Some remarks on the control of distributed automata

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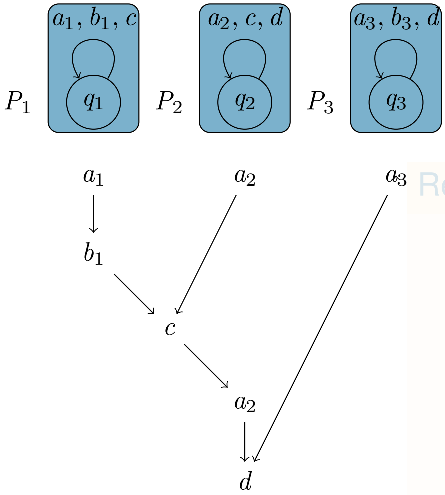
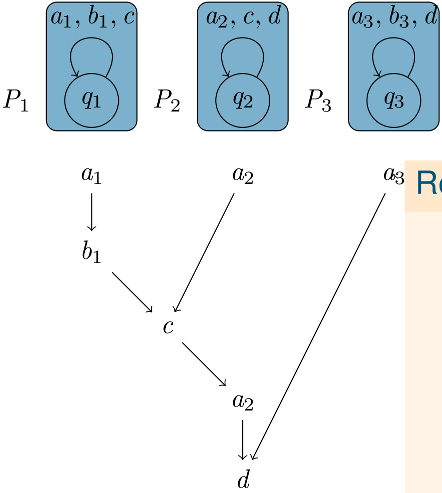
Asynchronous (Z-)automata, traces and event structures informally



Representing executions

- As a word:
  $a_1 a_2 a_3 b_1 c a_2 d$ or $a_2 a_3 a_1 b_1 c a_2 d$
- As a trace.
- The set of all executions can be represented as a tree,
- or as an *event structure* (richer: concurrency).

Asynchronous (Z-)automata, traces and event structures informally



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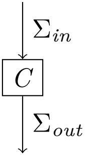
The synthesis problem



$K \subseteq (\Sigma_{in}\Sigma_{out})^*$

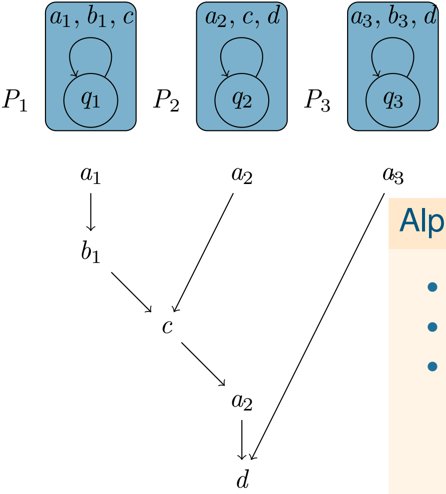
Centralized synthesis

- We are given a specification K.
- We want a finite automaton C with
  $$L(C) \subseteq K$$
  + additional requirements (e.g., inputs are unconstrained).

Distributed synthesis

- Comes along with a distributed architecture (e.g., distributed (trace) alphabet).
- In general undecidable (Peterson/Reif '79, Pnueli/Rosner 90).
- Important: use adequate specifications (e.g. trace closed ones for asynchronous automata).

Asynchronous automaton: example

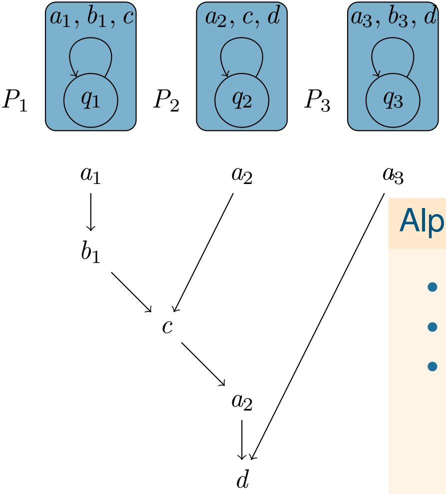


Alphabet

- $\mathbb{P}$: finite set of processes.
- Σ: finite set of letters.
- $loc : \Sigma \to (2^{\mathbb{P}} \setminus \emptyset)$: distribution of letters over processes.

