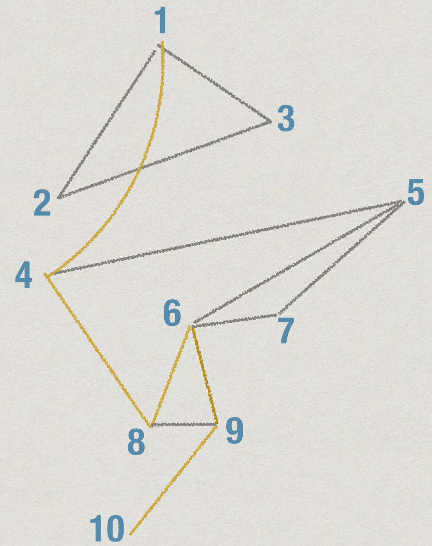
Graphs, formally

G = (V,E)

- Set of vertices V
- Set of edges E
  - E is a subset of pairs (v,v'): $E \subseteq V \times V$
  - Undirected graph: (v,v') and (v',v) are the same edge
  - Directed graph:
    - (v,v') is an edge from v to v'
    - Does not guarantee that (v',v) is also an edge

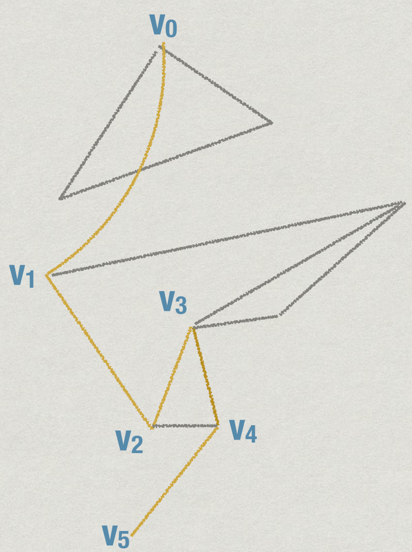
Adjacency matrix



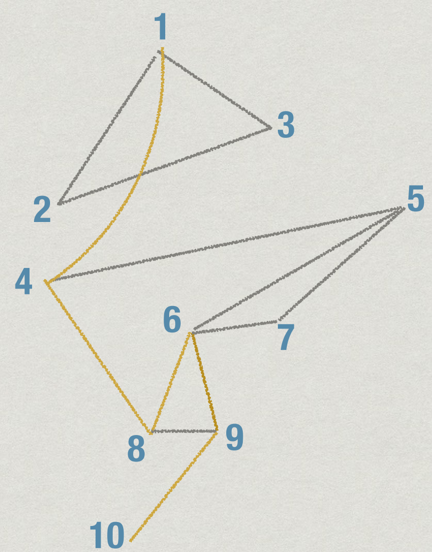
	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
2	1	0	1	0	0	0	0	0	0	0
3	1	1	0	0	0	0	0	0	0	0
4	1	0	0	0	1	0	0	1	0	0
5	0	0	0	1	0	1	1	0	0	0
6	0	0	0	0	1	0	1	1	1	0
7	0	0	0	0	1	1	0	0	0	0
8	0	0	0	1	0	1	0	0	1	0
9	0	0	0	0	0	1	0	1	0	1
10	0	0	0	0	0	0	0	0	1	0

Finding a route

- Find a sequence of vertices v_0, v_1, …, v_k such that
  - v_0 is **source**
  - Each (v_i,v_{i+1}) is an edge in E
  - v_k is **target**



Adjacency list



- For each vertex, maintain a list of its neighbours

1	2,3,4
2	1,3
3	1,2
4	1,5,8
5	4,6,7
6	5,7,8,9
7	5,6
8	4,6,9
9	6,8,10
10	9

Finding a path

- Mark vertices that have been visited
- Keep track of vertices whose neighbours have already been explored
  - Avoid going round indefinitely in circles
- Two fundamental strategies: breadth first and depth first

Breadth first search

- Recall that V = {1,2,…,n}
- Array visited[i] records whether i has been visited
- When a vertex is visited for the first time, add it to a **queue**
  - Explore vertices in the order they reach the queue

Breadth first search

- Explore the graph level by level
  - First visit vertices one step away
  - Then two steps away
  - …
- Remember which vertices have been **visited**
- Also keep track of vertices visited, but whose neighbours are yet to be **explored**

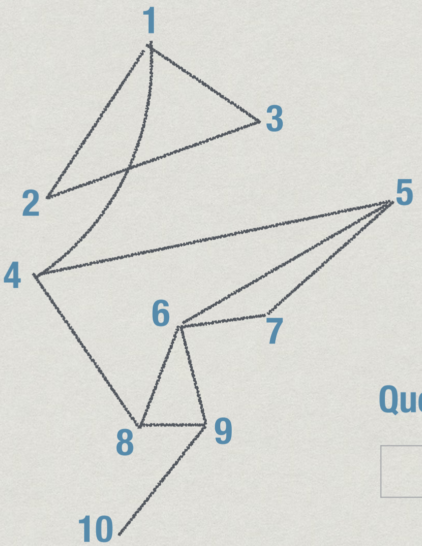
Breadth first search

- Exploring a vertex i:

```
for each edge (i,j)
  if visited[j] == 0
    visited[j] = 1
    append j to queue
```

- Initially, queue contains only source vertex
- At each stage, explore vertex at the head of the queue
- Stop when the queue becomes empty

Breadth first search

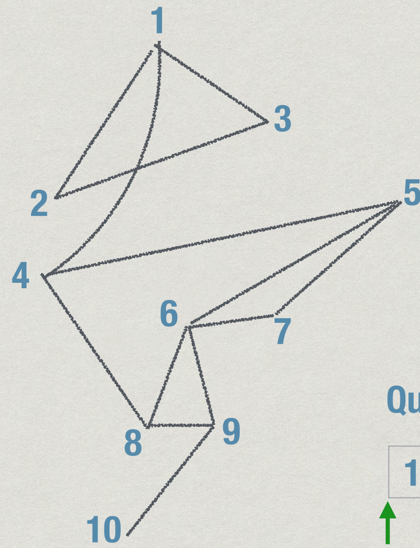


Visited

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Queue

Breadth first search



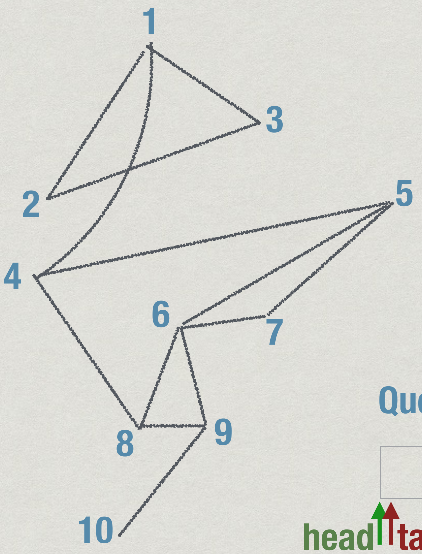
Visited

1	1
2	
3	
4	
5	
6	
7	
8	
9	
10	

Queue

1									

Breadth first search



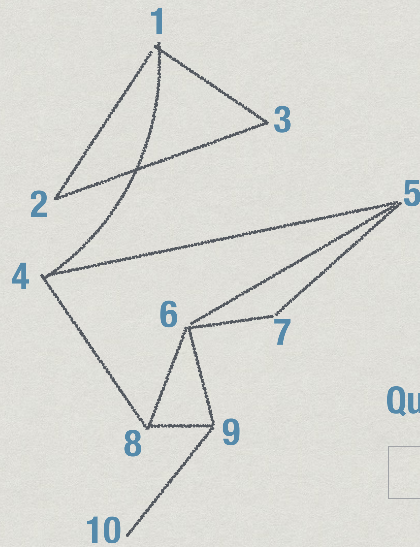
Visited

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Queue

head tail

Breadth first search

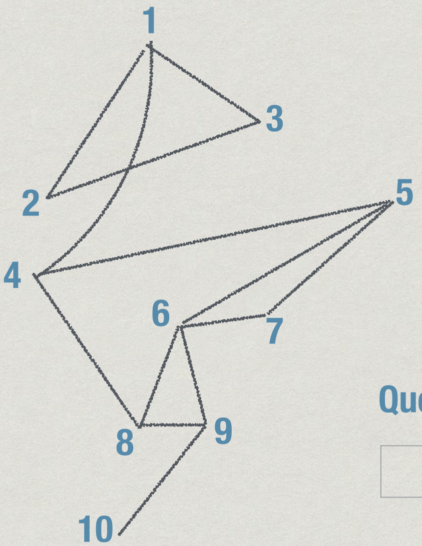


Visited

1	1
2	
3	
4	
5	
6	
7	
8	
9	
10	

Queue

Breadth first search



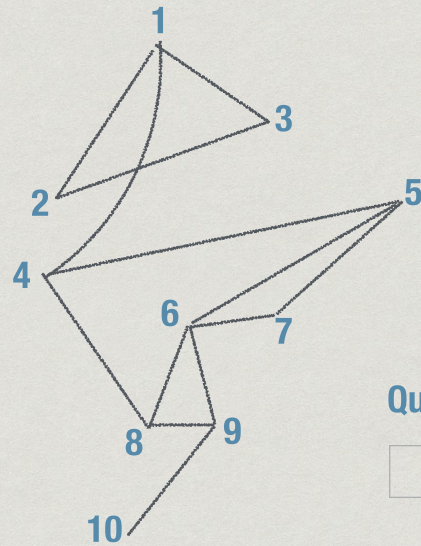
Visited

1	1
2	1
3	
4	
5	
6	
7	
8	
9	
10	

Queue

	2								

Breadth first search



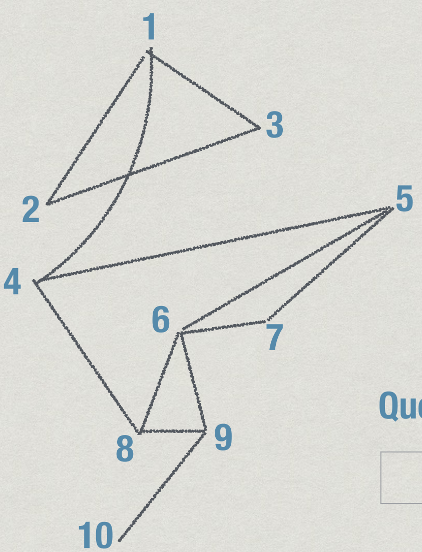
Visited

1	1
2	1
3	1
4	1
5	
6	
7	
8	
9	
10	

Queue

	2	3	4						

Breadth first search



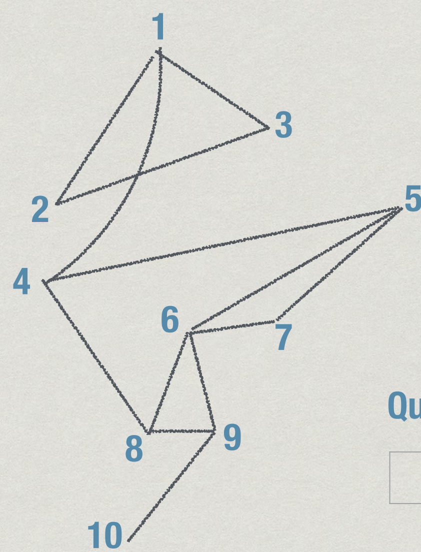
Visited

1	1
2	1
3	1
4	
5	
6	
7	
8	
9	
10	

Queue

	2	3							

Breadth first search



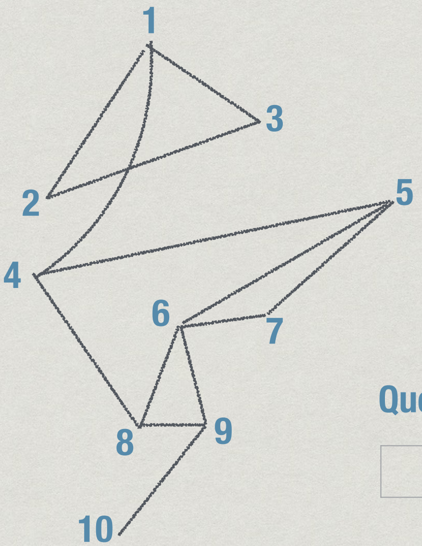
Visited

1	1
2	1
3	1
4	1
5	
6	
7	
8	
9	
10	

Queue

		3	4						

Breadth first search



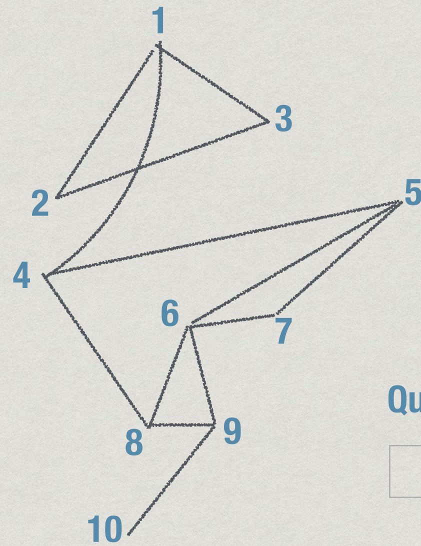
Visited

1	1
2	1
3	1
4	1
5	
6	
7	
8	
9	
10	

Queue

			4						

Breadth first search



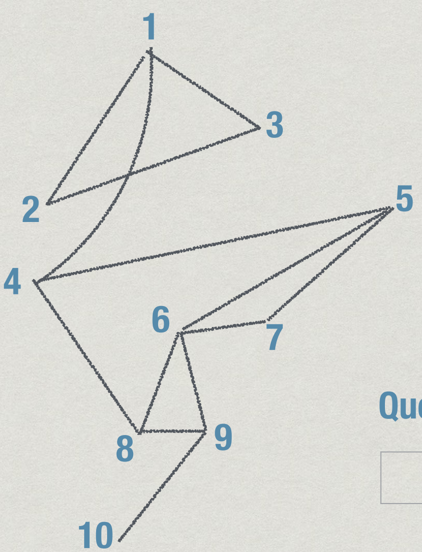
Visited

1	1
2	1
3	1
4	1
5	1
6	
7	
8	
9	
10	

Queue

				5					

Breadth first search

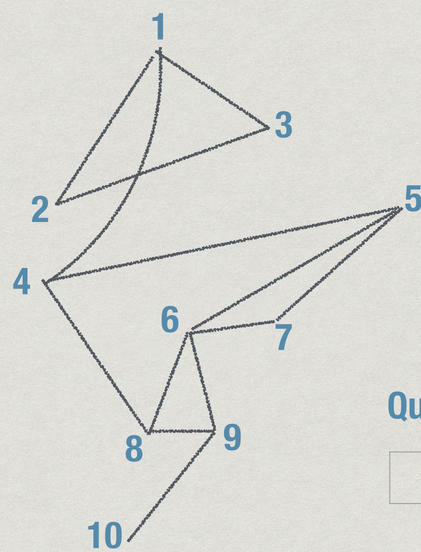


Visited

1	1
2	1
3	1
4	1
5	
6	
7	
8	
9	
10	

Queue

Breadth first search



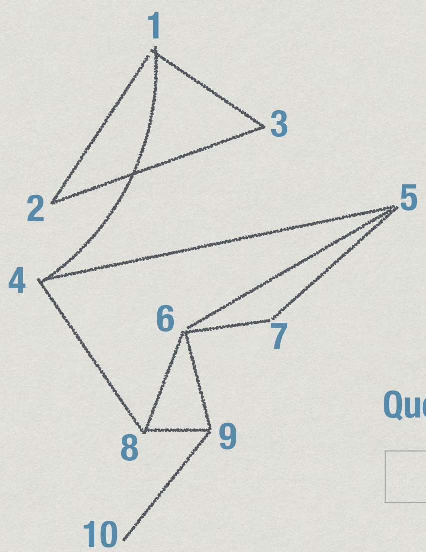
Visited

1	1
2	1
3	1
4	1
5	1
6	
7	
8	1
9	
10	

Queue

				5	8				

Breadth first search



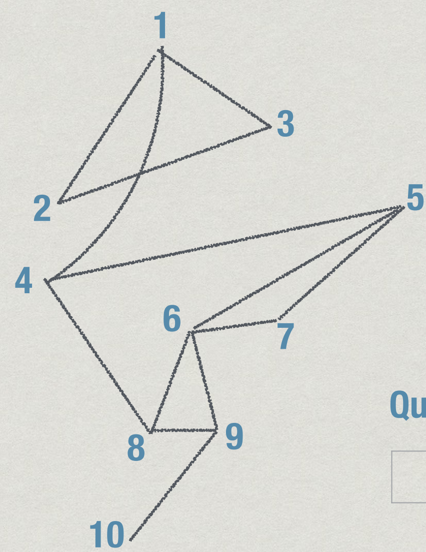
Visited

1	1
2	1
3	1
4	1
5	1
6	
7	
8	1
9	
10	

Queue

					8				

Breadth first search



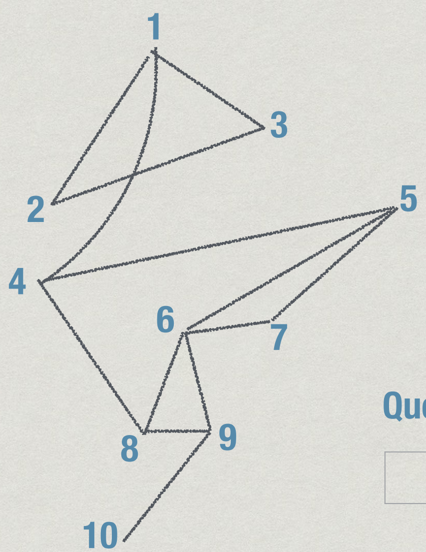
Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	
10	

Queue

					8	6	7		

Breadth first search



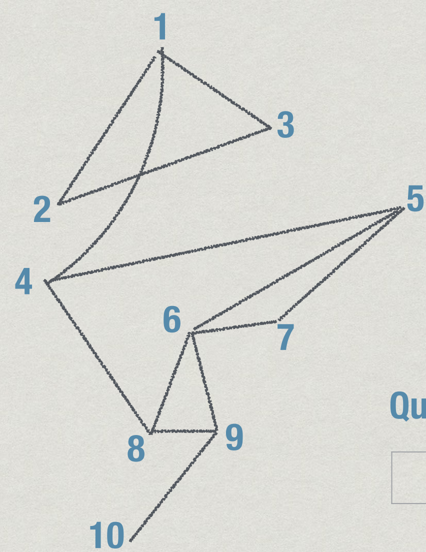
Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	
8	1
9	
10	

Queue

					8	6			

Breadth first search



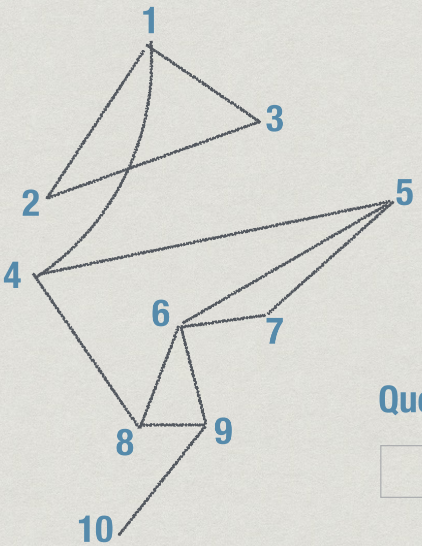
Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	
10	

