Two-player Reachability Games with Partial Observation on Both Sides

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- Reachability, safety (+Büchi) conditions.
- Determinacy, decidability. With Nathalie Bertrand and Blaise Genest, Rennes.

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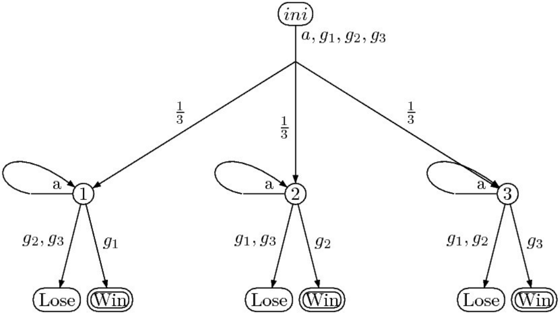
- Goal of player 2: the opposite.



Only one player.

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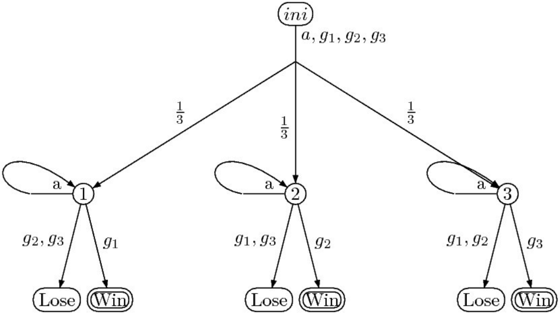

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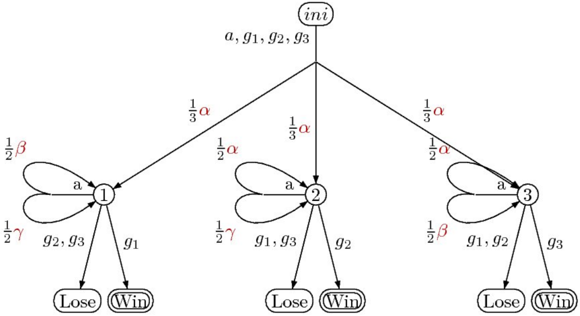
- A play is an infinite sequence of states, actions and signals in $(KIJCD)^\omega$



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- Signals contain actions.

Winning probability

- Initial distribution δ and strategies σ and τ. Markov chain. Probability measure $\mathbb{P}_{\delta}^{\sigma,\tau}(\cdot)$ on the set of infinite plays $(KIJCD)^{\omega}$.

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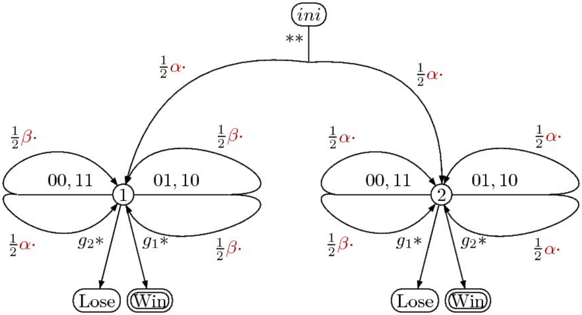
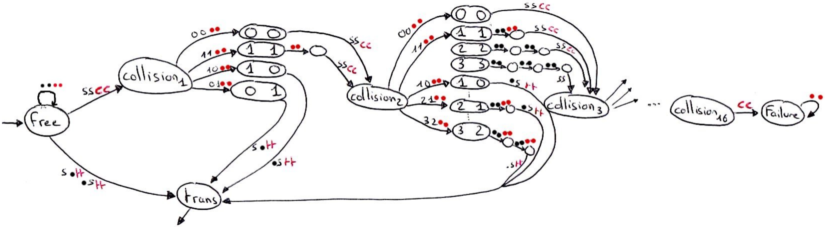
- Player 2 has the opposite goal.



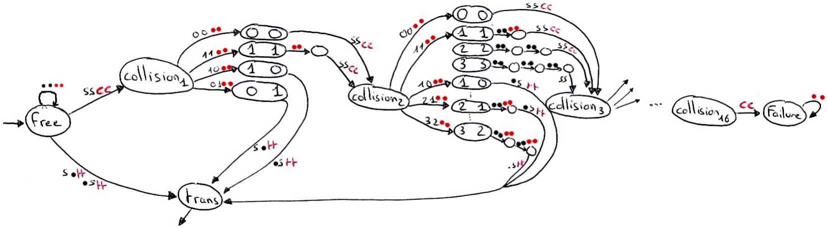
Two players.

- States K = {ini,1,2,Lose,Win}
- Actions $I = \{0, 1, g_1, g_2\}$ and $J = \{0, 1\}$
- Player 2 is blind.
- Signals $C = \{\alpha, \beta, \gamma\}$ and $D = \{\cdot\}$

- collisions occur on ethernet networks,



- collisions occur on ethernet networks,
- controllers choose to stay idle for some randomly chosen delay.



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- Player 2 wins positively: if $\exists \tau \ \forall \sigma \ \mathbb{P}_{\delta}^{\sigma,\tau}(\exists n, K_n \in T) < 1$.
- Randomized strategies are necessary, in practice (ethernet) and in theory (determinacy).

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- Yes, it holds.
- Requires knowledge about almost-surely and positively winning strategies.

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- False for Büchi games.