$$loc(a_1) = \{P_1\},\ loc(c) = \{P_1, P_2\}, \dots$$

Asynchronous automaton: example



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Asynchronous automata formally

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A (deterministic) *asynchronous automaton*

$$\mathcal{A} = \langle \{S_p\}_{p\in\mathbb{P}}, s_{in}, \{\delta_a\}_{a\in\Sigma} \rangle$$

- S_p states of process p
- $s_{in} \in \prod_{p\in\mathbb{P}} S_p$ is a (global) initial state,
- $\delta_a : \prod_{p\in loc(a)} S_p \dot{\to} \prod_{p\in loc(a)} S_p$ is a transition relation.

Language of an asynchronous automaton

The language of the automaton

The (regular) language of the product automaton.

Independence/Dependence

- Function $loc : \Sigma \to (2^{\mathbb{P}} \setminus \emptyset)$ implies an independence relation on letters:

$$(a, b) \in I \quad \text{iff} \quad loc(a) \cap loc(b) = \emptyset$$

- So the language is *closed* under commuting independent letters (trace closed):

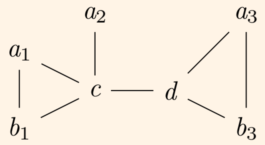
$$vabw \in L(\mathcal{A}) \quad \text{implies} \quad vbaw \in L(\mathcal{A})$$

- Dependence relation $D = (\Sigma \times \Sigma) \setminus I$. We will express it graphically:

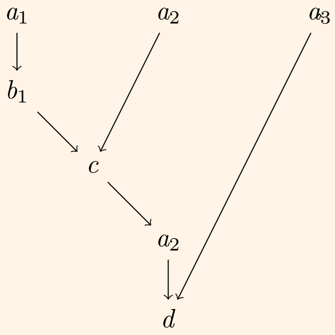
$$a - c - b$$

Traces: an example

Dependence relation



A trace



Structure on traces

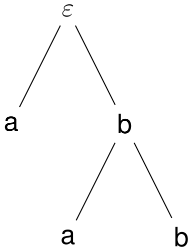
Prefix relation on traces

- The prefix relation on traces $\sqsubseteq$ is defined similarly as for words.
- Differently from words, a trace may have two prefixes that are themselves $\sqsubseteq$-incomparable.

$$t_1, t_2 \sqsubseteq t \text{ but } \quad t_1 \not\sqsubseteq t_2 \text{ and } t_1 \not\sqsubseteq t_2$$

  For example: a and b are both prefixes of abc when $(a, b) \in I$.
- We write $t_1 \# t_2$ if the two traces do not have a common extension.
  For example: $ac \# aac$ when $(a, c) \notin I$.

Event structures



From words to trees

A prefix-closed language $L \subseteq \Sigma^*$ defines a Σ-labeled tree:

- nodes are elements of L,
- the tree order is given by the prefix relation $\sqsubset$.
- the label of $w \in L$ is the last letter in L.

From traces to event structures

A prefix-closed language $L \subseteq \mathrm{Tr}(\Sigma)$ defines a Σ-labeled event structure:

- nodes are prime traces from L.
- the partial order is given by the prefix relation $\sqsubset$.
- relation $\#$ is called conflict relation.
- the label of t is the label of the maximal element of t.

$ES(\mathcal{A})$

We denote by $ES(\mathcal{A})$ the (trace) event structure of the language $L(\mathcal{A})$.

