

Analyzing time constrained MSGs

K Narayan Kumar

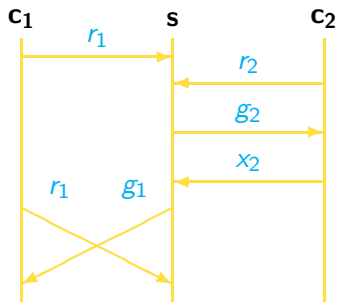
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Joint work with P Gastin, Madhavan Mukund

Chennai, 30 January 2009

Message Sequence Charts

Two clients and a server



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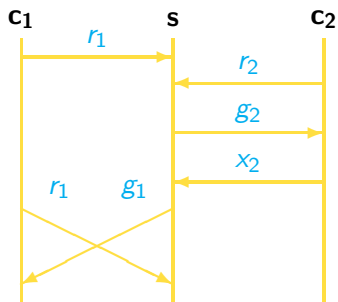
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Unfortunately, most of the results are negative.

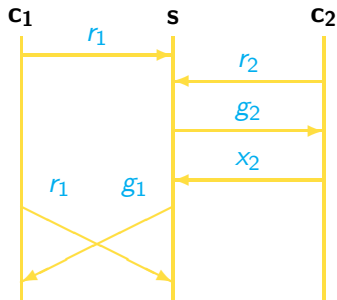
MSCs

Two clients and a server

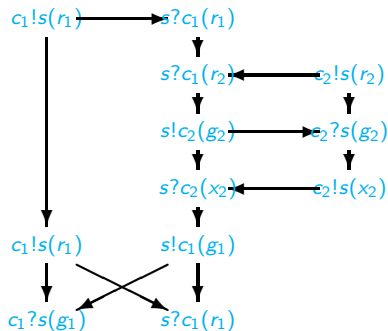


MSCs as Partial Orders

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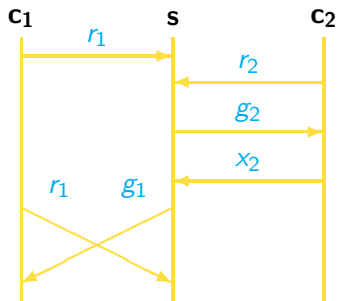


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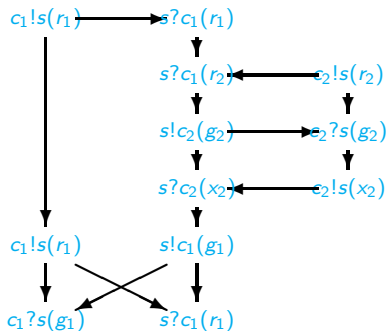


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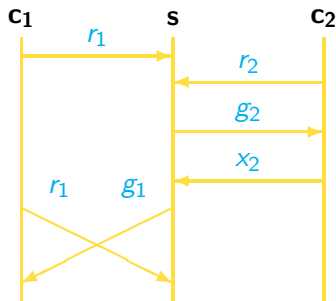
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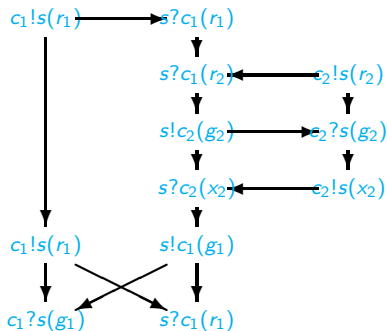
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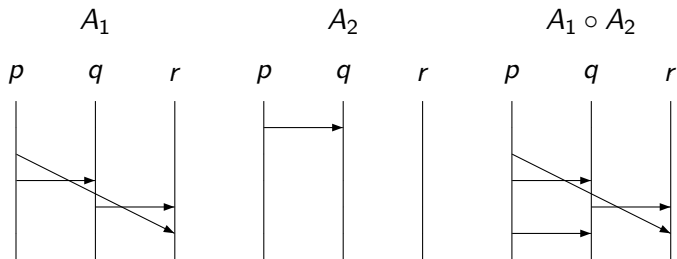


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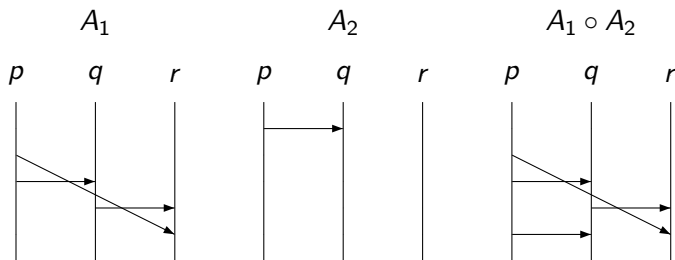


- ▶ All channels are assumed to be FIFO.
- ▶ An MSC can be regenerated from any one sequentialization.

Concatenation of MSCs



Concatenation of MSCs



$p!r \ p!q \ q?p \ q!r \ r?q \ p!q \ q?p \ r?p$

is a sequentialization of $A_1 \circ A_2$.