Queue

						6	7		

Breadth first search



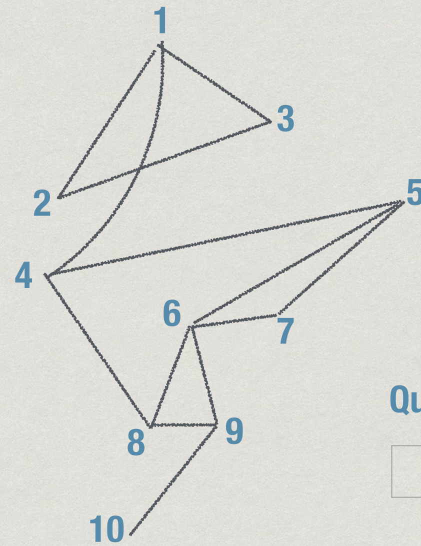
Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	

Queue

						6	7	9	

Breadth first search



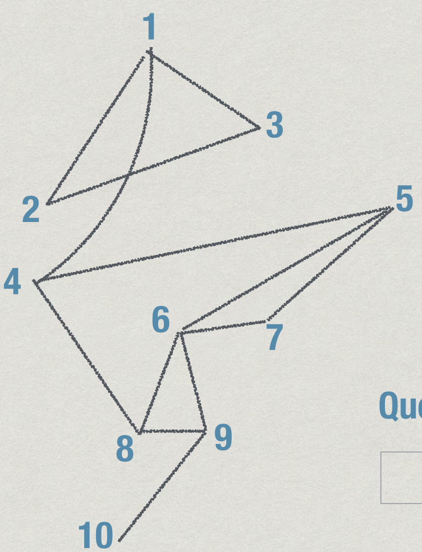
Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	

Queue

								9	

Breadth first search



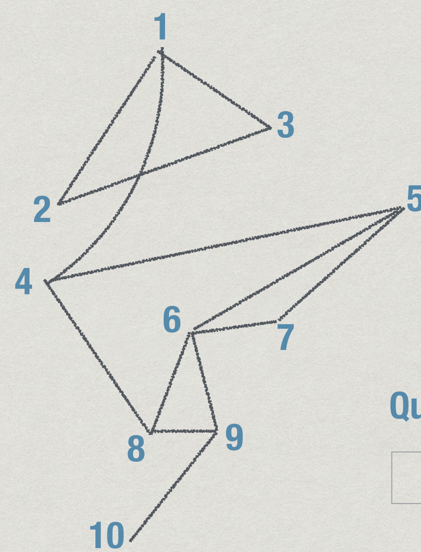
Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	

Queue

							7	9	

Breadth first search

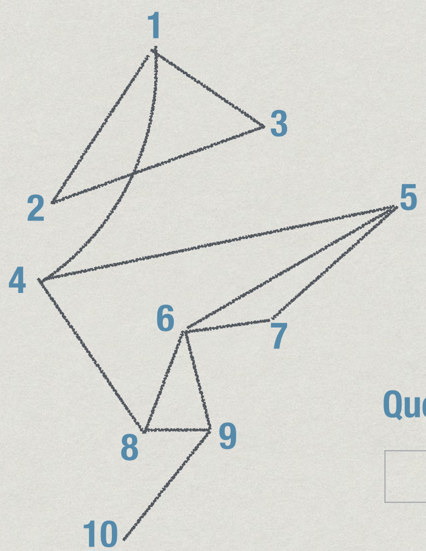


Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	

Queue

Breadth first search



Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

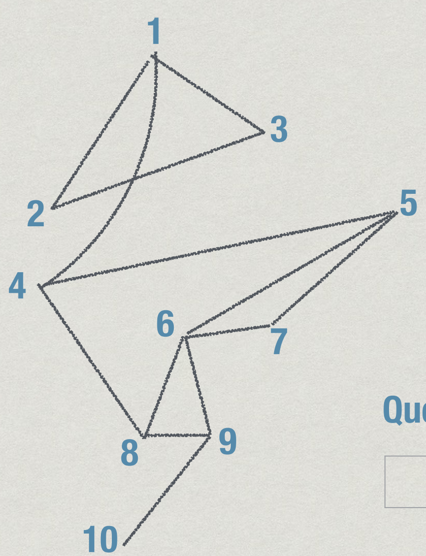
Queue

									10

Breadth first search

```
function BFS(i) // BFS starting from vertex i

  //Initialization
  for j = 1..n {visited[j] = 0}; Q = []

  //Start the exploration at i
  visited[i] = 1; append(Q,i)

  //Explore each vertex in Q
  while Q is not empty
     j = extract_head(Q)
     for each (j,k) in E
        if visited[k] == 0
           visited[k] = 1; append(Q,k)
```

Breadth first search



Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

Queue

Complexity of BFS

- Each vertex enters Q exactly once
- If graph is connected, loop to process Q iterated n times
  - For each j extracted from Q, need to examine all neighbours of j
  - In adjacency matrix, scan row j: n entries
- Hence, overall $O(n^2)$

Complexity of BFS

- Let m be the number of edges in E. What if $m << n^2$?
- Adjacency list: scanning neighbours of j takes time proportional to number of neighbours (**degree** of j)
- Across the loop, each edge (i,j) is scanned twice, once when exploring i and again when exploring j
  - Overall, exploring neighbours takes time O(m)
- Marking n vertices visited still takes O(n)
- Overall, O(n+m)

Enhancements to BFS

- If BFS(i) sets visited[j] = 1, we know that i and j are connected
- How do we identify a path from i to j
- When we mark visited[k] = 1, remember the neighbour from which we marked it
  - If exploring edge (j,k) visits k, set parent[k] = j

Complexity of BFS

- For graphs, O(m+n) is considered the best possible
  - Need to see each edge and vertex at least once
- O(m+n) is considered to be **linear** in the size of the graph

Breadth first search

```
function BFS(i) // BFS starting from vertex i

  //Initialization
  for j = 1..n {visited[j] = 0; parent[j] = -1}
  Q = []

  //Start the exploration at i
  visited[i] = 1; append(Q,i)

  //Explore each vertex in Q
  while Q is not empty
    j = extract_head(Q)
    for each (j,k) in E
      if visited[k] == 0
        visited[k] = 1; parent[k] = j; append(Q,k);
```

Reconstructing the path

- BFS(i) sets visited[j] = 1
- visited[j] = 1, so parent[j] = j' for some j'
- visited[j'] = 1, so parent[j'] = j" for some j"
- ...
- Eventually, trace back path to k with parent[k] = i

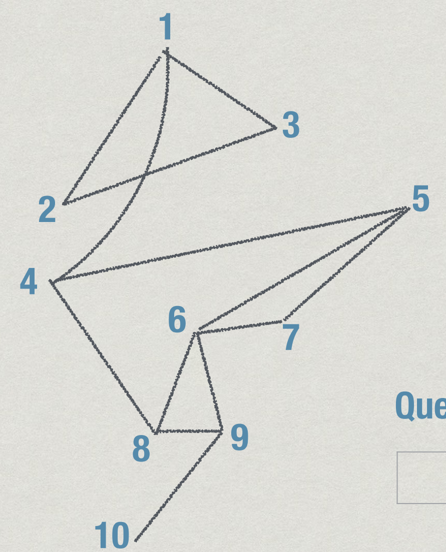
Breadth first search

```
function BFS(i) // BFS starting from vertex i

   //Initialization
   for j = 1..n {level[j] = -1; parent[j] = -1}
   Q = []

   //Start the exploration at i, level[i] set to 0
   level[i] = 0; append(Q,i)

   //Explore each vertex in Q, increment level for each new vertex
   while Q is not empty
      j = extract_head(Q)
      for each (j,k) in E
         if level[k] == -1
            level[k] = 1+level[j]; parent[k] = j;
            append(Q,k);
```

Recording distances

- BFS can record how long the path is to each vertex
- Instead of binary array visited[ ], keep integer array level[ ]
- level[j] = -1 initially
- level[j] = p means j is reached in p steps from i

Breadth first search

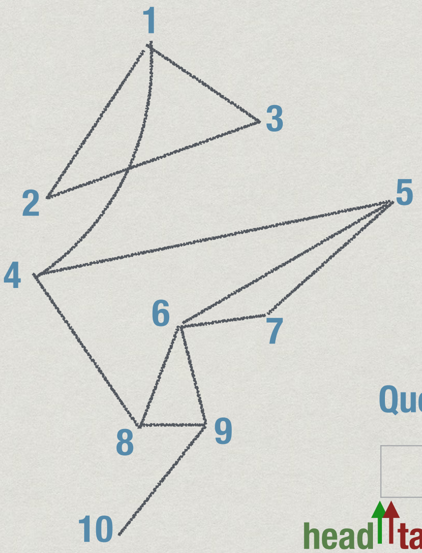


L : Level
P : Parent

	L	P
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Queue

Breadth first search



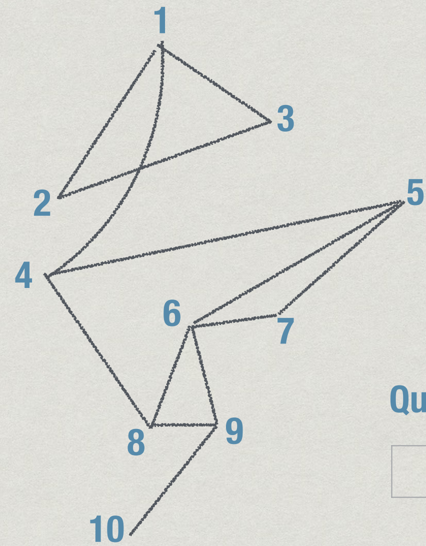
L : Level
P : Parent

	L	P
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Queue

head tail

Breadth first search

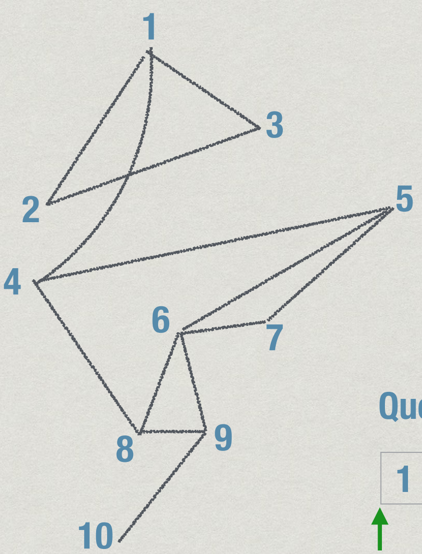


L : Level
P : Parent

	L	P
1	0	-
2		
3		
4		
5		
6		
7		
8		
9		
10		

Queue

Breadth first search



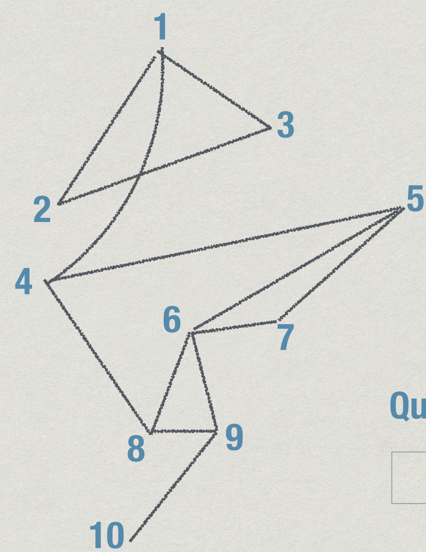
L : Level
P : Parent

	L	P
1	0	-
2		
3		
4		
5		
6		
7		
8		
9		
10		

Queue

1									

Breadth first search



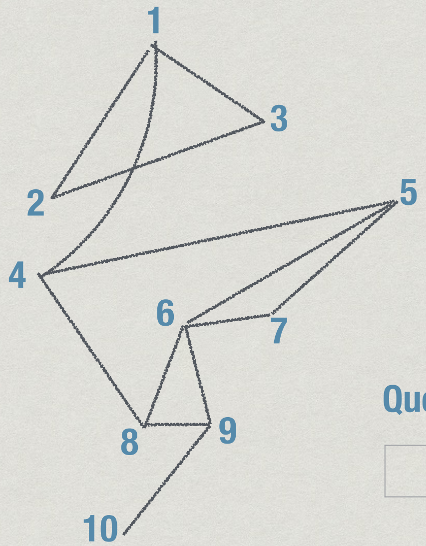
L : Level
P : Parent

	L	P
1	0	-
2	1	1
3		
4		
5		
6		
7		
8		
9		
10		

Queue

	2								

Breadth first search



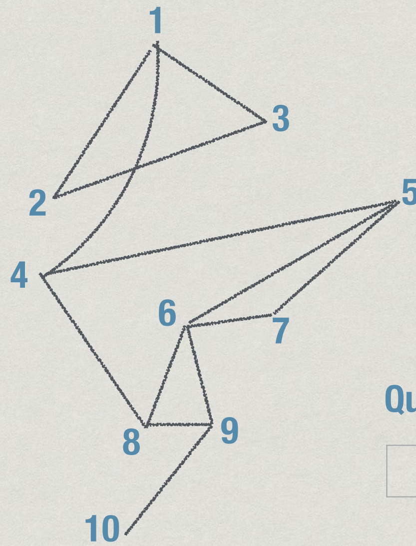
L : Level
P : Parent

	L	P
1	0	-
2	1	1
3	1	1
4		
5		
6		
7		
8		
9		
10		

Queue

	2	3							

Breadth first search



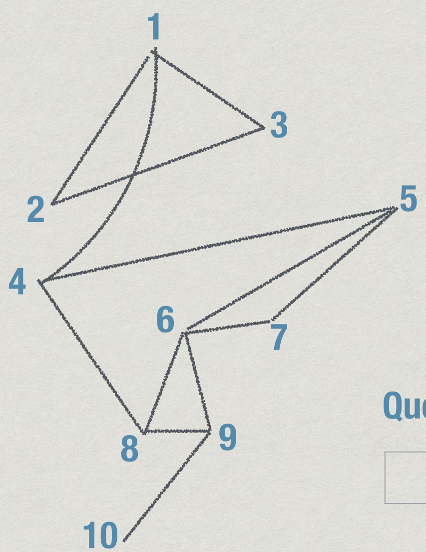
L : Level
P : Parent

	L	P
1	0	-
2	1	1
3	1	1
4	1	1
5		
6		
7		
8		
9		
10		

Queue

		3	4						

Breadth first search



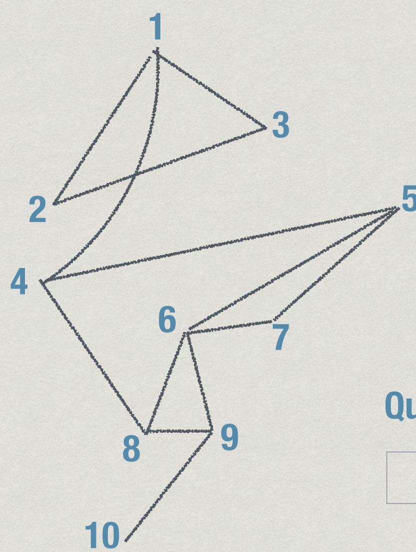
L : Level
P : Parent

	L	P
1	0	-
2	1	1
3	1	1
4	1	1
5		
6		
7		
8		
9		
10		

Queue

	2	3	4						

Breadth first search



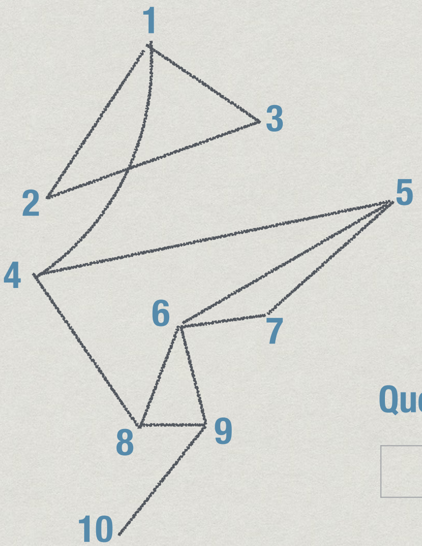
L : Level
P : Parent

	L	P
1	0	-
2	1	1
3	1	1
4	1	1
5		
6		
7		
8		
9		
10		

Queue

			4						

Breadth first search

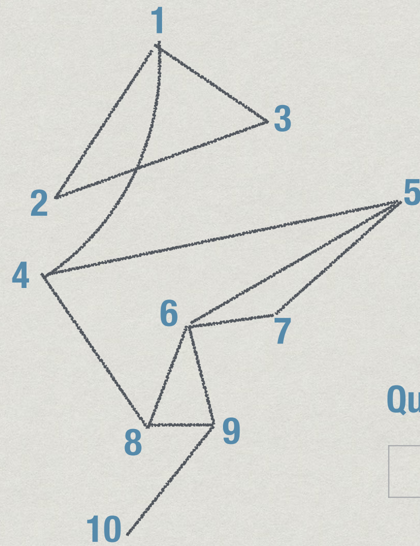


L : Level
P : Parent

	L	P
1	0	-
2	1	1
3	1	1
4	1	1
5		
6		
7		
8		
9		
10		

Queue

Breadth first search



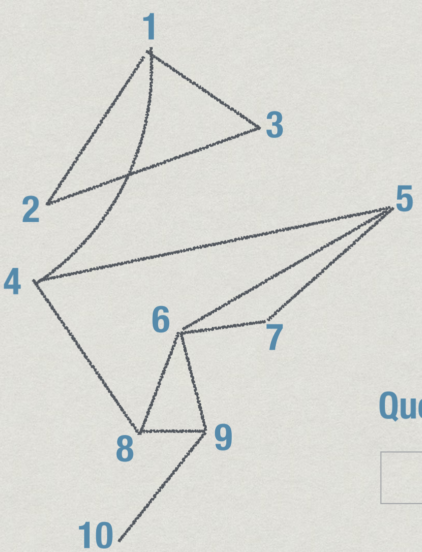
L : Level
P : Parent

	L	P
1	0	-
2	1	1
3	1	1
4	1	1
5	2	4
6		
7		
8	2	4
9		
10		

Queue

				5	8				

Breadth first search



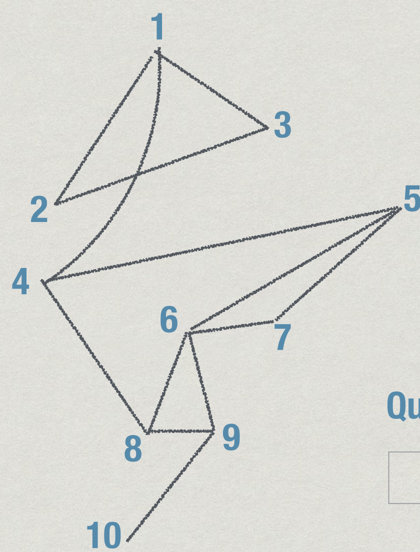
L : Level
P : Parent

	L	P
1	0	-
2	1	1
3	1	1
4	1	1
5	2	4
6		
7		
8		
9		
10		

Queue

				5					

Breadth first search



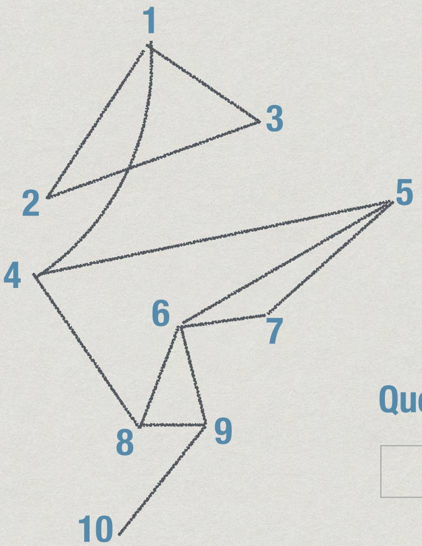
L : Level
P : Parent

	L	P
1	0	-
2	1	1
3	1	1
4	1	1
5	2	4
6		
7		
8	2	4
9		
10		

Queue

					8				

Breadth first search



L : Level
P : Parent

	L	P
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3	1	1
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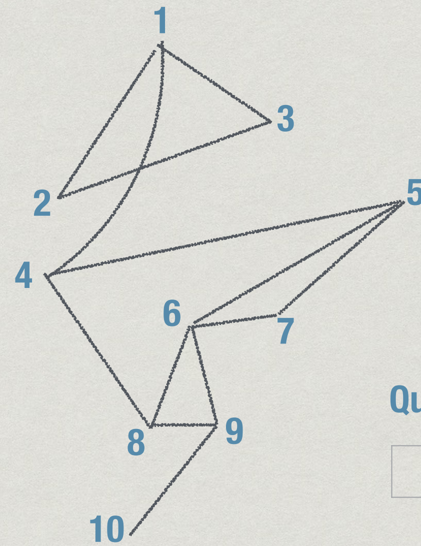
5	2	4
6	3	5
7		
8	2	4
9		
10		

