NCM IST, Mathematics for Computer Science
Multiplicative weight updates, Ellipsoid algorithm

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1. Recall that one can model the linear regression problem by setting up a likelihood function, assuming that the difference $y_i - \theta^T x_i$ is a Gaussian normal variable, and the variables are independent for different i. Write down the likelihood function and maximize this to get the value of θ. Assuming that you are performing gradient descent what is the gradient of the function at given point θ_o. What would the next point of the gradient descent algorithm be?

2. Repeat the above problem for the logistic regression problem using the function $\frac{1}{1+\exp^{-\theta^T x}}$ as our estimate of the probablity that the output is 1 for the input x assuming that the parameters of regression are θ.

3. Show that the strategy of following the opinion of the majority among the experts on a given day has very large regret. Here regret is $M^T - min_i m_i^T$, M^T is the number of mistakes the algorithm makes upto day T and m_i^T is the number of mistakes the i-th expert makes.

4. In class there was a discussion on whether the strategy of following the best expert so far will work. Show that the regret could be large in this case.

5. An ellipsoid is the image of a sphere under an invertible linear transformation. Show that a sphere in n-space of radius R centered at a point $c \in \mathcal{R}^n$ is given $B(c, R) = \{x | (x-c)^T (x-c) \leq R\}$. Now apply an invertible linear transformation A. Write down a closed form for the ellipsoid E, the image of $B(c, R)$ under A.

6. Let $G = (V, E)$ be a directed graph with a specified vertex r. Assume G an arborescense rooted at r. Show that the optimal solution to the LP, $min \sum_e w_e x_e$ with constraints, $\forall S \subseteq V - r, \sum_{e \in S^{-1}} x_e \geq 1$, where S^{-1} denotes the set of edges leaving S is an arborescense rooted at r. If there is no arborescense show that the constraints are not feasible.