Message Sequence Graphs

- ▶ A finite state automaton

Message Sequence Graphs

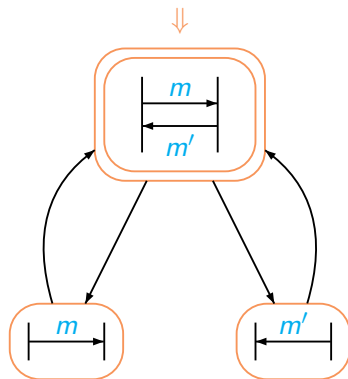
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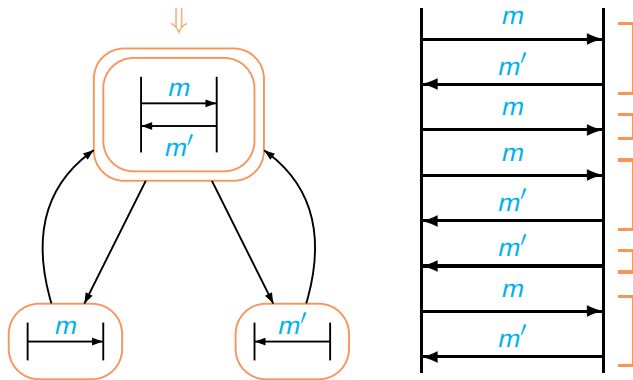
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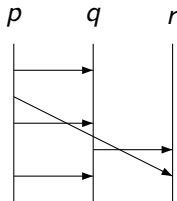


Boundedness

A sequentialization of an MSC is B -bounded if no channel has more than B messages at any point.

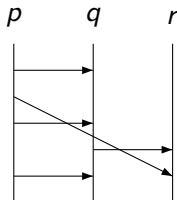
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The linearization

$p!q \ q?p \ p!r \ p!q \ q?p \ q!r \ r?q \ p!q \ q?p \ r?p$

is 1-bounded while the linearization

$p!q \ p!r \ p!q \ p!q \ q?p \ q?p \ p!r \ q?p \ r?q \ r?p$

is 3-bounded.

Boundedness ...

An MSC is **existentially** B -bounded if one of its sequentializations is B -bounded.

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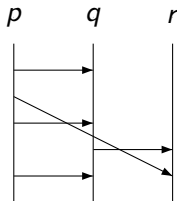
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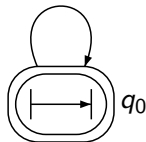
An existentially 1-bounded and universally 3-bounded MSC.

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An MSG is **existentially B -bounded** if every MSC it generates is existentially B -bounded.

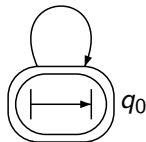
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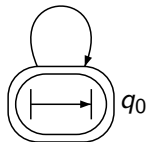
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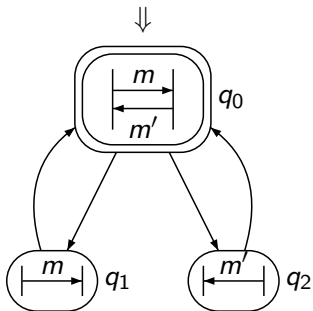
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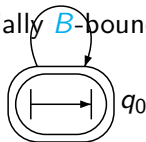


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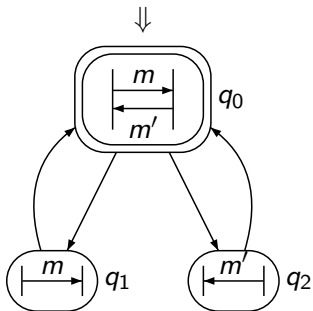


Boundedness ...

An MSG is **existentially bounded** if there exists a B such that every MSC it generates is existentially B -bounded.



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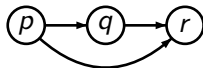
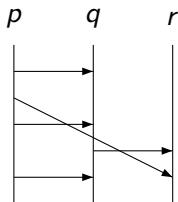
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- ▶ Checking whether an MSG is bounded is decidable.

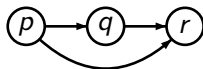
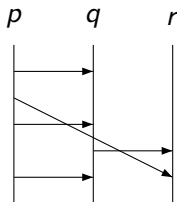
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An MSG is bounded if and only if every the MSC generated by every loop has a communication graph that is a disjoint union of SCCs.