Queue

					8	6			

Breadth first search



L : Level
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	L	P
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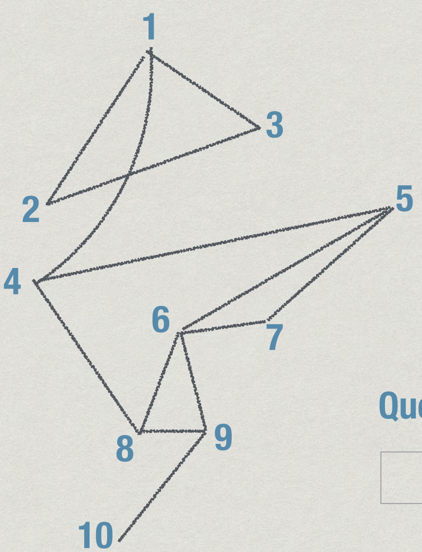
2	1	1
3	1	1
4	1	1
5	2	4
6	3	5
7	3	5
8	2	4
9		
10		

Queue

						6	7		

Breadth first search



L : Level
P : Parent

	L	P
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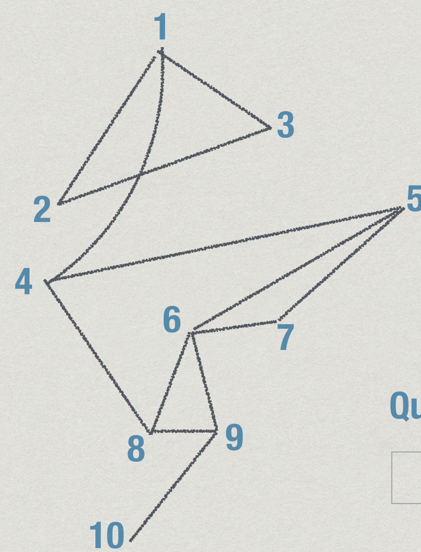
3	1	1
4	1	1
5	2	4
6	3	5
7	3	5
8	2	4
9		
10		

Queue

					8	6	7		

Breadth first search



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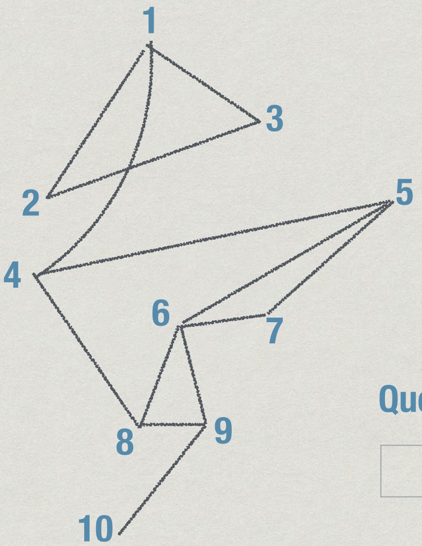
8	2	4
9	3	8
10		

Queue

						6	7	9	

Breadth first search



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3	1	1
4	1	1
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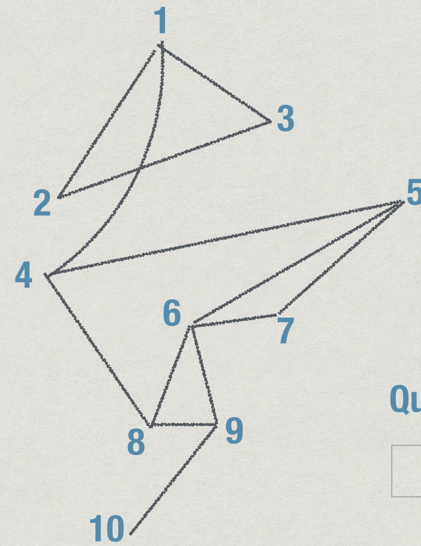
9	3	8
10		

Queue

							7	9	

Breadth first search



L : Level
P : Parent

	L	P
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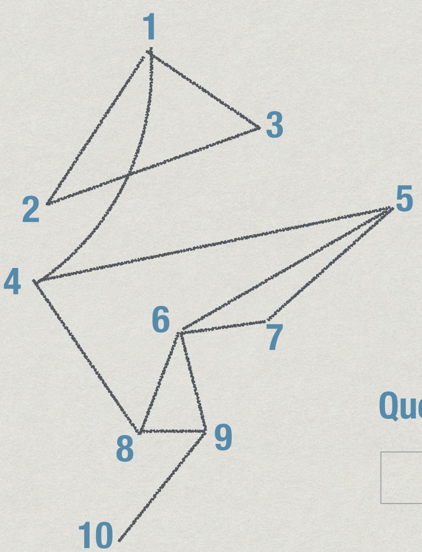
2	1	1
3	1	1
4	1	1
5	2	4
6	3	5
7	3	5
8	2	4
9	3	8
10		

Queue

Breadth first search



L : Level
P : Parent

	L	P
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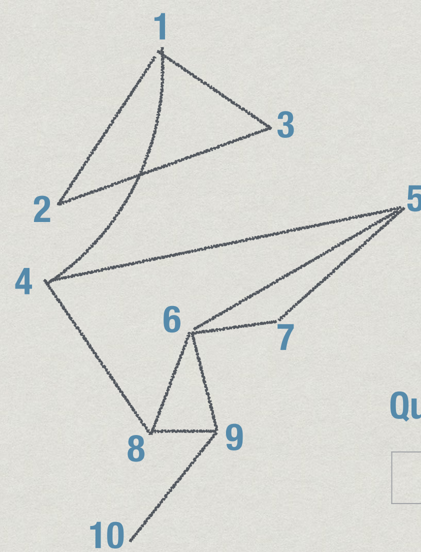
2	1	1
3	1	1
4	1	1
5	2	4
6	3	5
7	3	5
8	2	4
9	3	8
10		

Queue

								9	

Breadth first search



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	L	P
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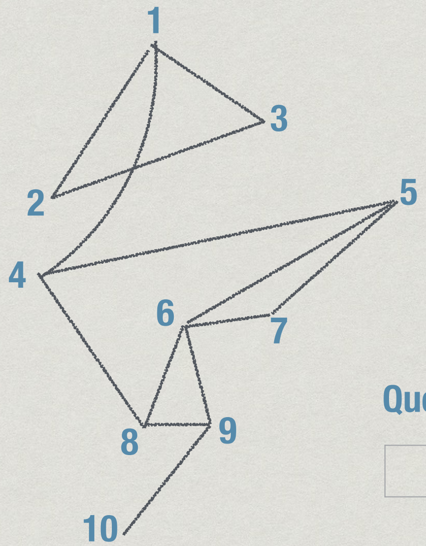
2	1	1
3	1	1
4	1	1
5	2	4
6	3	5
7	3	5
8	2	4
9	3	8
10	4	9

Queue

									10

Breadth first search



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1	0	-
2	1	1
3	1	1
4	1	1
5	2	4
6	3	5
7	3	5
8	2	4
9	3	8
10	4	9

Queue

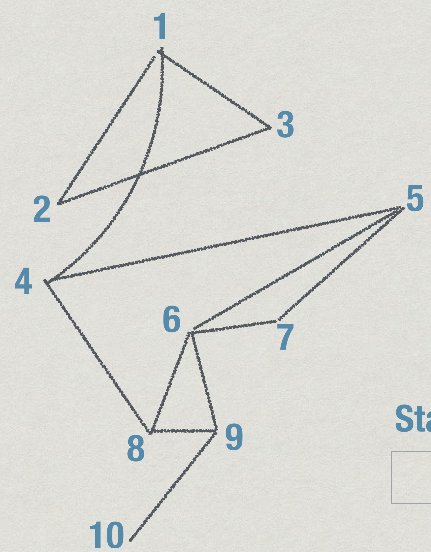
Depth first search

- Start from i, visit a neighbour j
- Suspend the exploration of i and explore j instead
- Continue till you reach a vertex with no unexplored neighbours
- Backtrack to nearest suspended vertex that still has an unexplored neighbour
- Suspended vertices are stored in a **stack**
  - Last in, first out: most recently suspended is checked first

Recording distances

- BFS with level[ ] gives us the shortest path to each node in terms of number of edges
- In general, edges are labelled by a cost (money, time, distance …)
  - Min cost path not same as fewest edges
- Will look at shortest paths in **weighted** graphs later
  - BFS computes shortest paths if all costs are 1

Depth first search



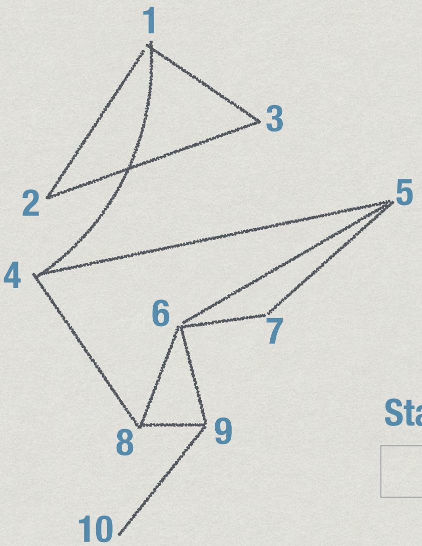
Visited

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

Depth first search

Start at 4



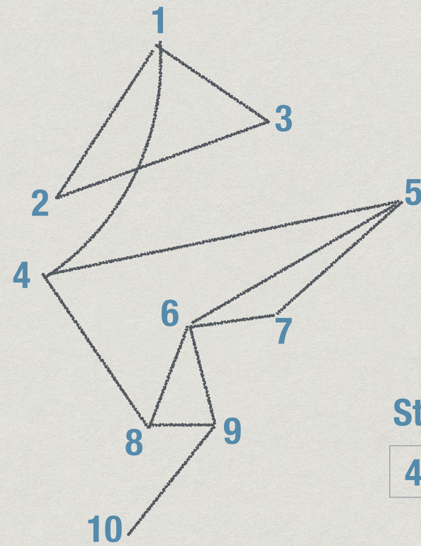
Visited

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

Depth first search

Start at 4



Visited

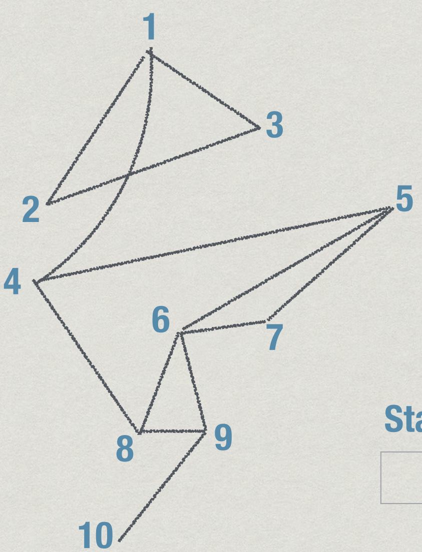
1	1
2	
3	
4	1
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

4									

Depth first search

Start at 4



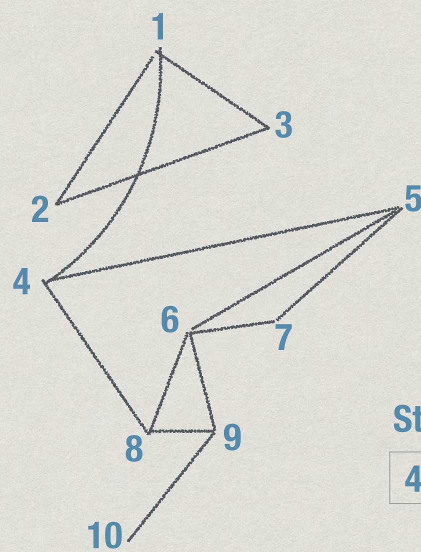
Visited

1	
2	
3	
4	1
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

Depth first search

Start at 4



Visited

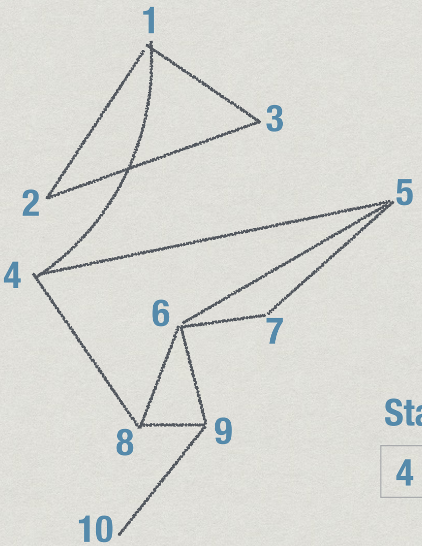
1	1
2	1
3	
4	1
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

4	1								

Depth first search

Start at 4



Visited

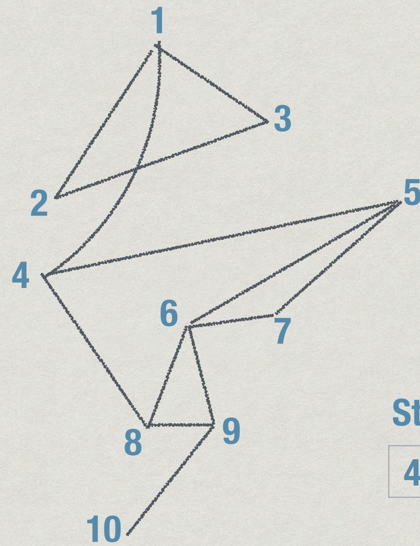
1	1
2	1
3	1
4	1
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

4	1	2							

Depth first search

Start at 4



Visited

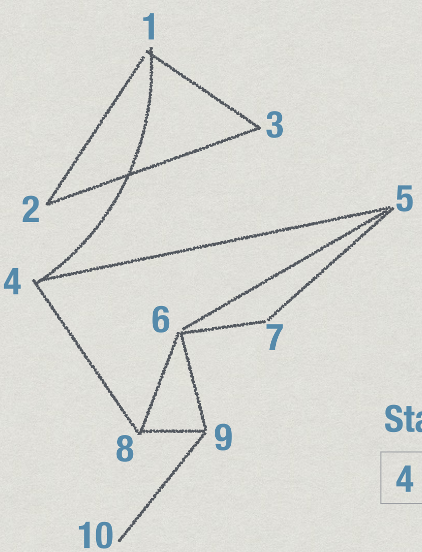
1	1
2	1
3	1
4	1
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

4									

Depth first search

Start at 4



Visited

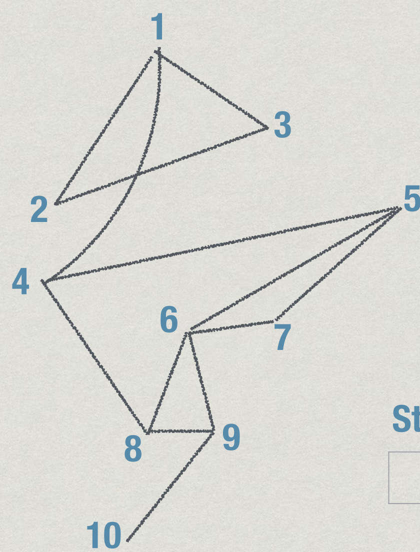
1	1
2	1
3	1
4	1
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

4	1								

Depth first search

Start at 4



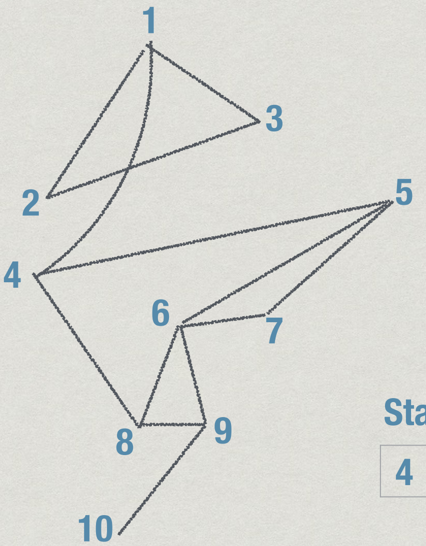
Visited

1	1
2	1
3	1
4	1
5	
6	
7	
8	
9	
10	

Stack of suspended vertices

Depth first search

Start at 4



Visited

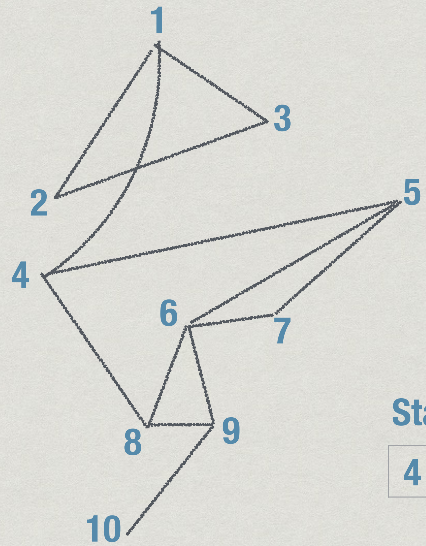
1	1
2	1
3	1
4	1
5	1
6	
7	
8	
9	
10	

Stack of suspended vertices

4									

Depth first search

Start at 4



Visited

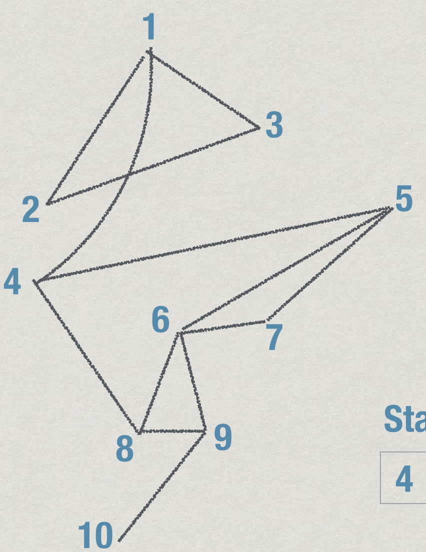
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	
9	
10	