Event structures: examples

From traces to event structures

A prefix-closed language $L \subseteq \mathrm{Tr}(\Sigma)$ defines a Σ-labeled event structure:

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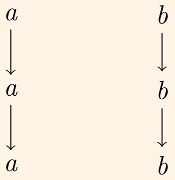
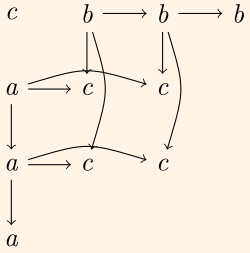
$\Sigma = \{a, b\}$, independent



$\Sigma = \{a, b, c\}$, $D:\ a - c - b$



Specifying event structures

Logics for event structures

First-order logic (FOL) over the signature $\leq$, $\#$, P_a for $a \in \Sigma$:

$$x \leq x' \mid x\#x' \mid P_a(x) \mid \neg\varphi \mid \varphi \vee \psi \mid \exists x.\varphi(x).$$

Monadic second-order logic (MSOL)

$$\ldots x \in Z \mid \exists Z.\varphi(Z).$$

Monadic trace logic (MTL): quantification restricted to conflict free sets.

Theorem (Madhusudan)

The problem if a given formula holds in a given trace event structure is decidable for FOL and MTL.

Remark

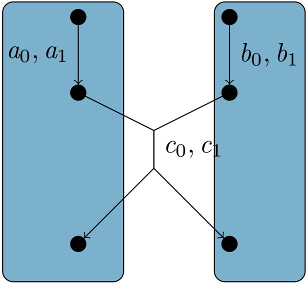
There are trace event structures with undecidable MSOL theory (grid).

Part 1

Controlling asynchronous automata

- Process and action-based control.
- Reduction from process-based to action-based control.
- Encoding into MSOL theory of event structures.

Controlling an asynchronous automaton: an example



Example specifications

1. $a_i b_j c_k$ with $k = i$.
2. $a_i b_j c_k$ with $k = i \cdot j$.

Two methods of control

- Process-based [Madhusudan et al.]: Process decides what actions it can do.
- Action-based [Gastin et al.]: Actions decide whether they can execute.

Process-based control

Plant over $\mathbb{P}$, $loc : \Sigma \to (2^{\mathbb{P}} \setminus \emptyset)$ and $\Sigma = \Sigma^{sys} \cup \Sigma^{env}$

A deterministic asynchronous automaton.

Views for a process $p \in \mathbb{P}$

- Let $view_p(t)$ be the smallest prefix of t containing all p-actions.
- Let $Plays_p(\mathcal{A}) = \{view_p(t) : t \in L(\mathcal{A})\}$.

Strategy

- A strategy is a tuple of functions $f_p : Plays_p(\mathcal{A}) \to 2^{\Sigma^{sys}}$ for $p \in \mathbb{P}$.
- Plays respecting $\sigma = \{f_p\}_{p \in \mathbb{P}}$. Assume $u \in Plays(\mathcal{A}, \sigma)$.
  - if $a \in \Sigma^{env}$ and $ua \in Plays(\mathcal{A})$ then ua is in $Plays(\mathcal{A}, \sigma)$.
  - if $a \in \Sigma^{sys}$ and $ua \in Plays(\mathcal{A})$ then $ua \in Plays(\mathcal{A}, \sigma)$ provided that $a \in f_p(view_p(u))$ for all $p \in loc(a)$.

Process-based control

Requirements

- We are given asynchronous automaton $\mathcal{A}$ and a regular trace language K.
- A strategy $\sigma = \{f_p\}_{p \in \mathbb{P}}$ gives us a set of traces $Plays^{\omega}(\mathcal{A}, \sigma)$.
- A strategy is non-blocking if every trace in $Plays(\mathcal{A}, \sigma)$ that has an extension in $Plays(\mathcal{A})$, also has an extension in $Plays(\mathcal{A}, \sigma)$.

The control problem

Given $\mathcal{A}$ and K, decide if there is a non-blocking strategy σ such that $Plays^{\omega}(\mathcal{A}, \sigma) \subseteq K$.