Adding time to scenarios

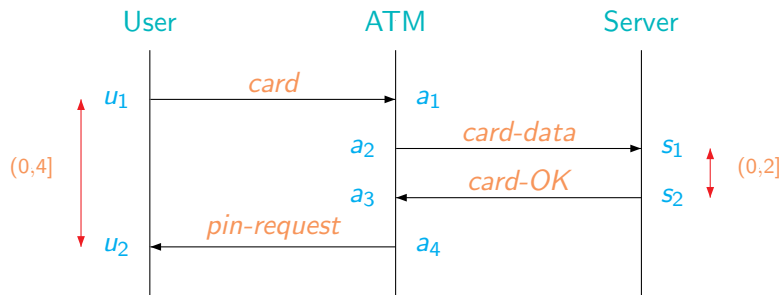
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- ▶ Time constrained Message Sequence Graphs
 - ▶ Generate infinite families of time constrained MSCs

MSCs with time constraints



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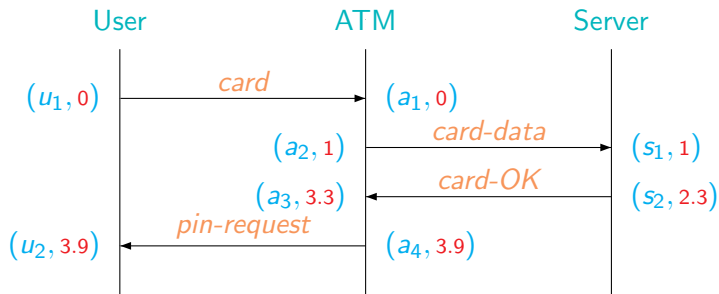
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 - ▶ e is $p!q(m)$ and e' is corresponding receive $q?p(m)$

A timed behaviour



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- ▶ TC-MSC $T \Rightarrow L(T)$, set of timed MSCs that cover T

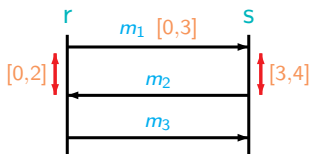
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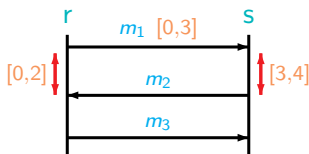
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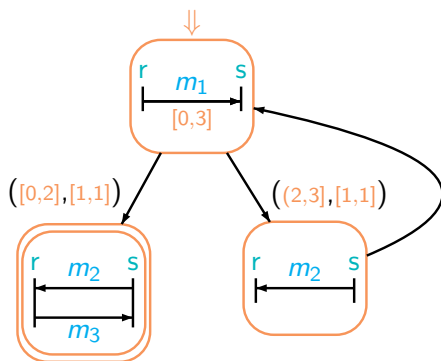
TC-MSCs and Timed MSCs

- ▶ The set of timed MSCs covering a TC-MSC may be empty.
- ▶ A TC-MSC is said to be **realizable** if it is covered by at least one timed MSC.



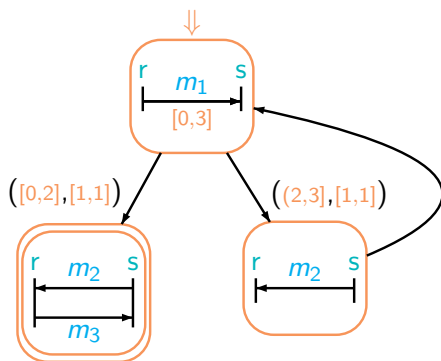
Time Constrained Message Sequence Graphs

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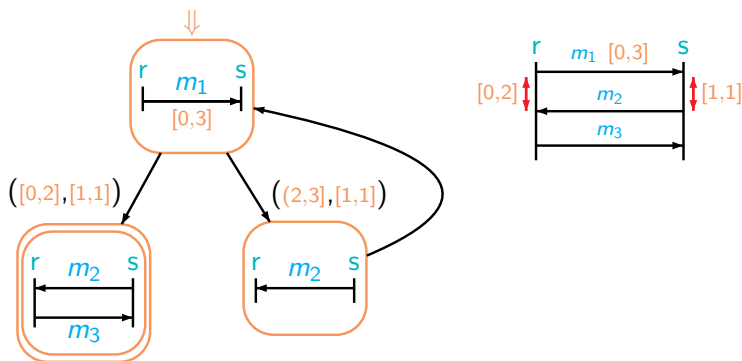
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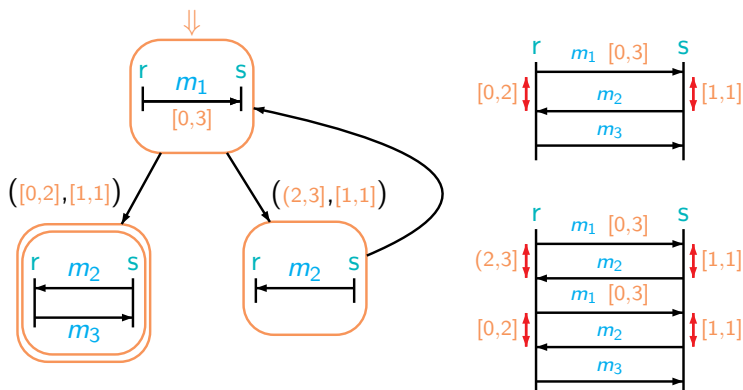
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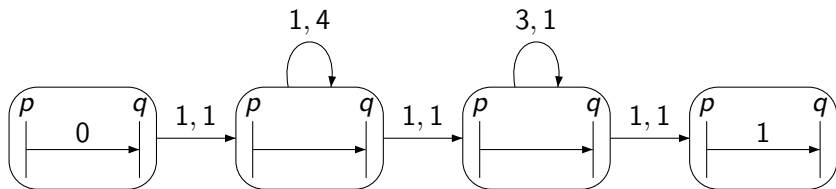
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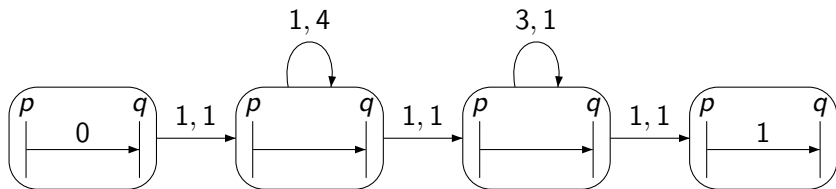
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This problem is trivial for ordinary MSGs.