Stack of suspended vertices

4	5	6							

Depth first search

Start at 4



Visited

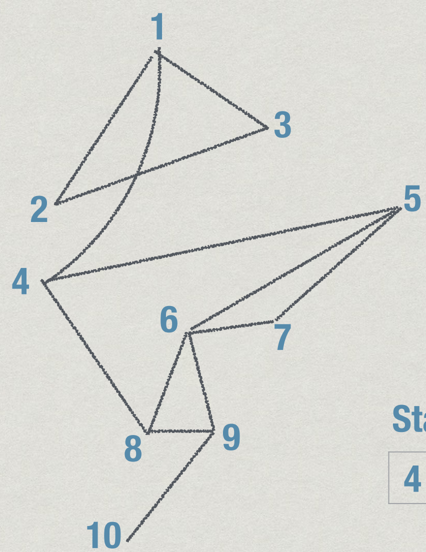
1	1
2	1
3	1
4	1
5	1
6	1
7	
8	
9	
10	

Stack of suspended vertices

4	5								

Depth first search

Start at 4



Visited

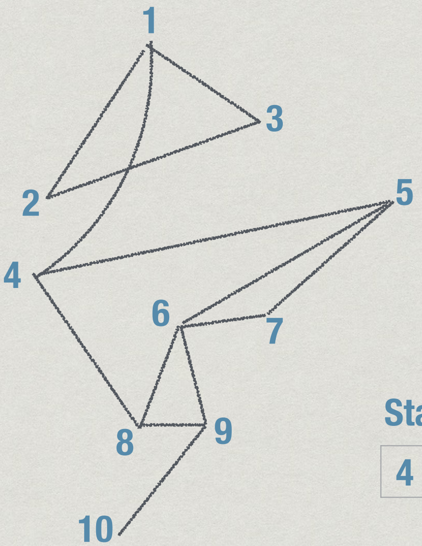
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	
9	
10	

Stack of suspended vertices

4	5								

Depth first search

Start at 4



Visited

1	1
2	1
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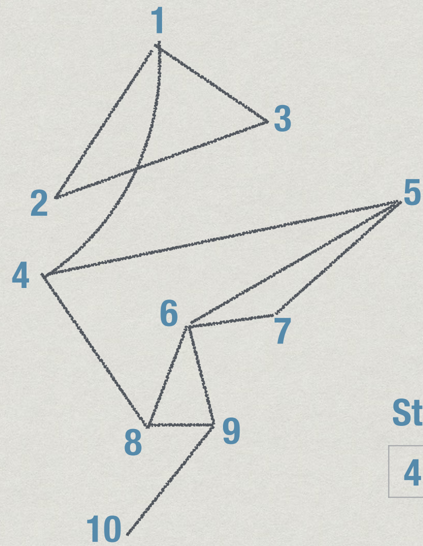
9	
10	

Stack of suspended vertices

4	5	6							

Depth first search

Start at 4



Visited

1	1
2	1
3	1
4	1
5	1
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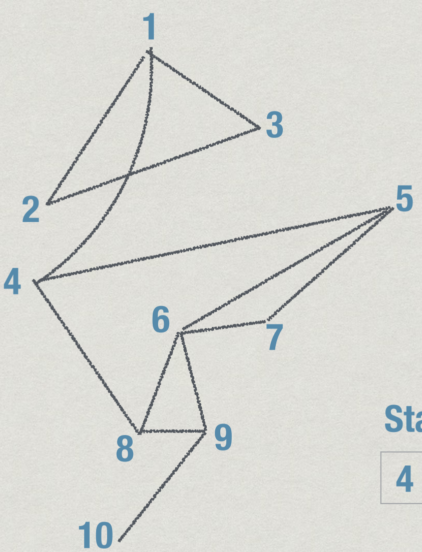
10	1

Stack of suspended vertices

4	5	6	8	9					

Depth first search

Start at 4



Visited

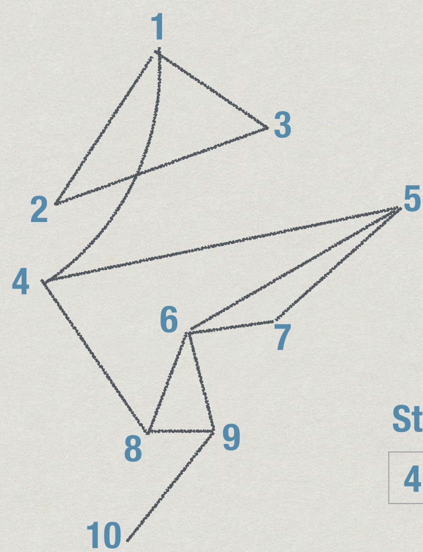
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	

Stack of suspended vertices

4	5	6	8						

Depth first search

Start at 4



Visited

1	1
2	1
3	1
4	1
5	1
6	1
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8	1
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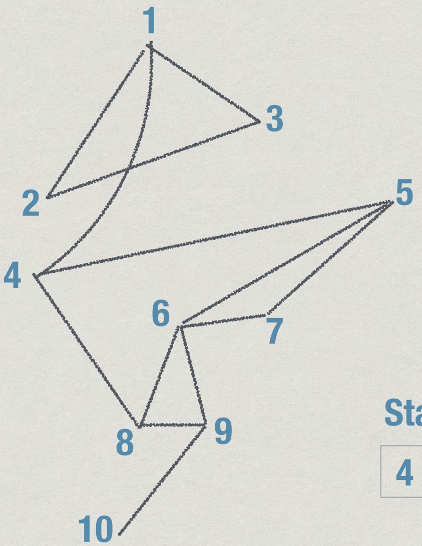
10	1

Stack of suspended vertices

4	5	6	8						

Depth first search

Start at 4



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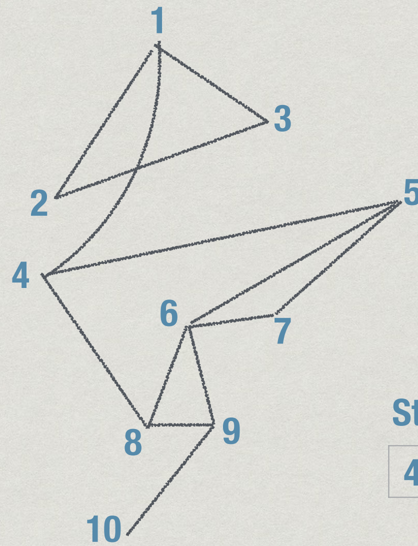
Stack of suspended vertices

4	5	6							

Depth first search

Start at 4



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1	1
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4									

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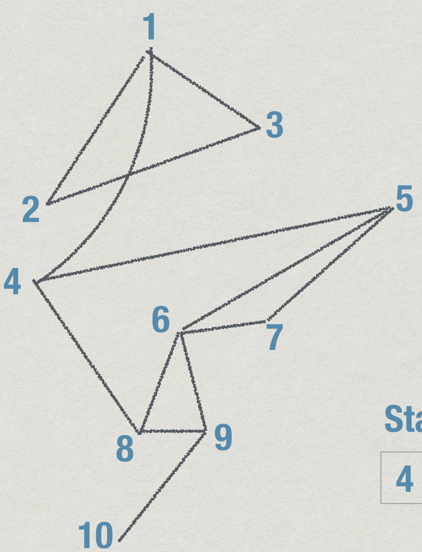

Visited

1	1
2	1
3	1
4	1
5	1
6	1
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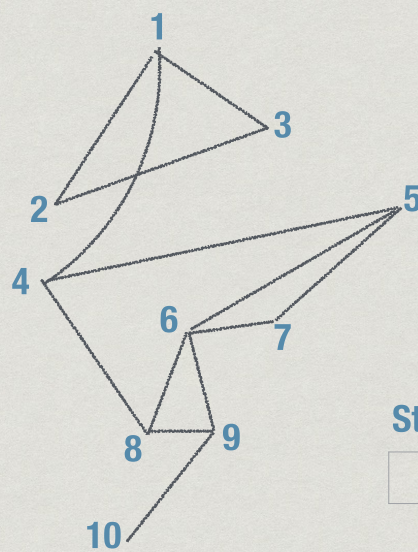
8	1
9	1
10	1

Stack of suspended vertices

4	5								

Depth first search

Start at 4



Visited

1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

Stack of suspended vertices

Depth first search

Depth first search

- DFS is most natural to implement recursively
  - For each unvisited neighbour j of i, call DFS(j)
- No need to explicitly maintain a stack
  - Stack is maintained implicitly by recursive calls

Depth first search

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  - For each unvisited neighbour j of i, call DFS(j)

Depth first search

```
//Initialization
   for j = 1..n {visited[j] = 0; parent[j] = -1}

function DFS(i) // DFS starting from vertex i

   //Mark i as visited
   visited[i] = 1

   //Explore each neighbour of i recursively
   for each (i,j) in E
      if visited[j] == 0
         parent[j] = i
         DFS(j)
```

Complexity of DFS

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- Each vertex marked and explored exactly once
- DFS(j) need to examine all neighbours of j

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- In adjacency matrix, scan row j: n entries
  - Overall $O(n^2)$

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- Each vertex marked and explored exactly once
- DFS(j) need to examine all neighbours of j
- In adjacency matrix, scan row j: n entries
  - Overall $O(n^2)$
- With adjacency list, scanning takes O(m) time across all vertices
  - Total time is O(m+n), like BFS

Properties of DFS

- Paths discovered by DFS are not shortest paths, unlike BFS

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- Paths discovered by DFS are not shortest paths, unlike BFS
- Why use DFS at all?
- Many useful features can be extracted from recording the order in which DFS visited vertices
  - **DFS numbering**
  - Maintain a counter
  - Increment and record counter value when entering and leaving a vertex.

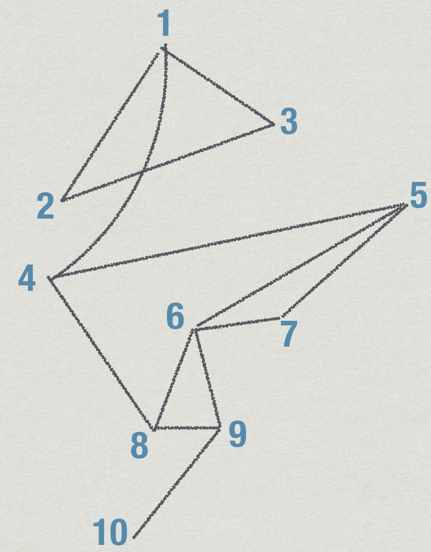
Depth first search

```
//Initialization
   for j = 1..n {visited[j] = 0; parent[j] = -1}
   count = 0

function DFS(i) // DFS starting from vertex i

   //Mark i as visited
   visited[i] = 1; pre[i] = count; count++

   //Explore each neighbours of i recursively
   for each (i,j) in E
      if visited[j] == 0
         parent[j] = i
         DFS(j)
         post[i] = count; count++
```

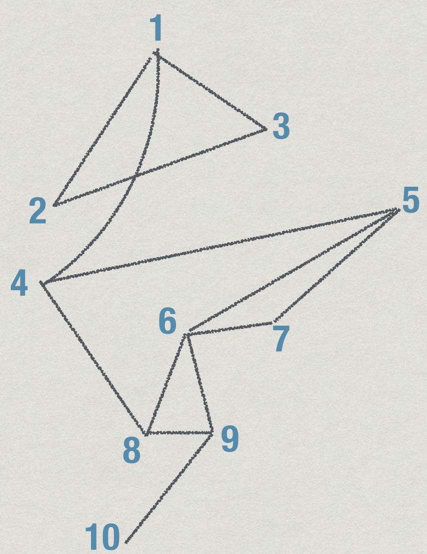
DFS numbering

pre[i] and post[i] can be used to find

- if the graph has a **cycle** — i.e., a loop
- **cut vertex** — removal disconnects the graph
- …



DFS numbering



Summary

- BFS and DFS are two systematic ways to explore a graph
  - Both take time linear in the size of the graph with adjacency lists
- Recover paths by keeping parent information
- BFS can compute shortest paths, in terms of number of edges
- DFS numbering can reveal many interesting features

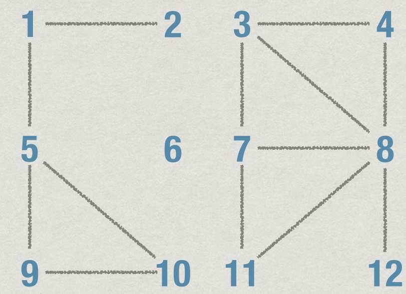
Graphs, formally

G = (V,E)

- Set of vertices V
- Set of edges E
  - E is a subset of pairs (v,v’): $E \subseteq V \times V$
  - Undirected graph: (v,v’) and (v’,v) are the same edge
  - Directed graph:
    - (v,v’) is an edge from v to v’
    - Does not guarantee that (v’,v) is also an edge

Connectivity

Connected graph



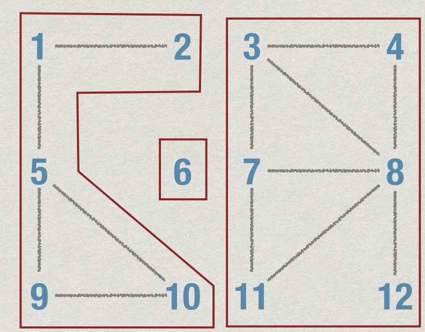
Disconnected graph

Exploring graph structure

- Breadth first search
  - Level by level exploration
- Depth first search
  - Explore each vertex as soon as it is visited
  - DFS numbering
- What can we find out about a graph using BFS/DFS?

Connectivity

Connected graph



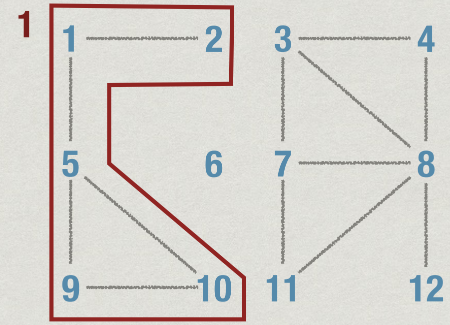
Disconnected graph

Connected components

Identifying connected components

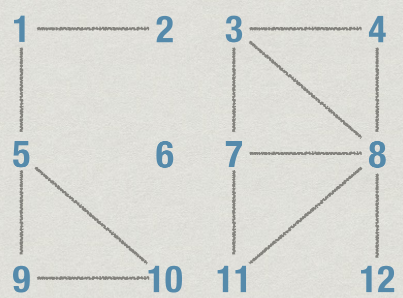
- Vertices {1,2,…,N}
- Start BFS or DFS from 1
  - All nodes marked Visited form a connected component
  - Pick first unvisited node, say j, and run BFS or DFS from j
  - Repeat till all nodes are visited
- Update BFS/DFS to label each visited node with component number

Connected components



- Add a counter `comp` to number components
- Increment counter each time a fresh BFS/DFS starts
- Label each visited node `j` with `component[j] = comp`

Connected components



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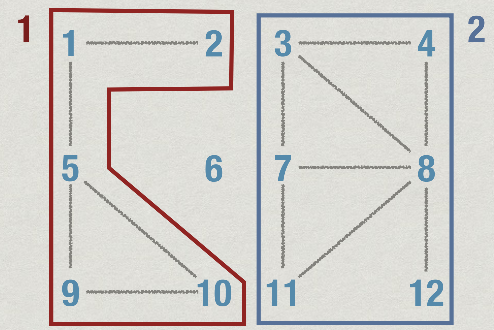
Connected components



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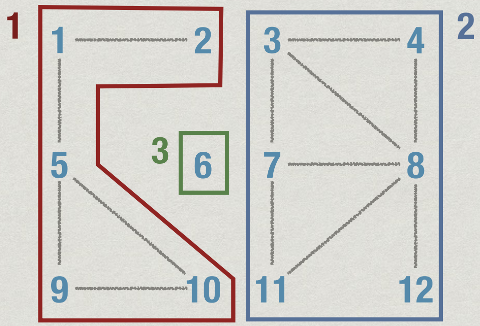
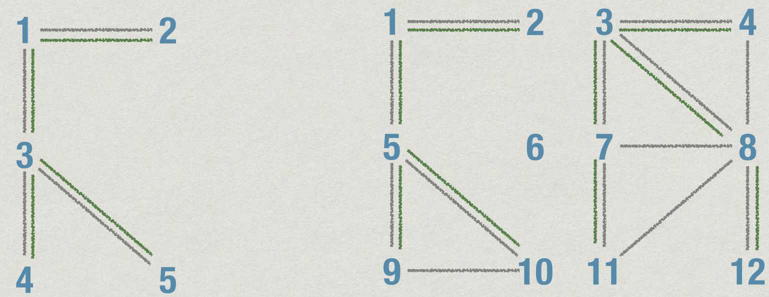
- Label each visited node `j` with `component[j] = comp`

Connected components



- Add a counter `comp` to number components
- Increment counter each time a fresh BFS/DFS starts
- Label each visited node `j` with `component[j] = comp`

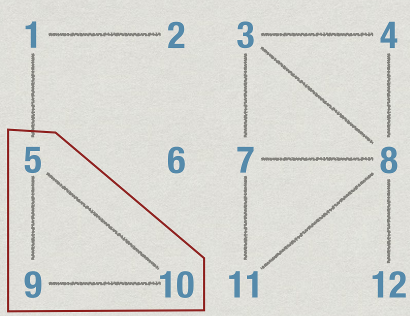
BFS tree



- Edges explored by BFS form a tree
  - Acyclic graph = connected, with n-1 edges