Action-based control

Process based	Action based
$view_p(t)$	$view_a(t) = \bigcup\{view_p(t) : p \in loc(a)\}$
$Plays_p(\mathcal{A})$	$Plays_a(\mathcal{A}) = \{view_a(t) : t \in L(\mathcal{A})\}$
$f_p : Plays_p(\mathcal{A}) \to 2^{\Sigma^{sys}}$	$g_a : Plays_a(\mathcal{A}) \to \{tt, ff\}$
$\sigma = \{f_p\}_{p\in\mathbb{P}}$	$\rho = \{g_a\}_{a\in\Sigma^{sys}}$

$Plays^\omega(\mathcal{A}, \rho)$

- if $a \in \Sigma^{env}$ and $ua \in Plays(\mathcal{A})$ then ua is in $Plays(\mathcal{A}, \rho)$.
- if $a \in \Sigma^{sys}$ and $ua \in Plays(\mathcal{A})$ then $ua \in Plays(\mathcal{A}, \rho)$ provided that $g_a(view_a(u)) = tt$.

Reduction “process-based” to “action-based”

Observation 1

If there is a process-based controller then there is an action-based controller.

Observation 2

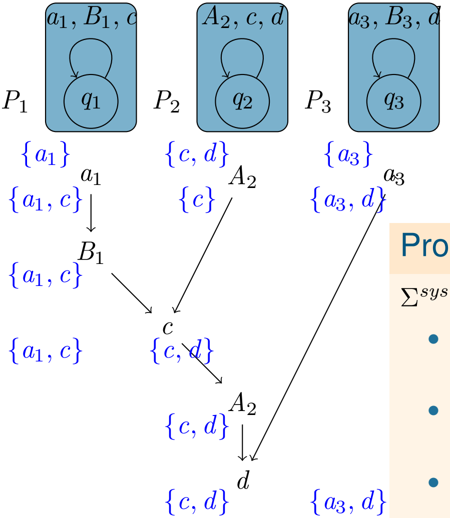
This does not in principle imply that process-based control is easier than action-based control (nor vice-versa).

Fact

For every asynchronous automaton $\mathcal{A}$ and MSOL specification α, one can construct $\overline{\mathcal{A}}$ and $\overline{\alpha}$ such that:

process-based controller for $(\mathcal{A}, \alpha)$ exists
iff
action-based controller for $(\overline{\mathcal{A}}, \overline{\alpha})$ exists.

Reduction: example

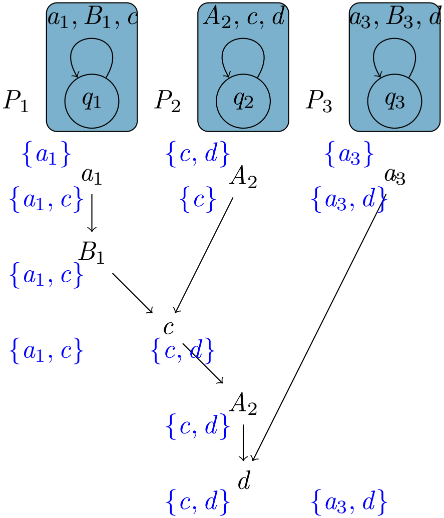


Process-based strategy

$\Sigma^{sys} = \{a_1, a_3, c, d\}$

- P_1: a_1 always possible, c only after a_1
- P_2: c always possible, d after c or if no A_2 before
- P_3: a_3 always possible, d only after a_3

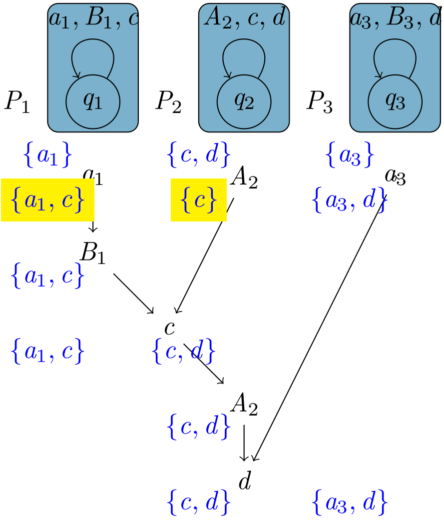
Reduction: example (cont.)