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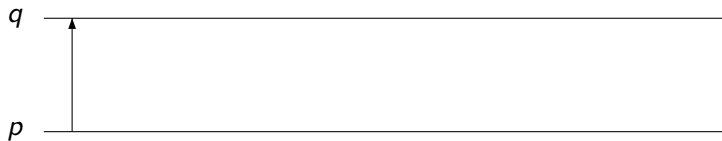
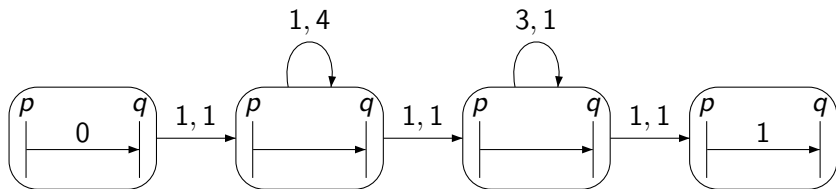
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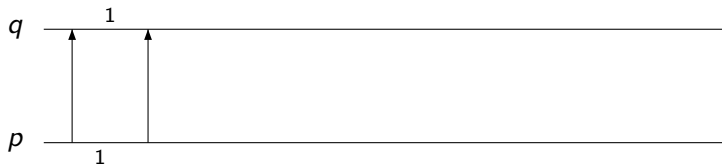
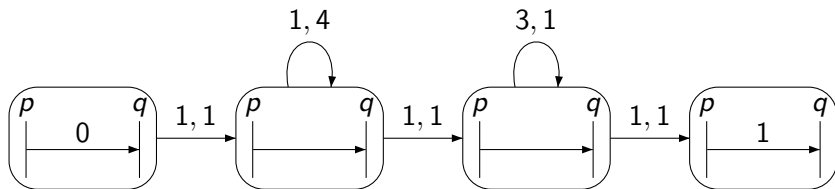
q _____