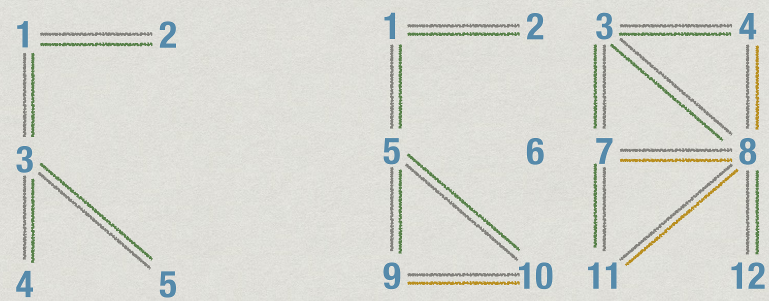
Cycles

Acyclic graph



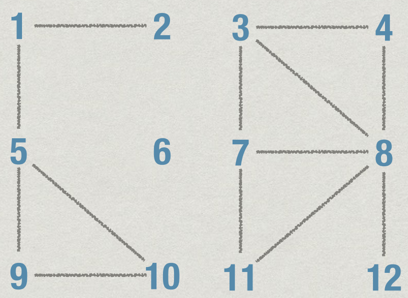
Graph with cycles

BFS tree

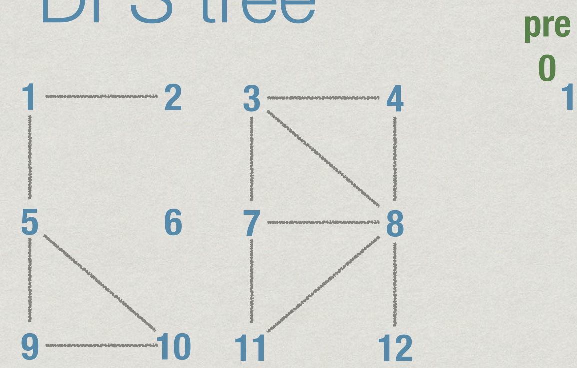


- Edges explored by BFS form a tree
  - Acyclic graph = connected, with n-1 edges
- Any non-tree edge generates a cycle

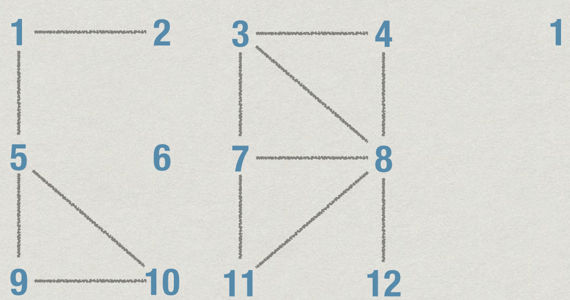
DFS tree



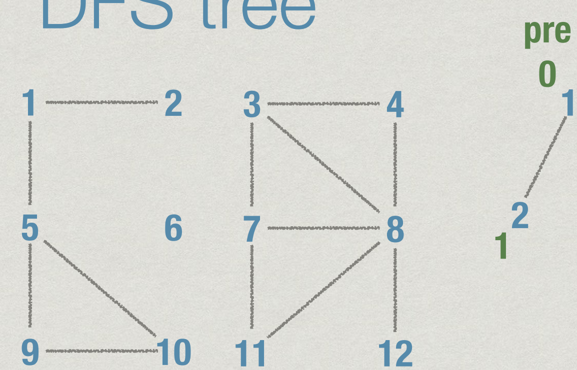
DFS tree



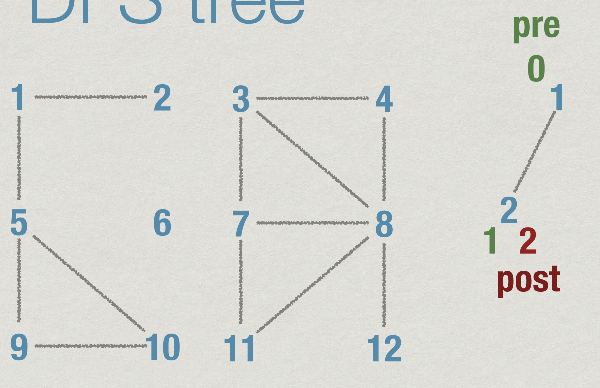
DFS tree



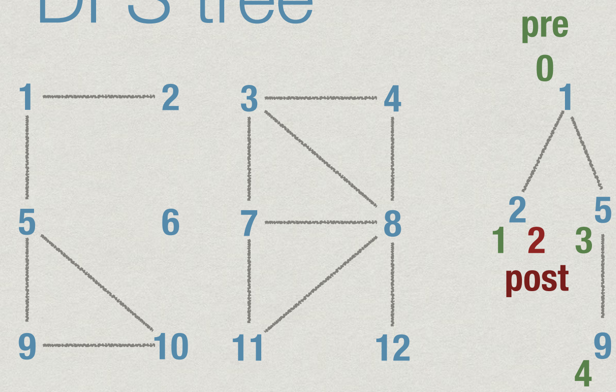
DFS tree



DFS tree



DFS tree



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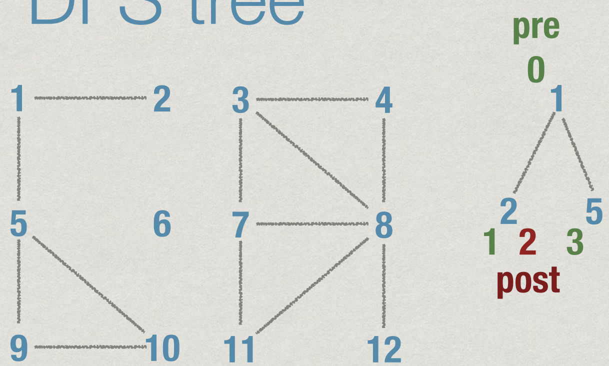

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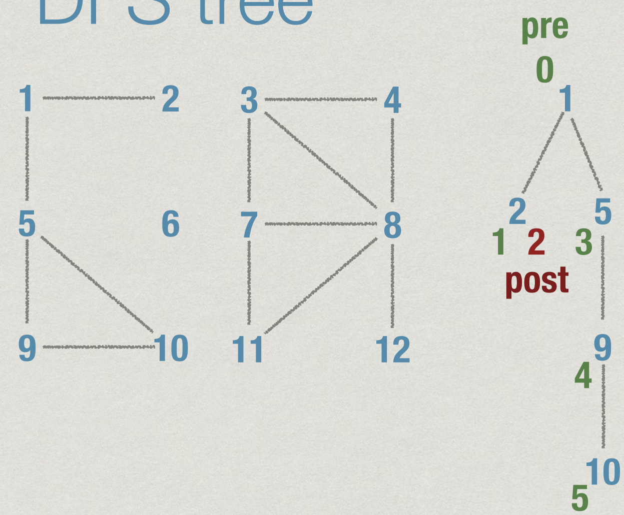
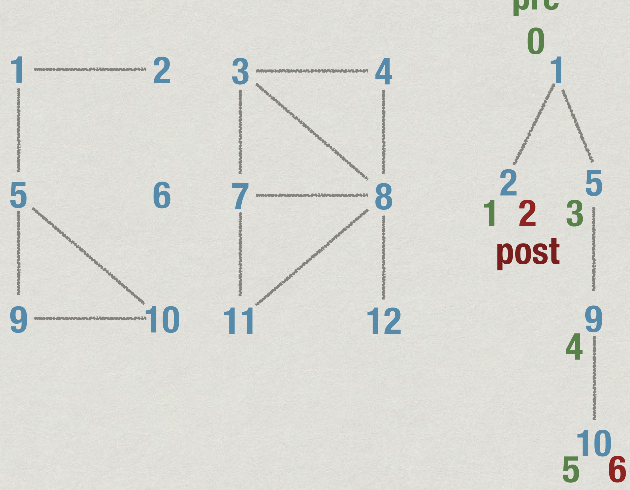
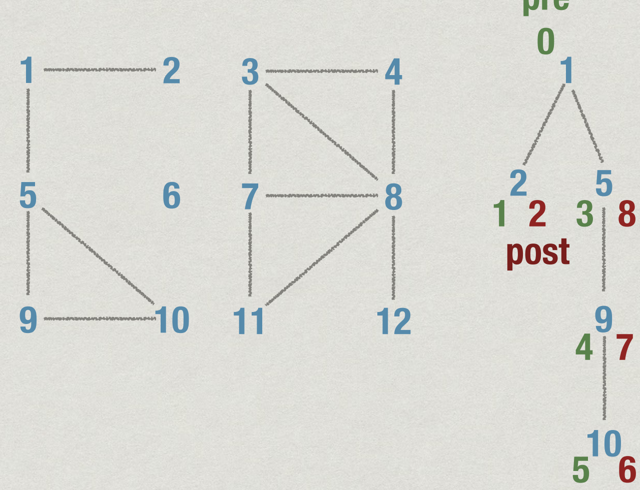