New arena for action-based strategy

- New (local) system actions: $\top, \{a_1\}, \{a_1, c\}, \{c, d\}, \{c\}, \ldots$
- New (local) environment actions: $\bot, (a, P_1), (c, P_1), (d, P_2), \ldots$
- $\top$ winning and $\bot$ losing (for system)

Reduction: example (cont.)



New arena for action-based strategy

- System proposes its set of local actions in form of new actions (process-wise), e.g. $\{a_1, c\}, \{c\}$. If proposed sets have empty $\cap$ (although actions are possible) then $\perp$ is possible.
- Environment chooses one of the proposed actions (process-wise). If it chooses maliciously (e.g. $(a_1, P_1), (c, P_2)$) then $\top$ is possible.

Encoding process-based control

MSOL encoding (Madhusudan et al.)

For a MSOL specification α there is a MSOL formula φ_α such that $ES(\mathcal{A}) \vDash \varphi_\alpha$ iff the process-based control problem for $(\mathcal{A}, \alpha)$ has a solution.

Remark

The same can be done for action-based control.

Writing the formula φ_α

Encoding strategies

- Take $\sigma = \{f_p\}_{p\in\mathbb{P}}$ where each $f_p : Plays_p(\mathcal{A}) \to 2^{\Sigma^{sys}}$.
- Encode σ with the help of variables Z_p^a for $a \in \Sigma^{sys}$ and $p \in \mathbb{P}$.

$$\text{for every } e \in ES(\mathcal{A}) \qquad e \in Z_p^a \quad \text{iff} \quad a \in f_p(e)$$

Encoding action-based control

- Write a formula $\pi(X, Z_p^a, \dots)$ defining $Plays(\mathcal{A}, \sigma)$.
- Write a formula $\pi^\omega(X, Z_p^a, \dots)$ defining $Plays^\omega(\mathcal{A}, \sigma)$.
- Say that all paths in $Plays^\omega(\mathcal{A}, \rho)$ satisfy the specification: $\forall X.\, \pi^\omega(X, Z_p^a, \dots) \Rightarrow \alpha(X)$.
- The required formula is: $\exists Z_p^a \dots \forall X.\, \pi^\omega(X, Z_p^a, \dots) \Rightarrow \alpha(X)$.

Decidability of MSOL is not necessary

Definition

A trace alphabet is a co-graph if it does not contain the induced subgraph $a - b - c - d$.

Theorem (Gastin, Lerman, Zeitoun)

The action-based control problem is decidable for automata over co-graph trace alphabets.

Remark

Alphabet $\Sigma = \{a, b, c\}$ with $a - c - b$ is a co-graph. There is $\mathcal{A}$ over this alphabet whose $ES(\mathcal{A})$ has undecidable MSOL theory.

Part 2

MSOL and Thiagarajan's conjecture

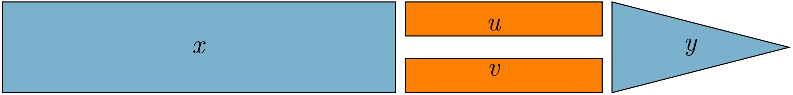
- Thiagarajan's conjecture
- Co-graph dependence alphabets

(Latest?) Thiagarajan's conjecture

Synchronizing automata

An automaton $\mathcal{A}$ is not synchronizing if there are traces x, u, v, y such that

- u, v are nonempty and independent from each other.
- $xuvy$ is a prime trace.
- $xu^*v^*y \subseteq L(\mathcal{A})$.



Remark

If $\mathcal{A}$ is not synchronizing then $ES(\mathcal{A})$ has undecidable MSOL theory.

Conjecture

If $\mathcal{A}$ is synchronizing then the MSOL theory of $ES(\mathcal{A})$ is decidable.