p _____

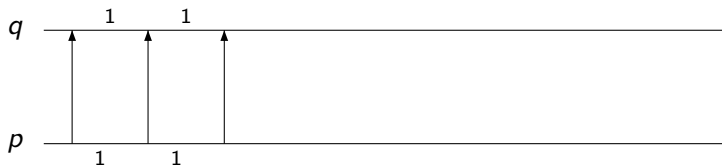
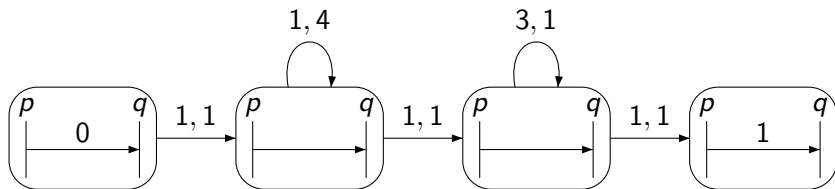
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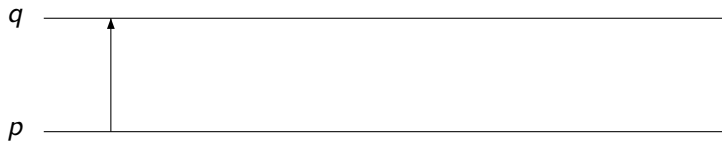
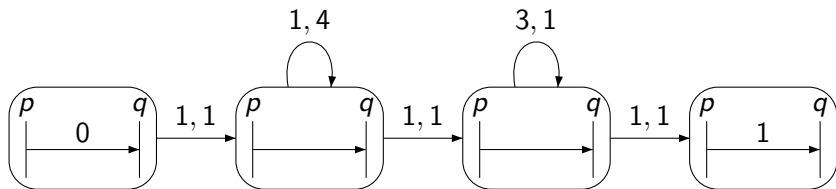
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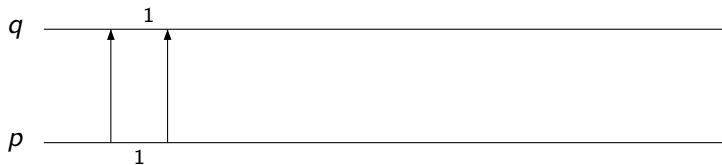
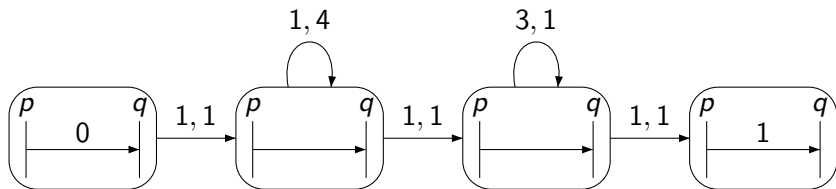
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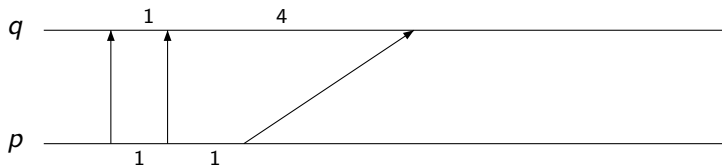
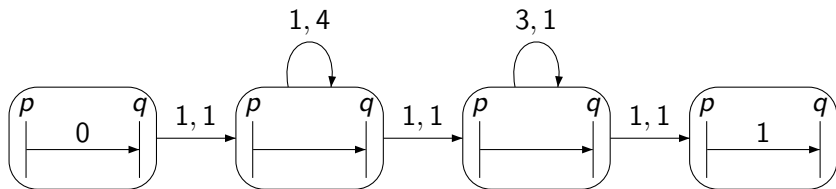
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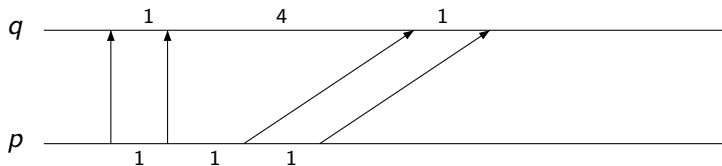
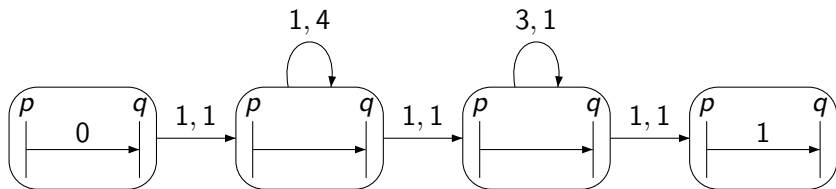
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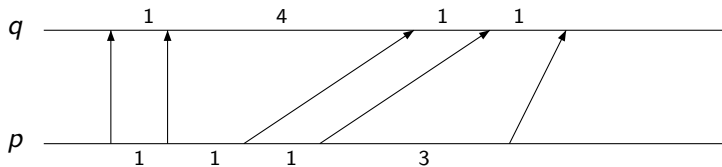
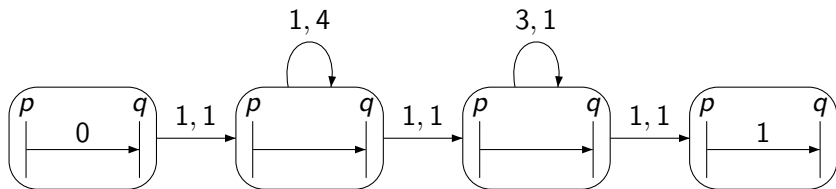
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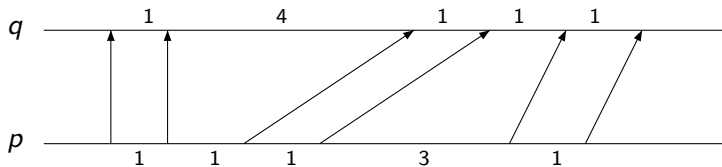
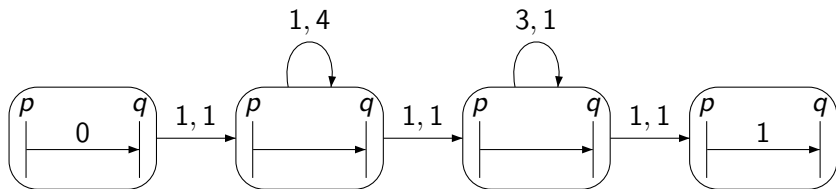
Reachability ...



