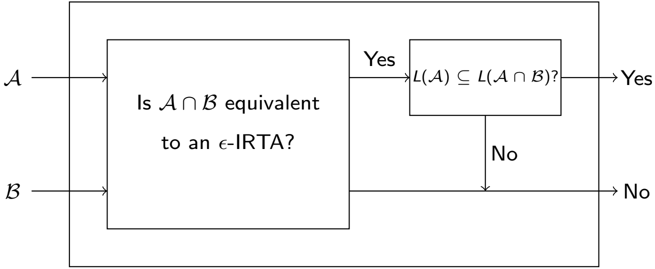

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Thank You

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