Strongly strongly-synchronizing automata

Strongly synchronizing automaton

An asynchronous automaton $\mathcal{A}$ is strongly synchronizing if in every prime trace of $L(\mathcal{A})$, each of its events has at most $|\mathcal{A}|$ many concurrent events.

Theorem (Madhusudan, Thiagarajan, Yang)

If $\mathcal{A}$ is strongly synchronizing then the MSOL theory of $ES(\mathcal{A})$ is decidable.

Corollary

Both process- and action-based control are decidable for strongly synchronizing automata.

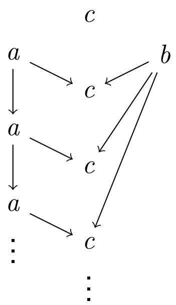
Strongly synchronizing are too strong

Remark

There are automata $\mathcal{A}$ that are not strongly synchronizing but still MSOL theory of $ES(\mathcal{A})$ is decidable.

Example: $\Sigma = \{a, b, c\}$, $D : \ a - c - b$, $L(\mathcal{A}) = a^* b a^* c + c$

- This event structure is not strongly synchronizing
- It has decidable MSOL theory.



- Encode prime trace $[a^m bc]$ by the *word* $a^m bc$, etc.
- Translate MSOL over event structure into MSOL over $\{a, b, c\}$-tree.
- Ex: partial order $[a^n] < [a^n bc]$ translates to $a^n < a^n bc$ (word prefix).
- Rem: encoding $[a^n bc]$ by $ba^n c$ does not work, since a^n and ba^n far apart in the tree.

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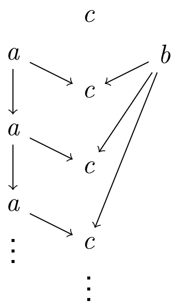

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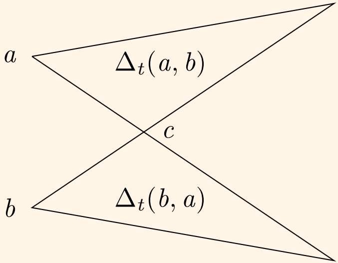
Towards a solution: co-graphs

Trace normal form

We look for trace normal forms $nf(t)$ that behave well w.r.t. prefix relation:

for all traces $t < t'$ there are words p, s, s' s.t.
$nf(t') = ps'$, $nf(t) = ps$ and s is small

Co-graphs



- In trace t: $a \parallel b$ and $a\uparrow \cap\, b\uparrow \neq \emptyset$.
- For co-graphs (and $\mathcal{A}$ sychronizing) there is no extension t' of t such that $\Delta_{t'}(a, b) > \Delta_t(a, b)$ *or* $\Delta_{t'}(b, a) > \Delta_t(b, a)$.

Normal form

Dynamical lexicographic form

- Enforce more order to the trace partial order:

  For $a \parallel b$ *let* $a \prec b$ *if* $|\Delta_t(a,b)| > N$, $N = |\mathcal{A}|$.

- Trace partial order t plus $\prec$ is acyclic: $t_\prec$.
- The priority normal form is the lexicographic normal form of $t_\prec$.
- If $\mathcal{A}$ is strongly synchronizing then it coincides with the lexicographic normal form.

Priority normal form and reduction

- Priority normal form has the desired property:

  for all traces $t < t'$ there are words p, s, s' s.t.

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- Reduction of MSOL over $ES(\mathcal{A})$ to MSOL over Σ-tree works by identifying t with word p in the tree and checking that small s fits correctly into s'.

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Conclusions

- While traces are relatively well understood, event structures are much less.
- From the synthesis point of view, event structures are more fundamental than traces.
- Thiagarajan's conjecture is an important milestone in understanding the decidability frontier.
- Thiagarajan's conjecture is true for co-graphs. The general case remains open.
- It may well be the case that action based control is decidable for all asynchronous automata.