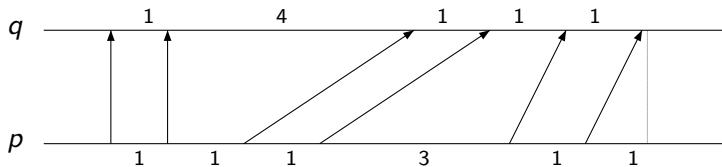
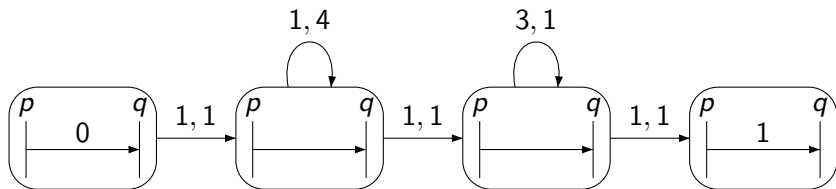
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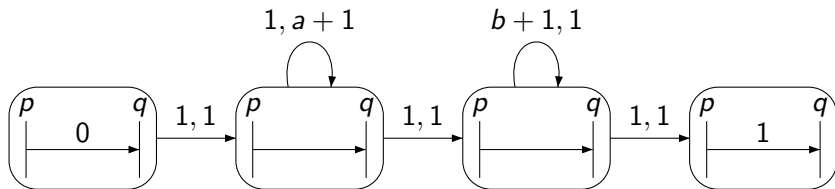
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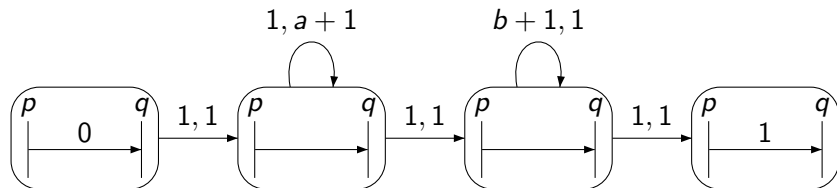
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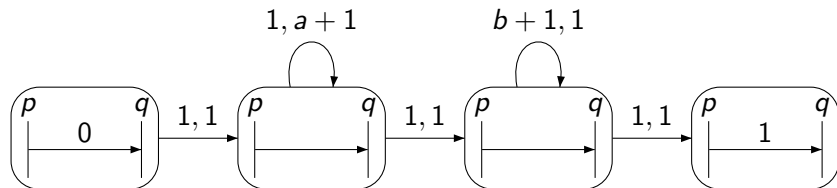


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- ▶ The first loop is to be executed k times and the second one l times such that $a.k - b.l = 1$.
- ▶ Simple paths may not be realizable while those with loops may be.

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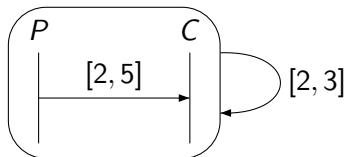
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Boundedness ...

Time constraints may ensure boundedness.

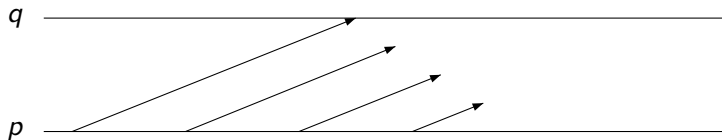
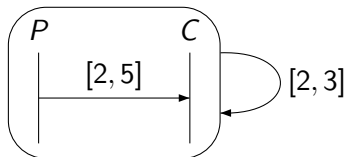
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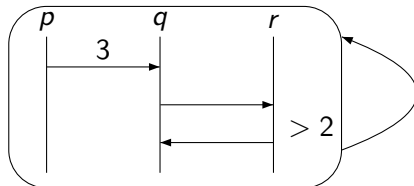


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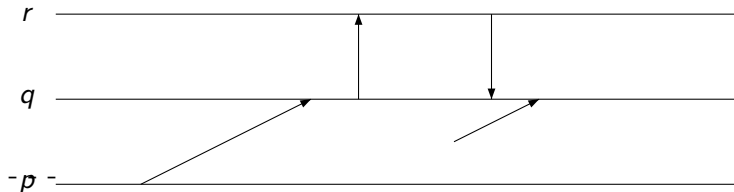
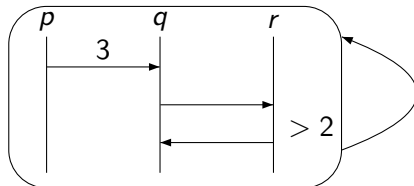
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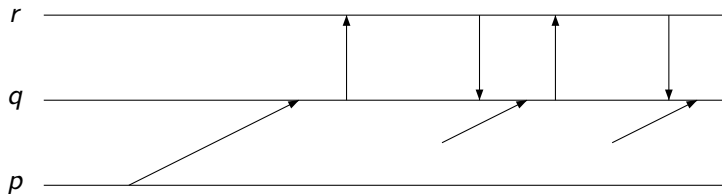
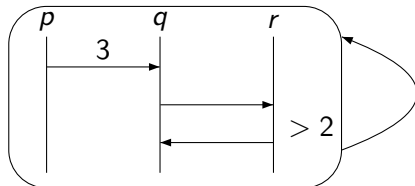
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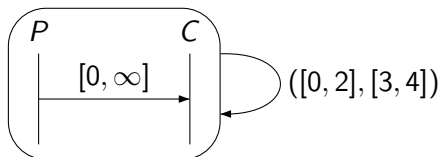
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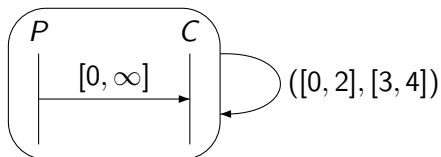
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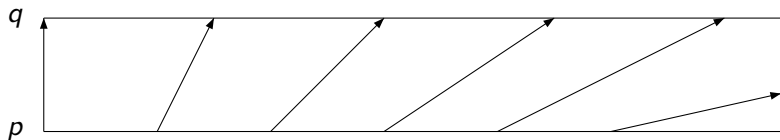
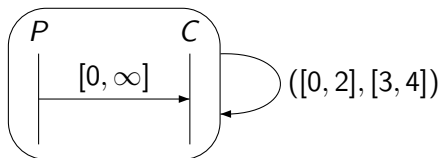