Values

Definition: a game has a value $0 \leq v \leq 1$, if player 1 can defend probability $v + \epsilon$ and player 2 can secure probability $v - \epsilon$, for every ϵ.

$$\sup_\sigma \inf_\tau \mathbb{P}_\delta^{\sigma,\tau}(\exists n, K_n \in T) = \leq v \leq \inf_\tau \sup_\sigma \mathbb{P}_\delta^{\sigma,\tau}(\exists n, K_n \in T).$$

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- Proposition [Renaut, Sorin, G.]: two players zero-sum reachability games with partial observation have a value.
- Theorem [Paz 71, Bertoni 75]: values are not computable, even for one player and no observations.

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- Existence of almost-surely or positively winning strategies decidable in polynomial time (deAlfaro, Henzinger, 00).
- One player perfectly informed + deterministic transitions: Existence of almost-surely or positively strategies decidable in EXPTIME (Chatterjee, Doyen, Henzinger, Raskin 03).

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- Beliefs of 2 about beliefs of 1. When player 2 receives signal d, she makes hypothesis about signal c received by player 1.
- Very different from perfect-information games, concurrent games, even games with a perfectly informed opponent.

Doubly exponential memory is needed

Proposition: Player 2 needs double-exponential memory for winning positively.

Special case: exponential memory if player 1 perfectly informed, because guessing current state = guessing belief of 1.

Special case: exponential memory if player 2 observes signals of player 1 because player 2 can compute the belief of player 1.

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- Implementable with polynomial number of states.

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- Moreover a counter is used for storing one of the positions m where a "1" was sent.

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- Player 1 may cheat without being caught, but he has positive probability to get caught.

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- A simple counter would use exponentially many states.
- Instead we force player 1 to count himself, by sending to the system the value of the counter, in binary form, using actions in $\{0,1\}^*$.
- The system is able to detect with positive probability whenever player 1 is cheating with the counter.

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Deciding almost-sure, sure and positive winning

Theorem: [Bertrand, Genest, G.] there is an algorithm running in exponential time $(G \cdot 2^K)$ for deciding whether the game is winning positively for player 1 or surely for player 2.

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Previous Work: [Chatterjee, Doyen, Henzinger, Raskin, 07] One player perfectly informed. No determinacy result. EXPTIME algorithm for deciding whether player 1 can win almost-surely a Büchi objective.

Lower bounds: EXPTIME and 2-EXPTIME complete.

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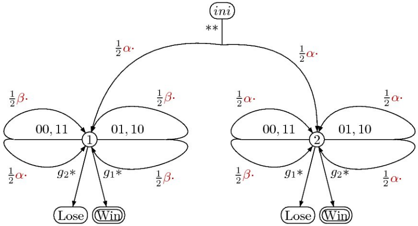

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  The pessimistic belief of player 1 after L and signal c is:

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- Proposition: if player 1 plays an almost-surely winning strategy, his pessimistic beliefs are almost-surely winning. If player 2 plays a surely winning strategy, her beliefs are surely winning.

Player 2 has a belief strategy for winning surely

Proposition: For winning surely, player 2 has a strategy with finite memory of exponential size.

- Monotonic operator Φ on $\mathcal{P}(\mathcal{P}(S \setminus T))$ with $\Phi(\mathcal{L}) = \{L \in \mathcal{L} \mid \exists j_L \in J, \forall d \in D, (j = j(d)) \implies (\mathcal{B}(L, d) \in \mathcal{L})\}$.

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- Let $\mathcal{L}_\infty$ be the largest fix-point of Φ.
- Strategy for player 2 consisting in playing actions j_L such that her belief L stays in $\mathcal{L}_\infty$.
- Conversely, playing randomly, player 1 wins positively from any support in $\Phi^n(\mathcal{P}(K)) \setminus \Phi^{n+1}(\mathcal{P}(K))$.

Player 2 has a double exponential memory strategy for wining positively

Proposition: Let $\mathcal{L} \subseteq \mathcal{P}(K)$ be a set of supports. Suppose player 1 can enforce his pessimistic beliefs to stay outside $\mathcal{L}$. Then,

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- (a) $\mathcal{L}'$ is not empty and does not intersect $\mathcal{L}$,
- (b) player 1 can enforce his pessimistic beliefs to stay outside $\mathcal{L} \cup \mathcal{L}'$,
- (c) player 2 has a strategy τ such that for every σ and initial distribution δ with support in $\mathcal{L}'$,

$$\mathbb{P}_\delta^{\sigma,\tau}\left(\forall n \in \mathbb{N}, K_n \notin T \mid \forall n \in \mathbb{N}, \mathcal{B}_p^1(L, C_1, \ldots, C_n) \notin \mathcal{L}\right) > 0 \ .$$

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Positively winning strategy of player 2

- The collection $\mathcal{L}_\infty$ of positively winning supports for player 2 is computed as a smallest fix-point $\emptyset = \mathcal{L}_0 \subset \mathcal{L}_1 \subset \ldots \subset \mathcal{L}_\infty$.

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  - Requires to make hypotheses about pessimistic beliefs of player 1, doubly-exponential size memory is required.
- For winning almost-surely from the complement of $\mathcal{L}_\infty$, player 1 plays randomly any action that ensures his pessimistic belief stays in the complement of $\mathcal{L}_\infty$.