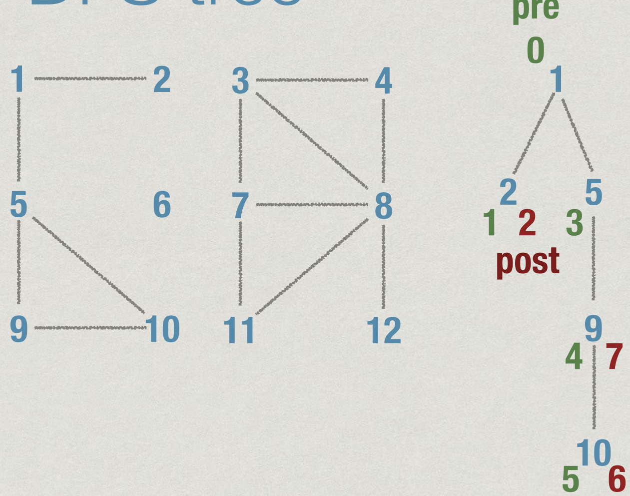
DFS tree



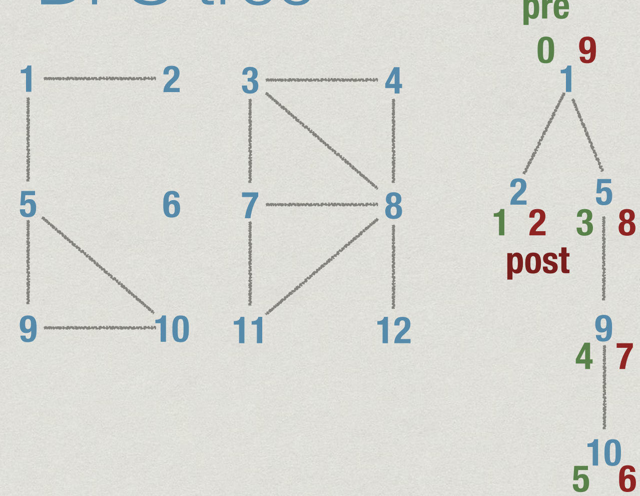
DFS tree



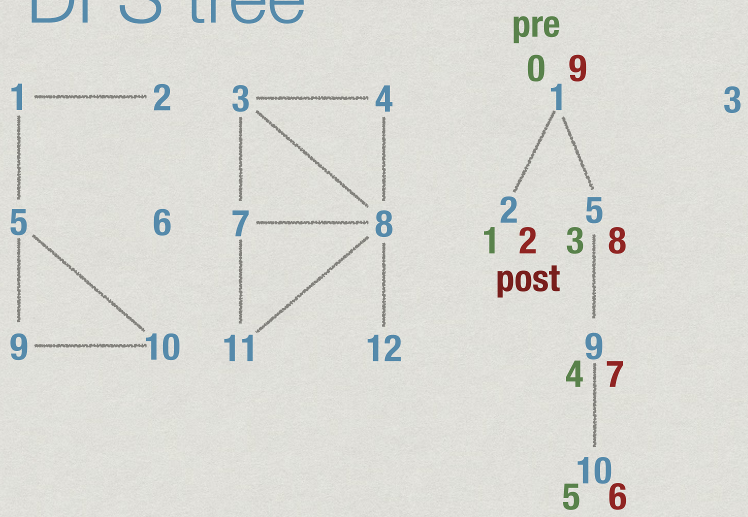
DFS tree



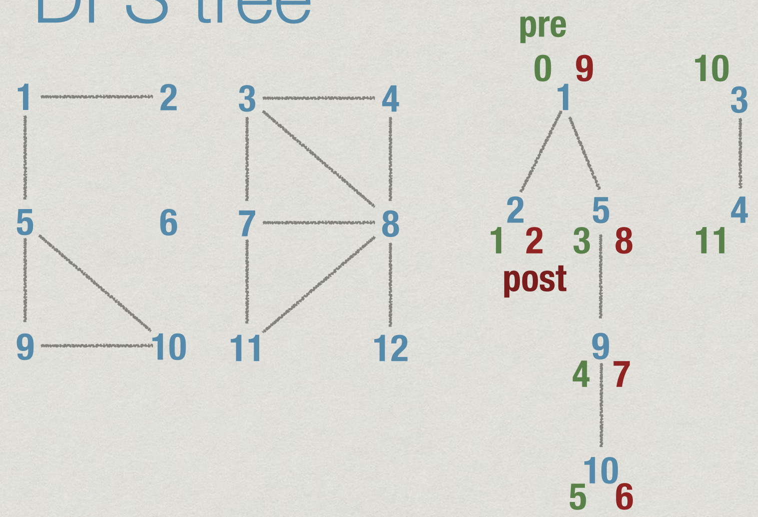
DFS tree



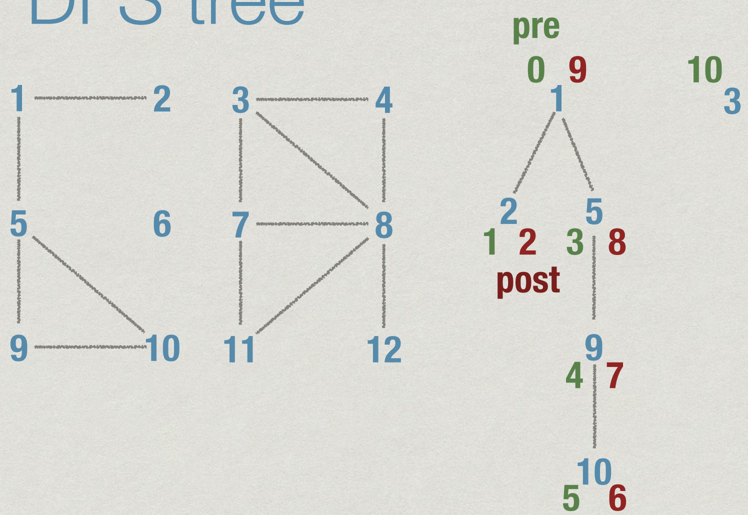
DFS tree



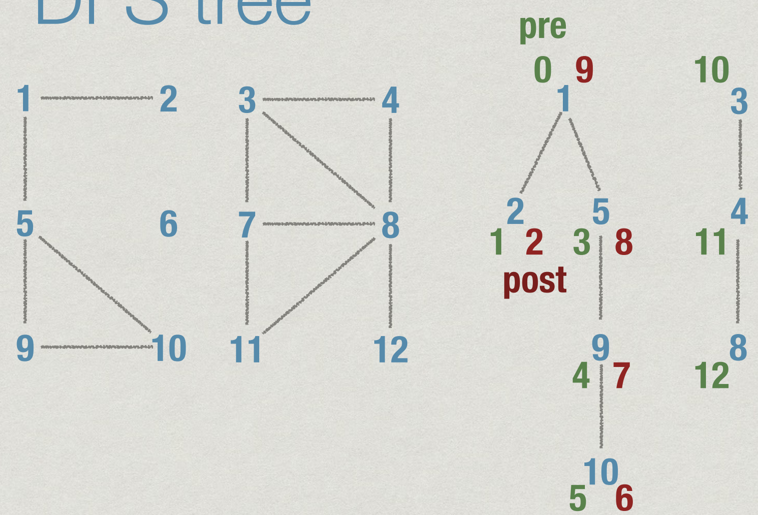
DFS tree



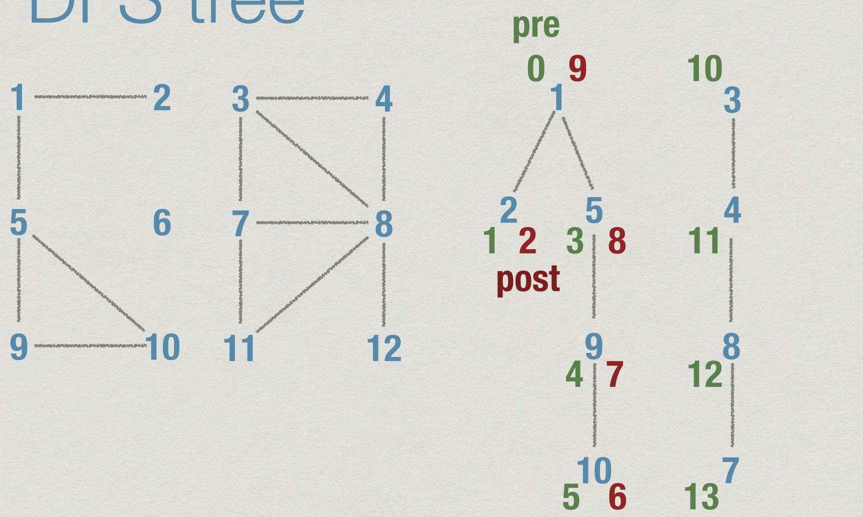
DFS tree



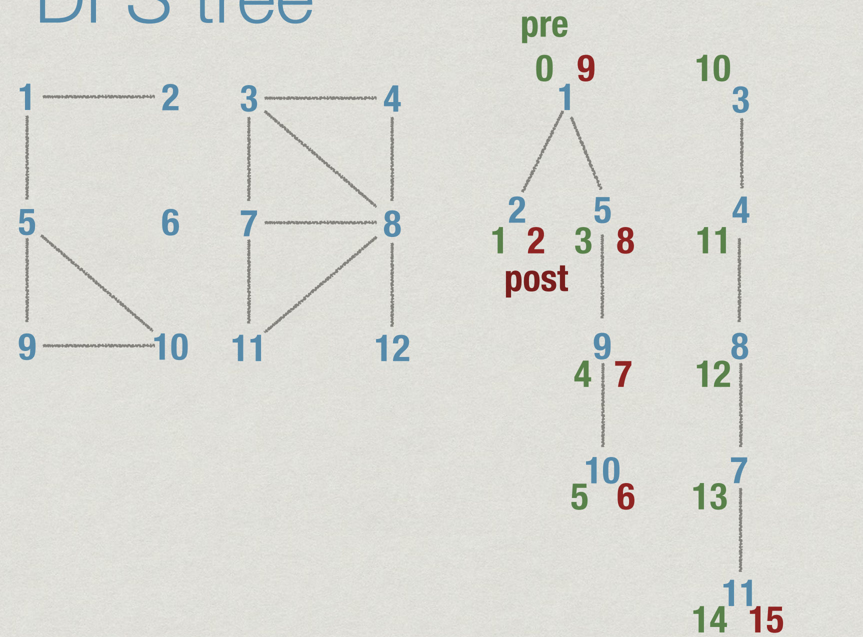
DFS tree



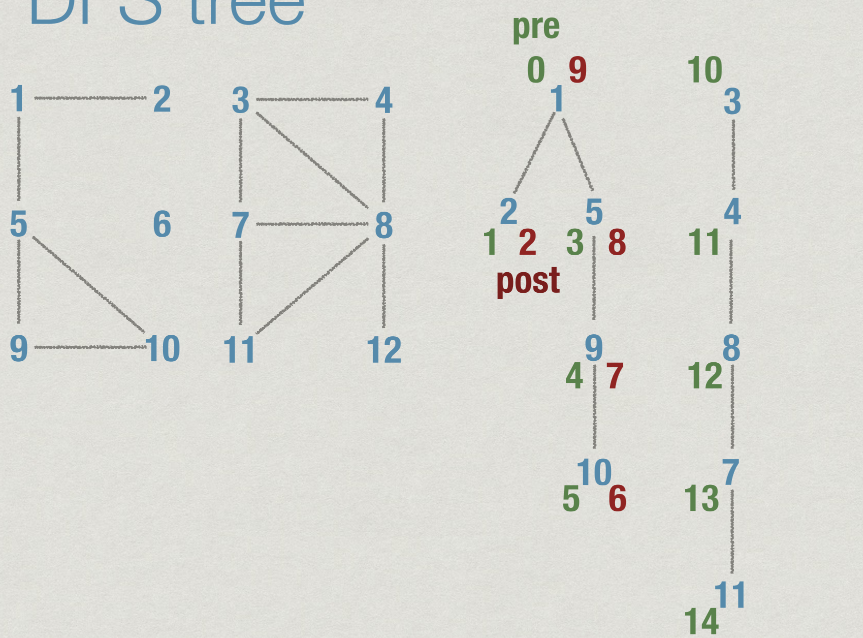
DFS tree



DFS tree



DFS tree



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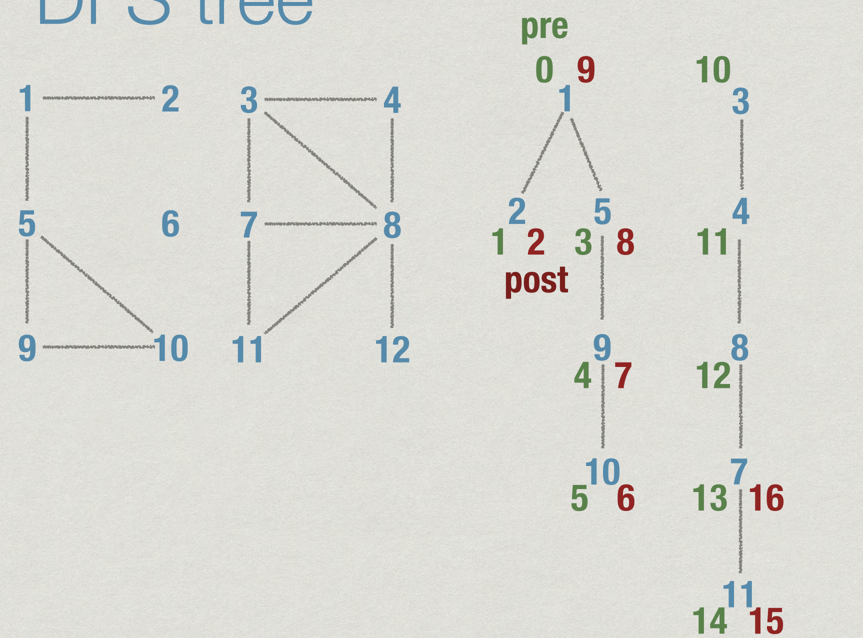
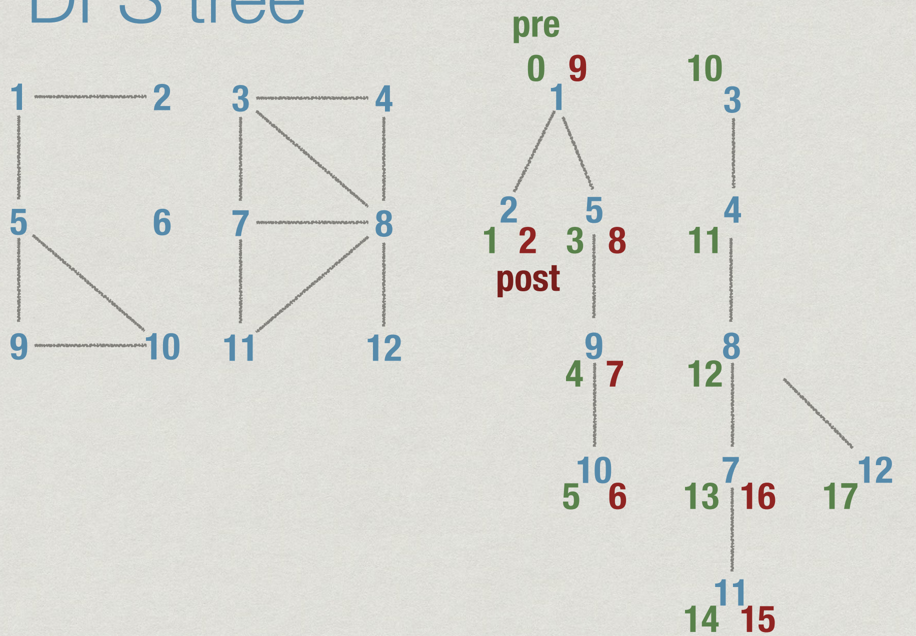


DFS tree



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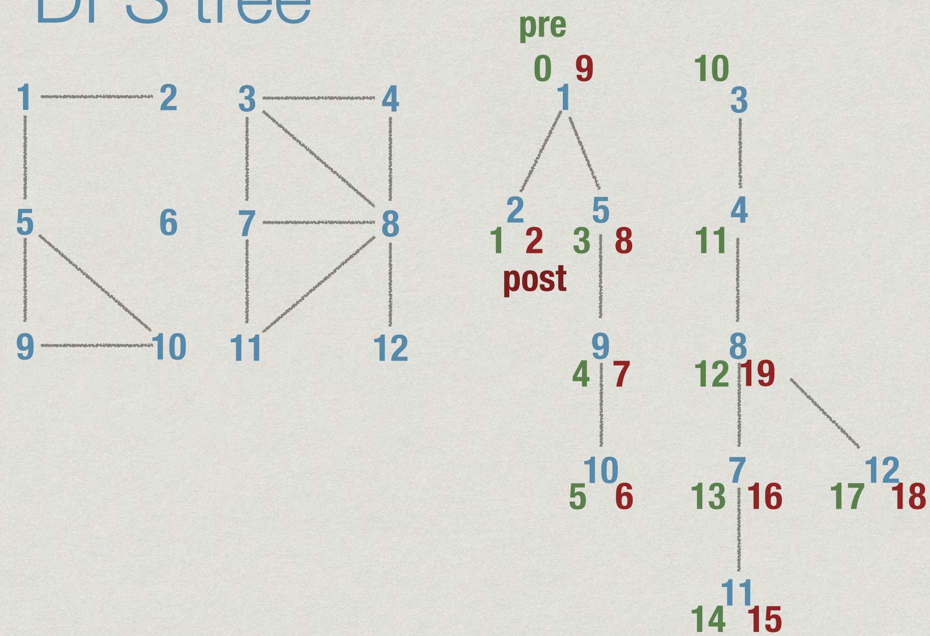

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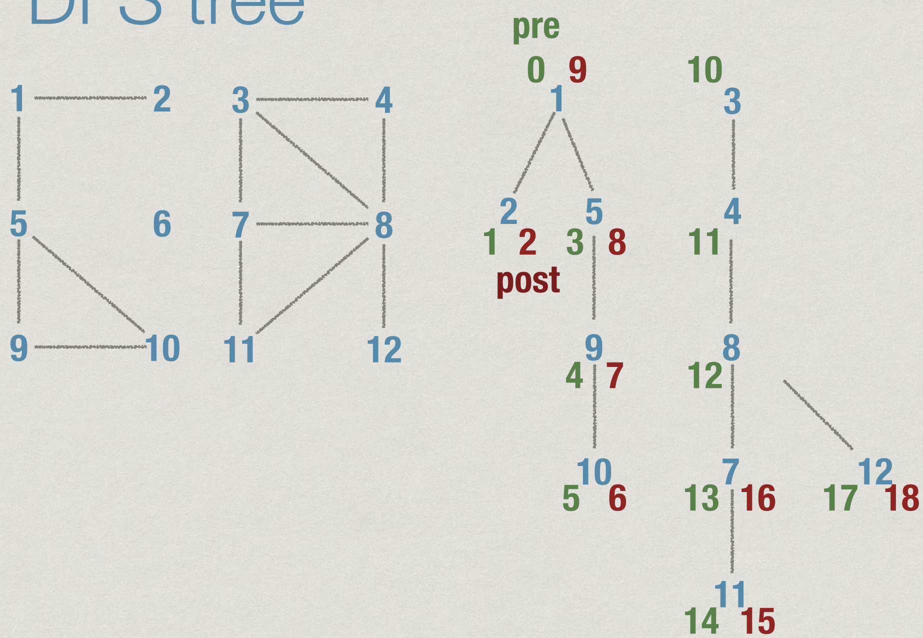

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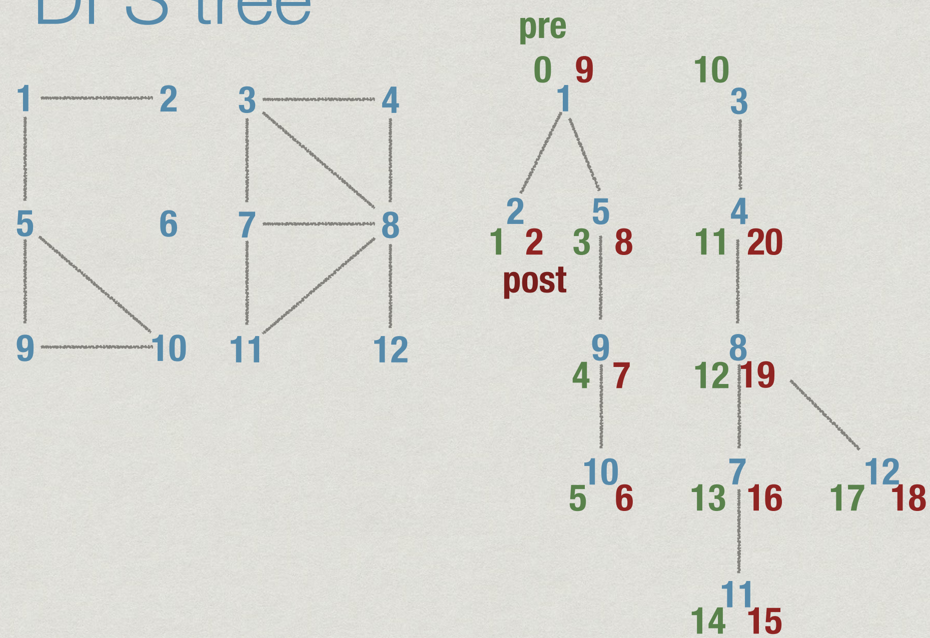
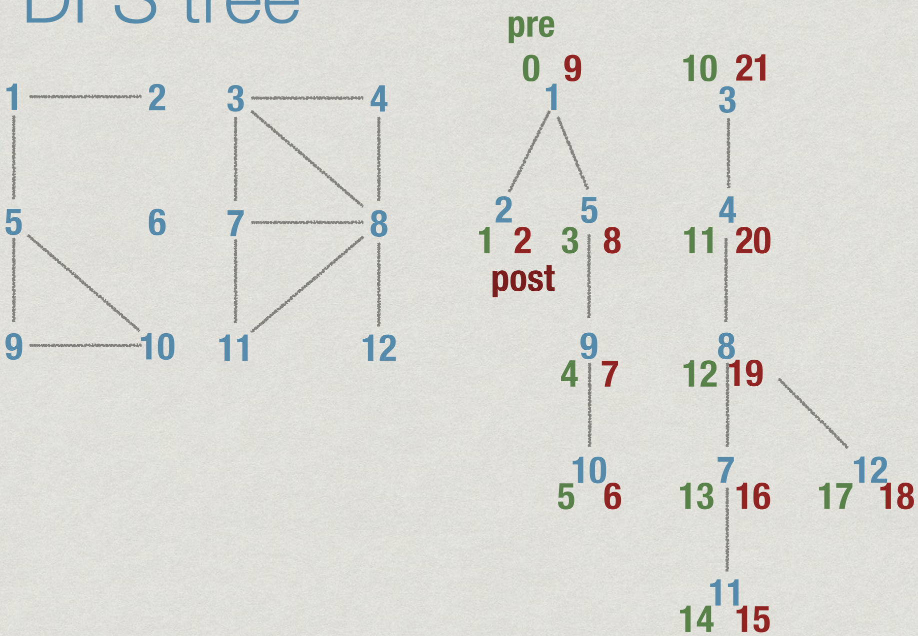
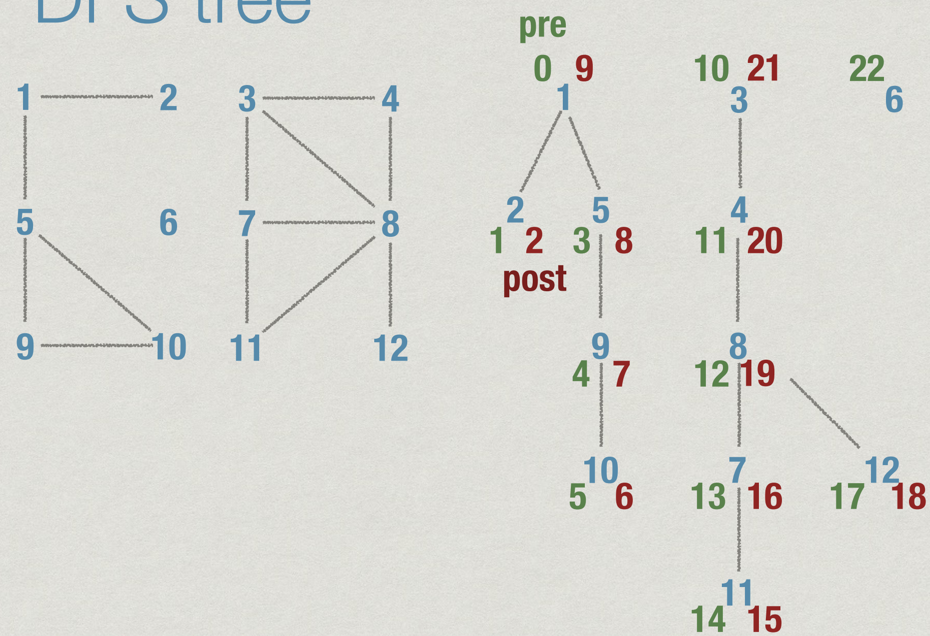