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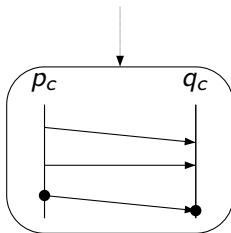
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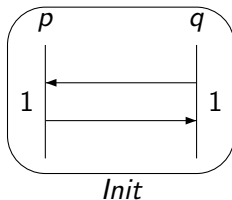
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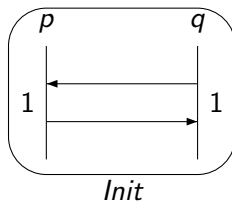
The reduction

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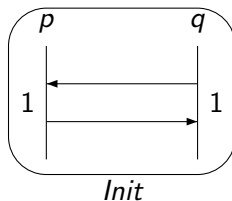
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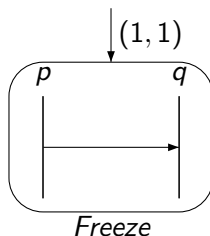
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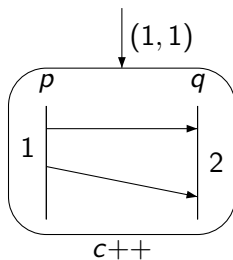


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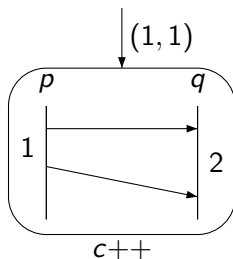
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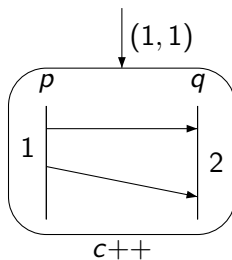
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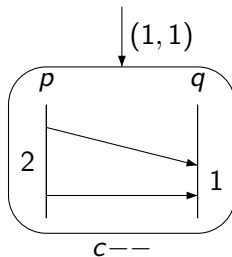
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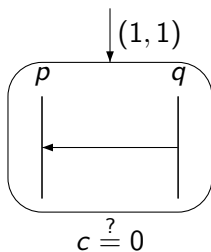


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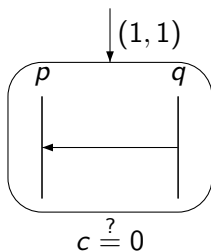
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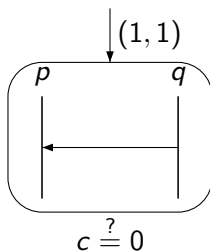
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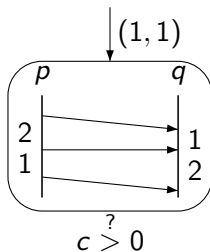
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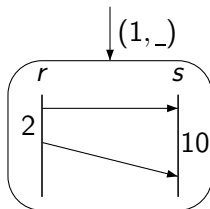
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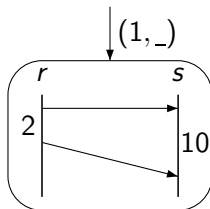
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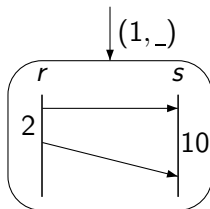
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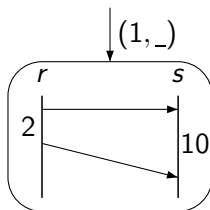
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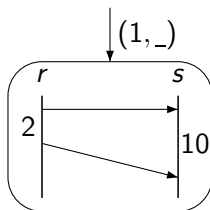
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Checking boundedness for TC-MSGs is undecidable

More Undecidability – 1

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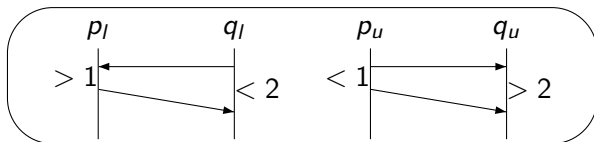
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- ▶ The value of $p_u - q_u$ is used to check for 0.