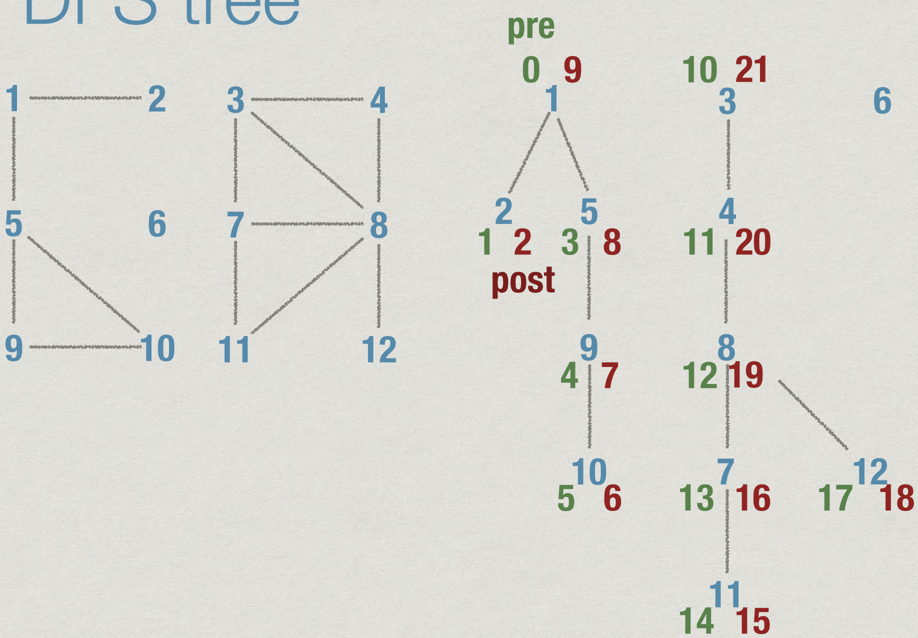
DFS tree



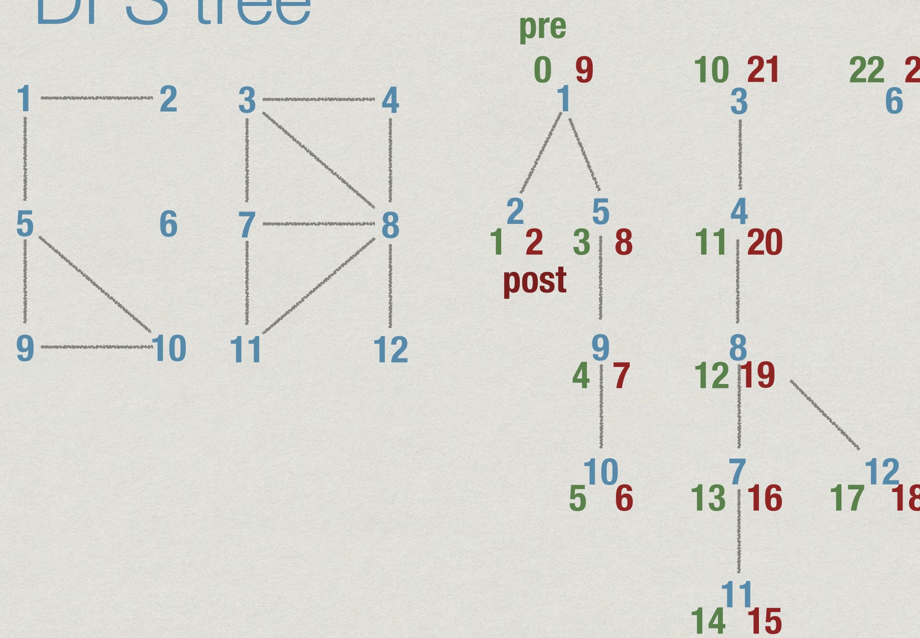
DFS tree



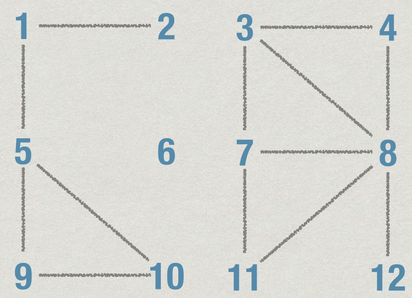
DFS tree

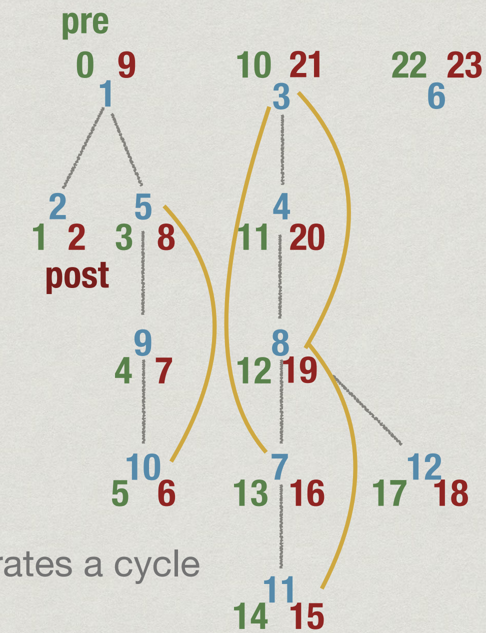


DFS tree



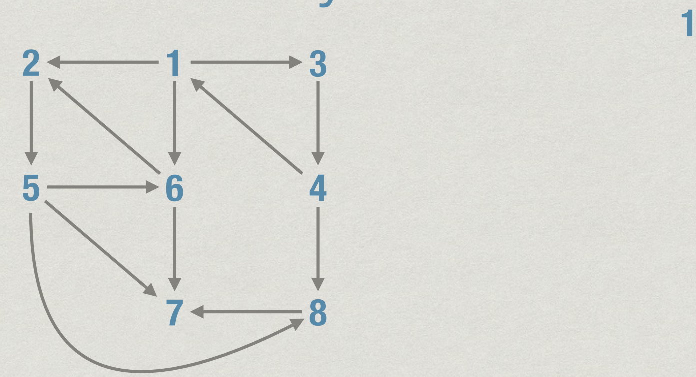
DFS tree



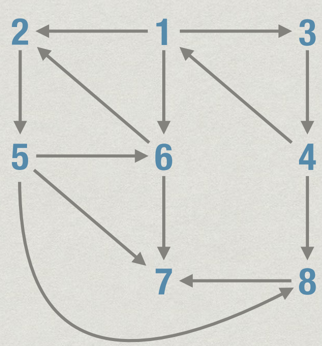


- Any non-tree edge generates a cycle

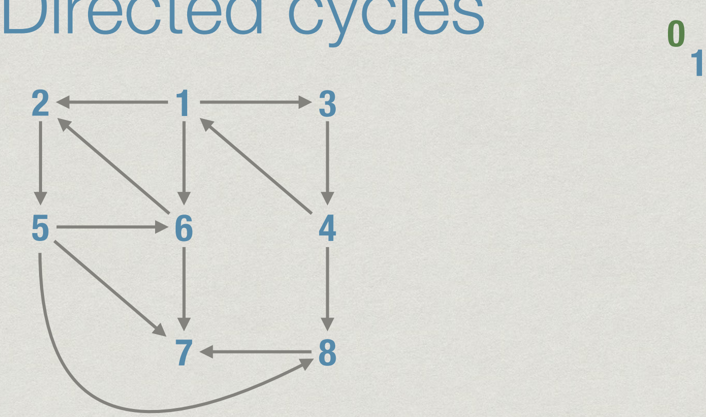
Directed cycles



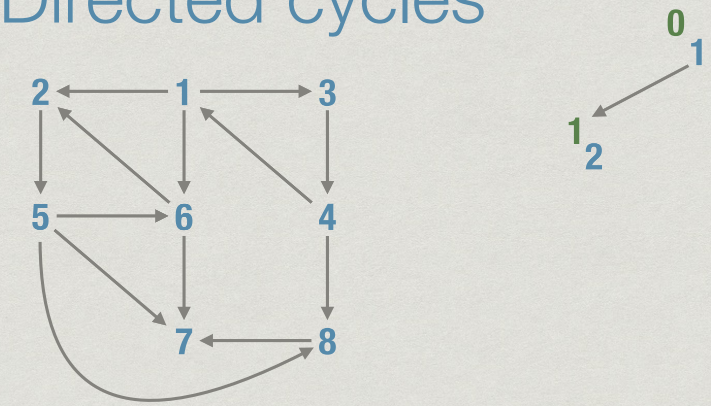
Directed cycles



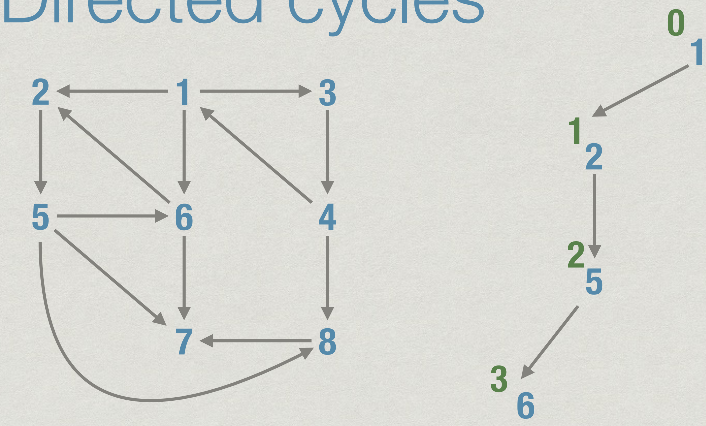
Directed cycles



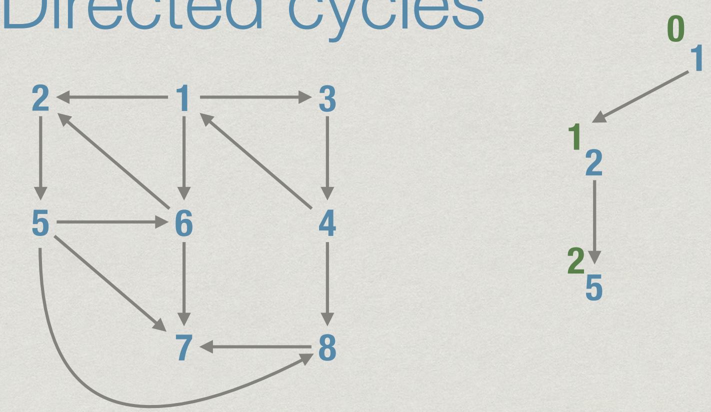
Directed cycles



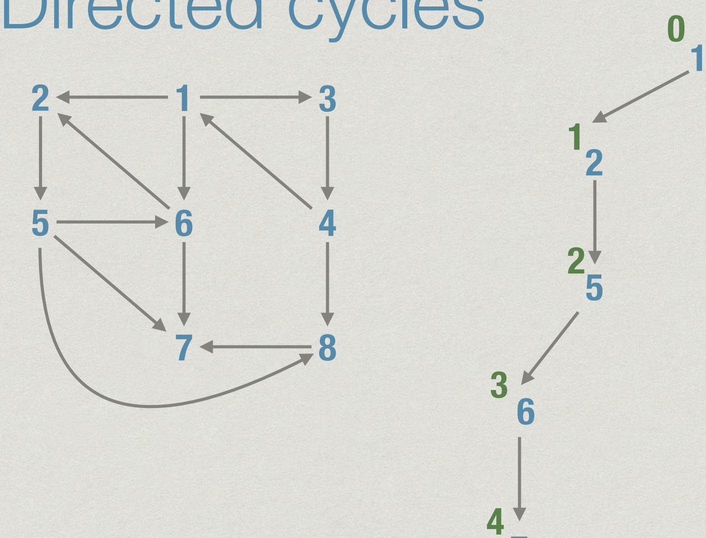
Directed cycles



Directed cycles



Directed cycles



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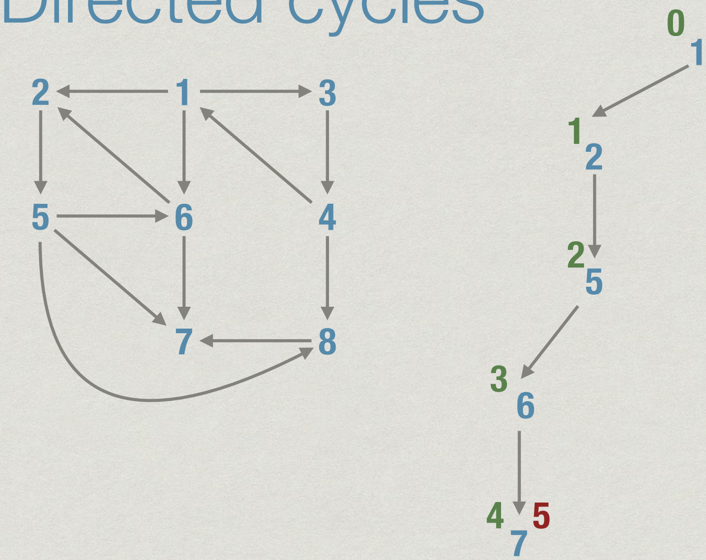
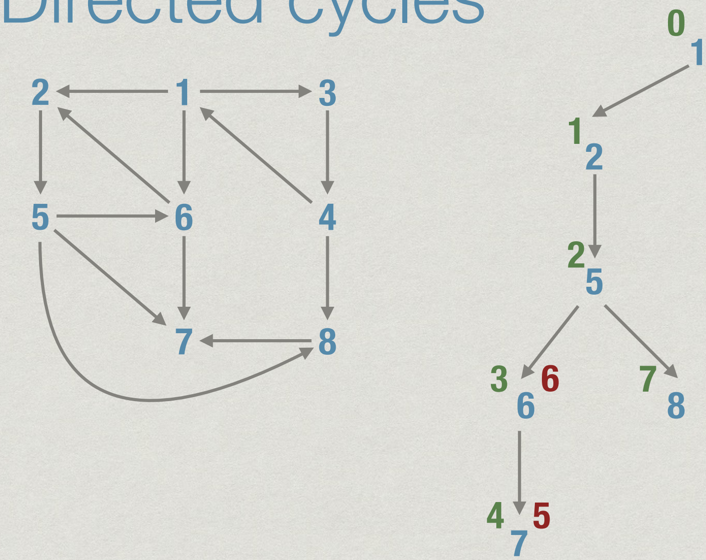


Directed cycles



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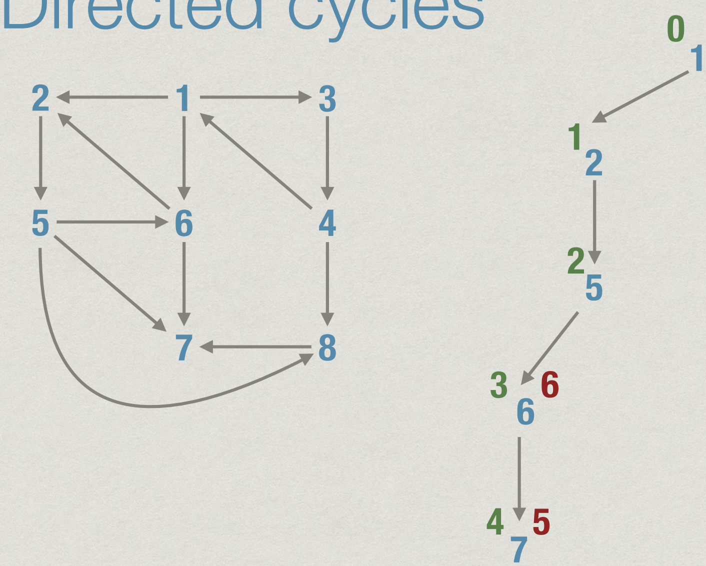
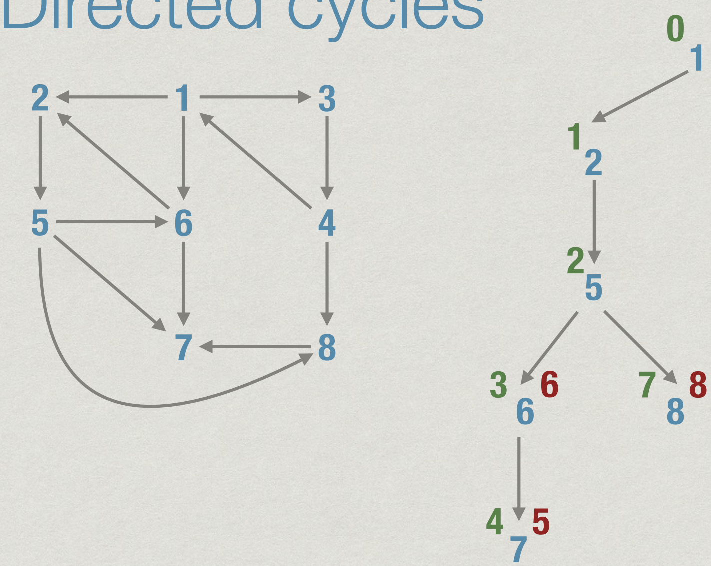


Directed cycles



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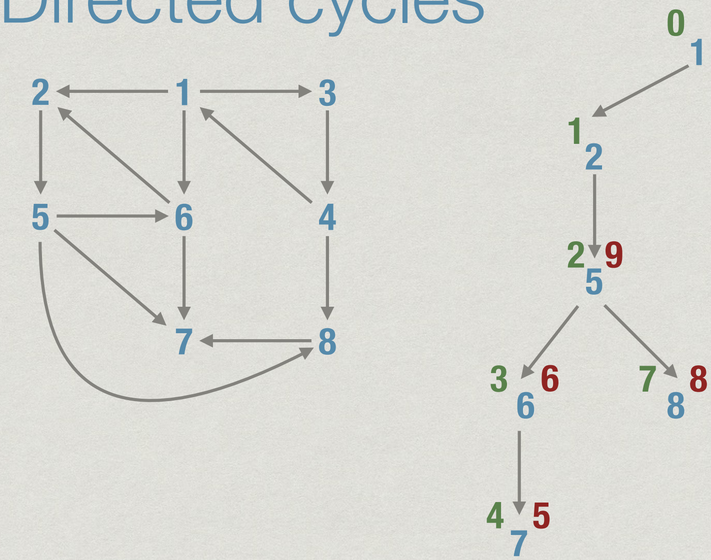

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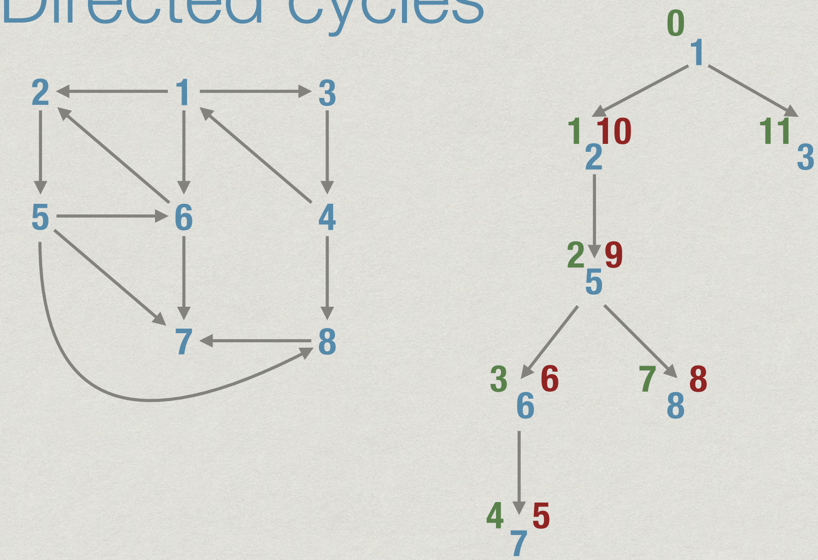
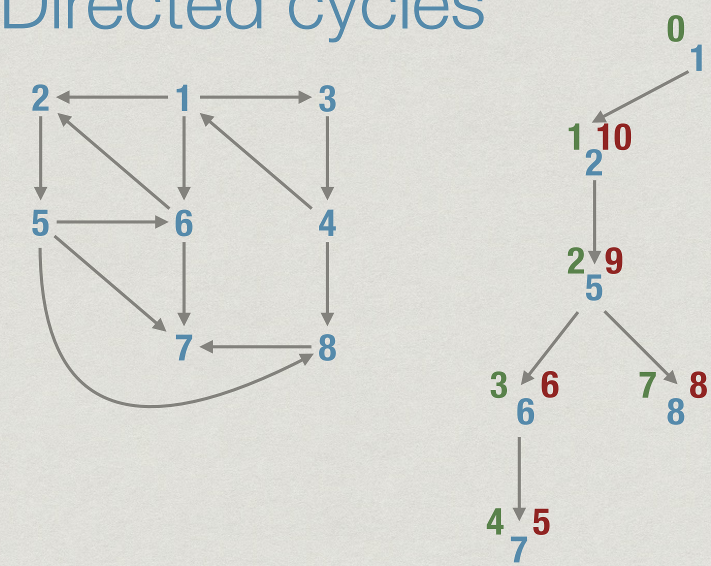
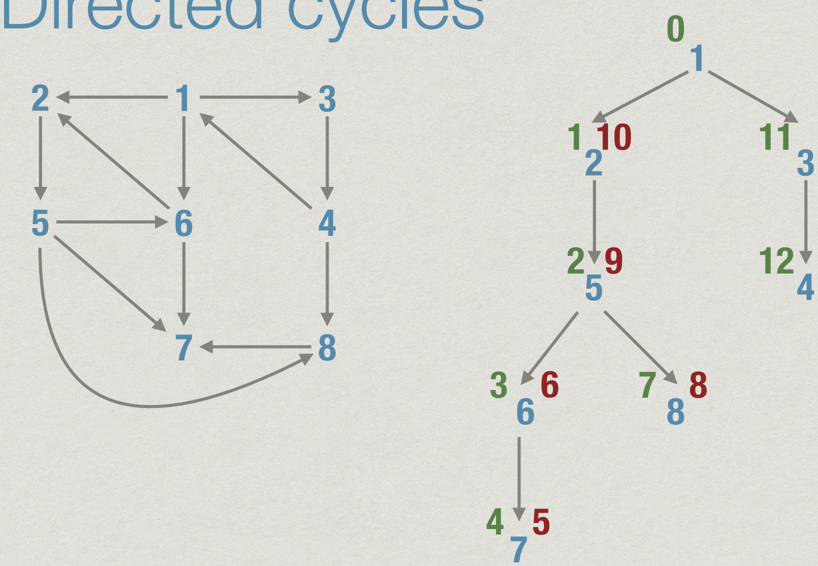