Open Intervals ...

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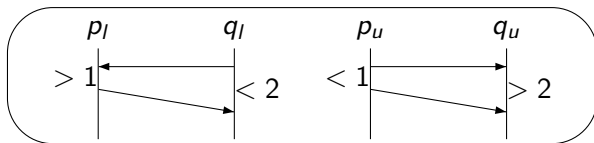
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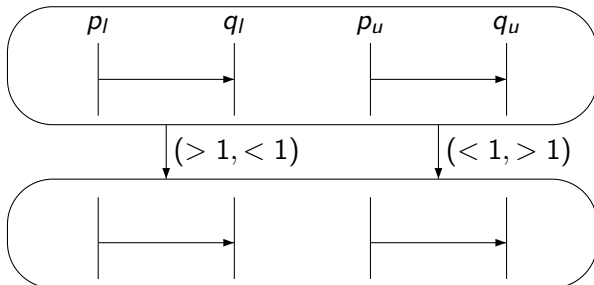


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Composition between Nodes

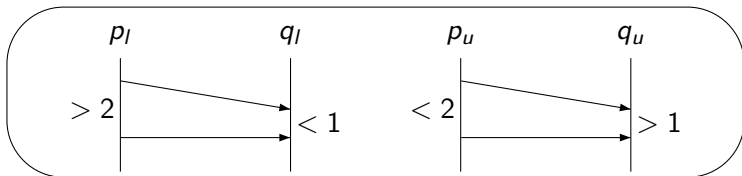


Open intervals ...

The decrement instruction

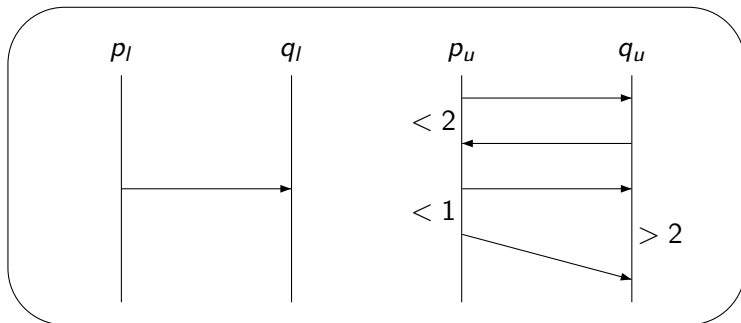
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More Undecidability – 2

What about the reachability problem for channel bounded TC-MSGs?

More Undecidability – 2

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The reachability problem for channel bounded TC-MSGs is also undecidable.

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Even with the restriction that constraints across nodes are permitted only on a fixed process, the reachability and boundedness problems for TC-MSGs remain undecidable.

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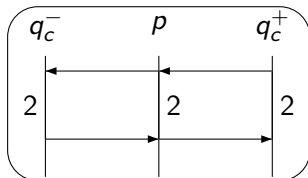
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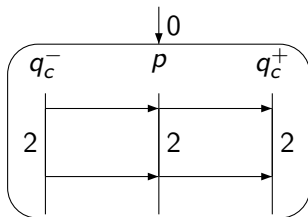
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More Undecidability - 3

Initialize

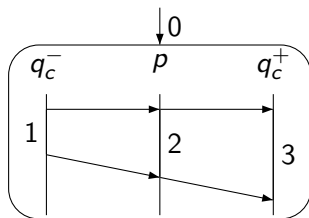


Freeze

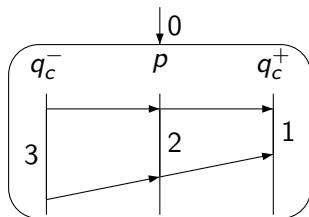


More Undecidability – 3

Increment

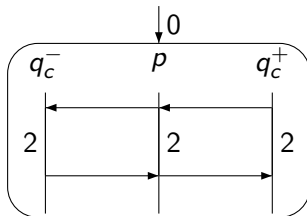


Decrement



More Undecidability – 3

Check for Zero



Locally synchronized MSGs

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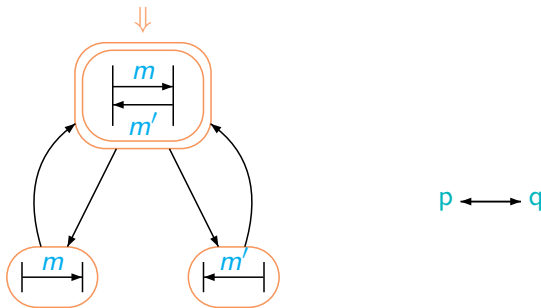
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 - ▶ Works like an event-clock automaton (upto some extra labelling).

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