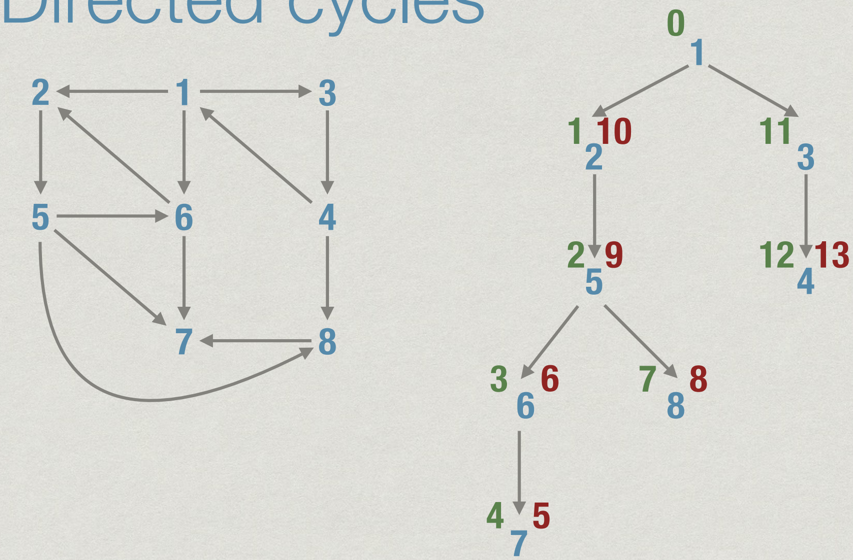
Directed cycles



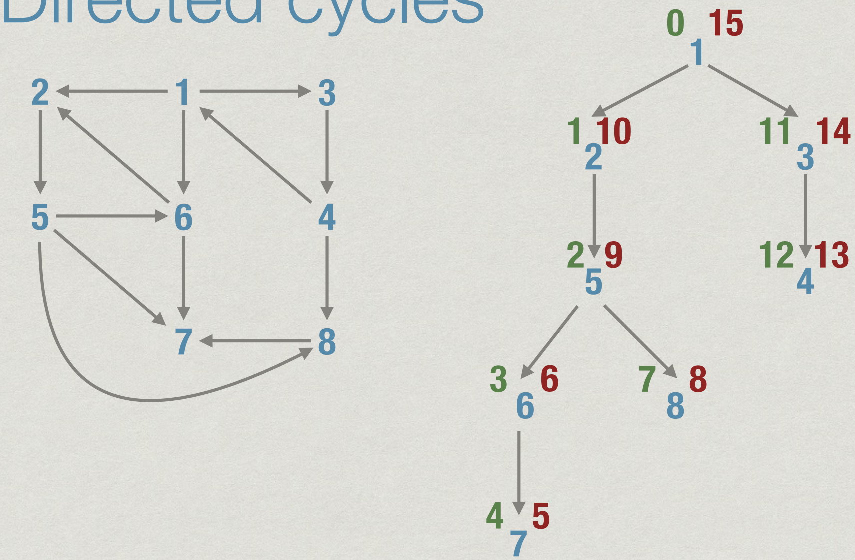
Directed cycles



Directed cycles



Directed cycles



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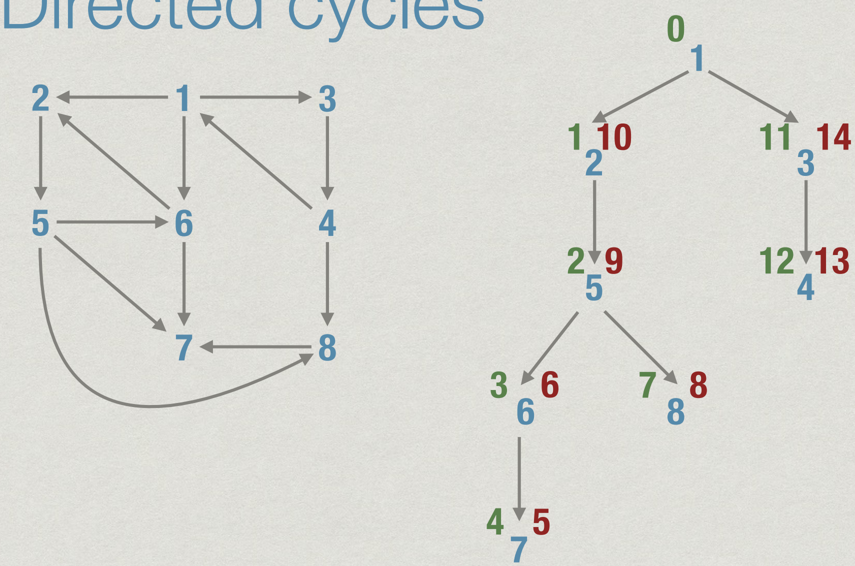

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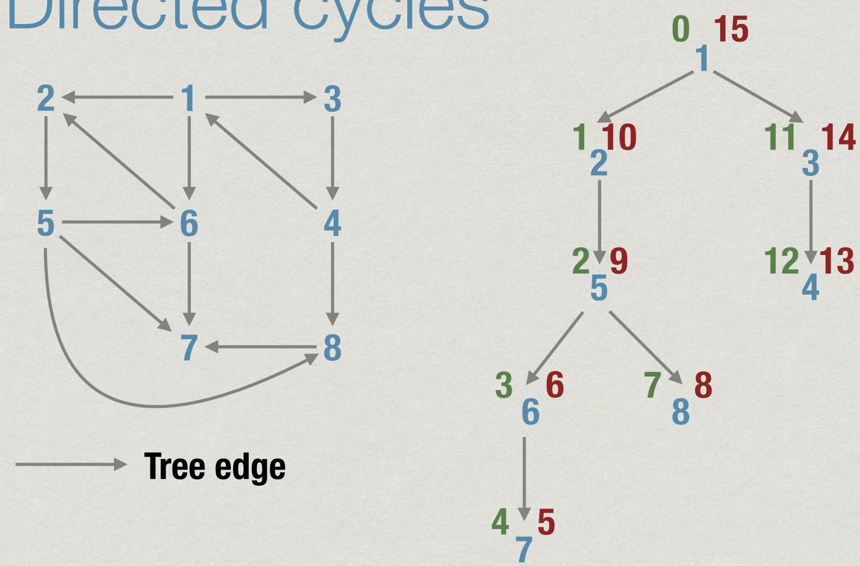

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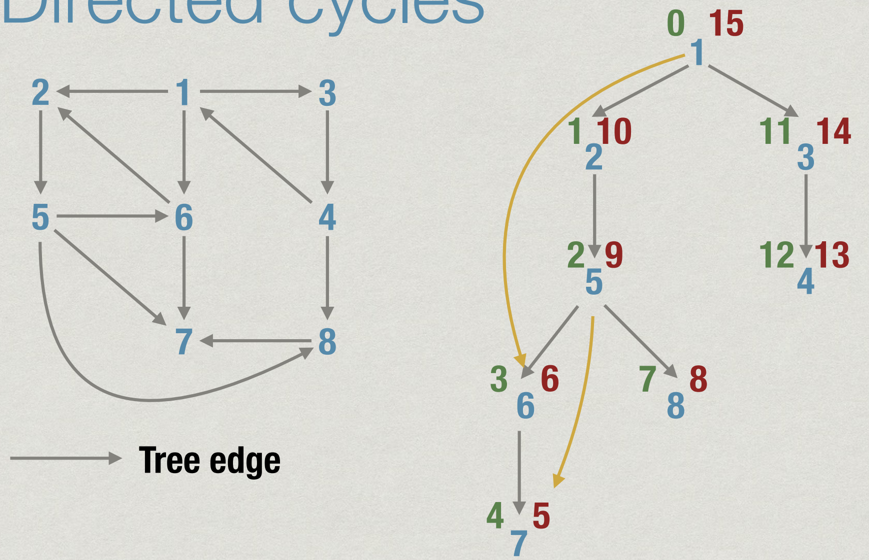

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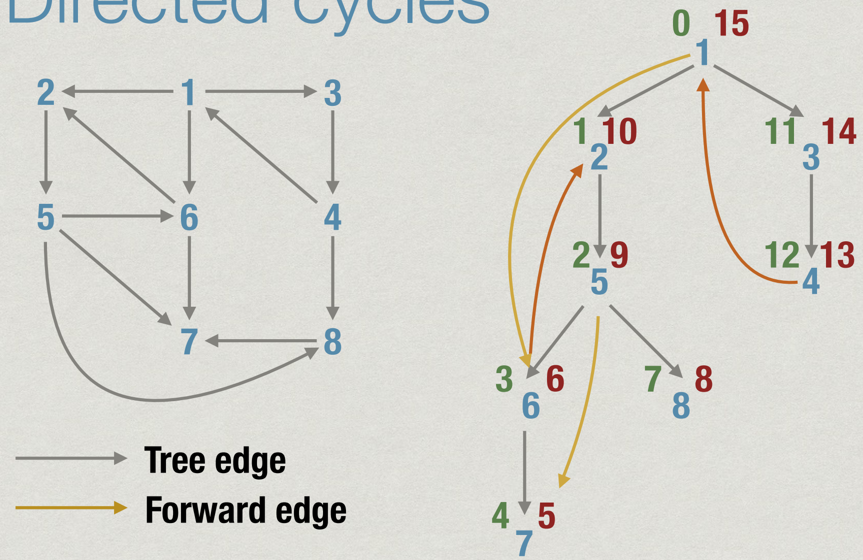

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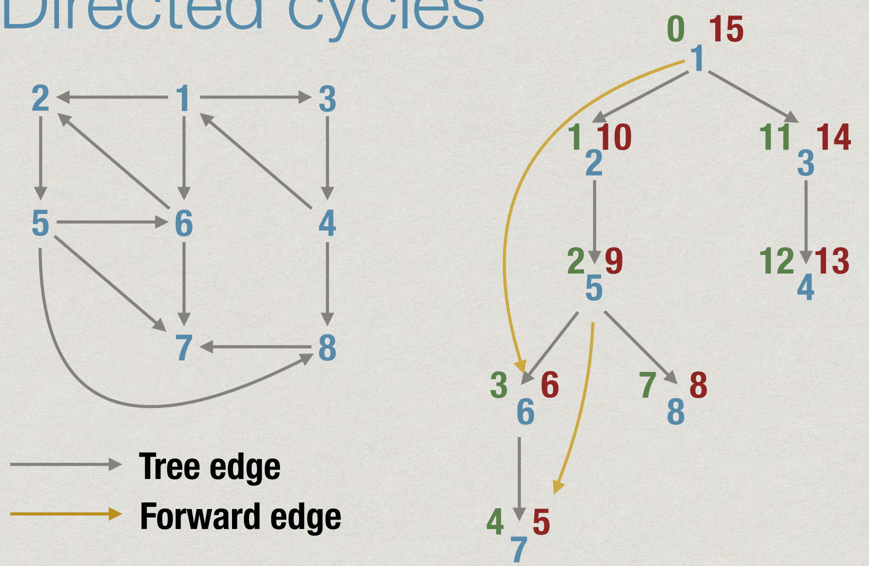

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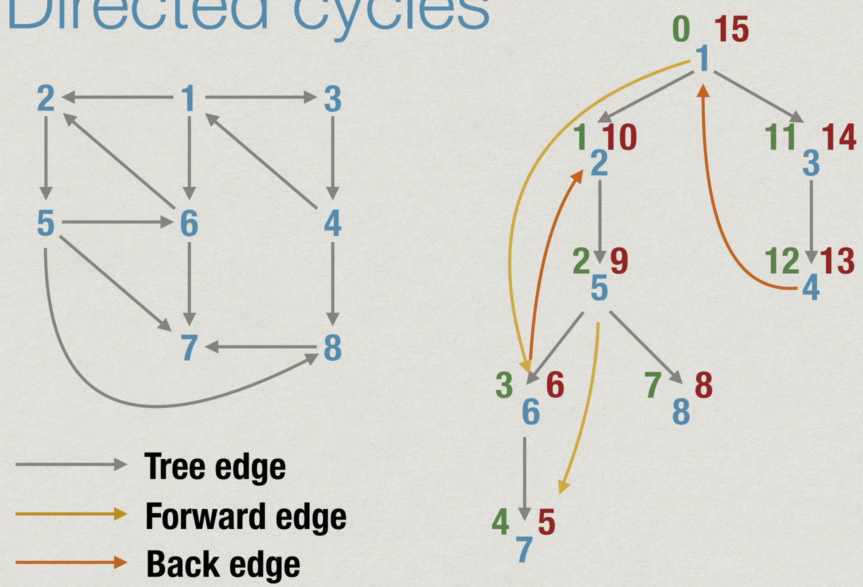
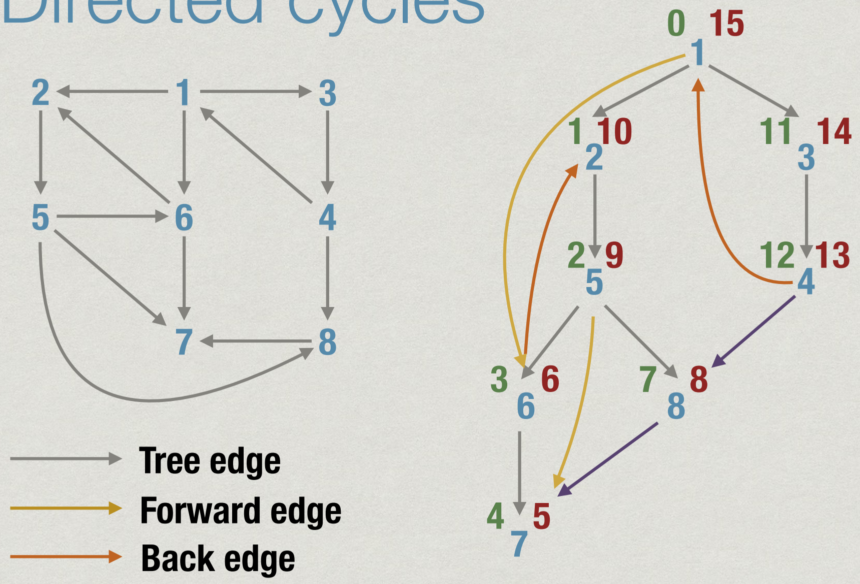
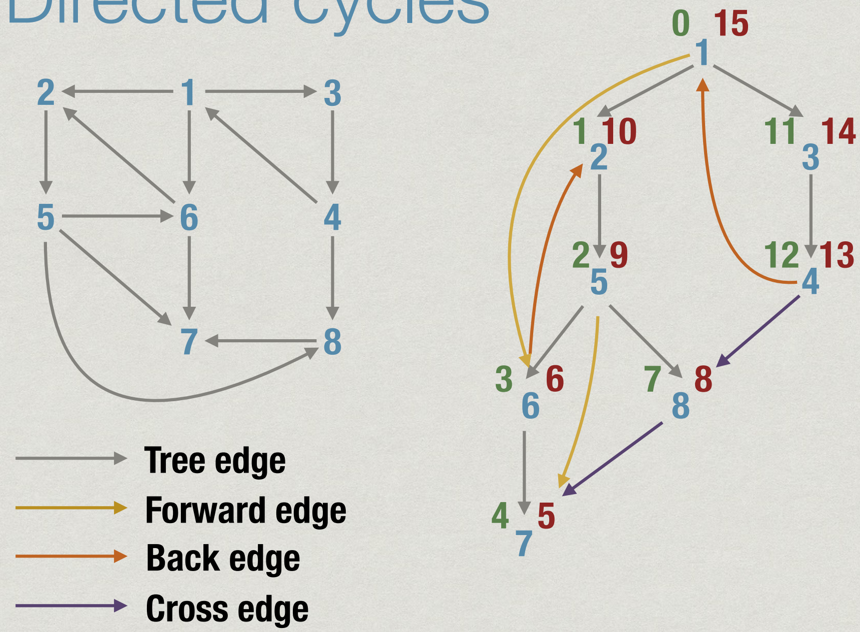


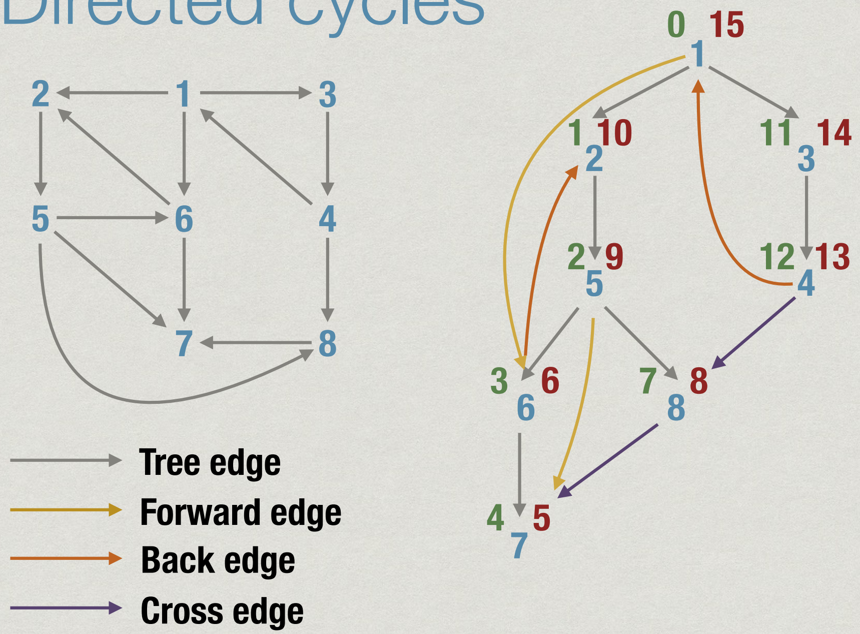
Directed cycles



Directed cycles



Directed cycles



Directed cycles

- A directed graph has a cycle if and only if DFS reveals a back edge
- Can classify edges using pre and post numbers
  - Tree/Forward edge (u,v) :
    Interval [pre(u),post(u)] contains [(pre(v),post(v)]
  - Backward edge (u,v):
    Interval [pre(v),post(v)] contains [(pre(u),post(u)]
  - Cross edge (u,v):
    Intervals [(pre(u),post(u)] and [(pre(v),post(v)] disjoint

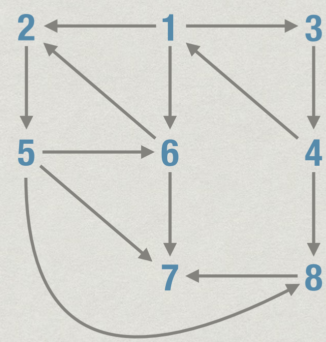
Directed acyclic graphs

- Directed graphs without cycles are useful for modelling dependencies
  - Courses with prerequisites
  - Edge (Algebra,Calculus) indicates that Algebra is a prerequisite for Calculus
- Will look at Directed Acyclic Graphs (DAGs) soon

Connectivity in directed graphs

- Need to take directions into account
- Nodes i and j are strongly connected if there is a path from i to j and a path from j to i
- Directed graph can be decomposed into strongly connected components (SCCs)
  - All pairs of nodes in an SCC are strongly connected

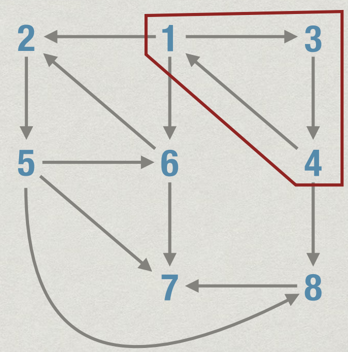
Computing SCCs



- DFS numbering (pre and post) can be used to compute SCCs

[Dasgupta, Papadimitriou,Vazirani]

Computing SCCs



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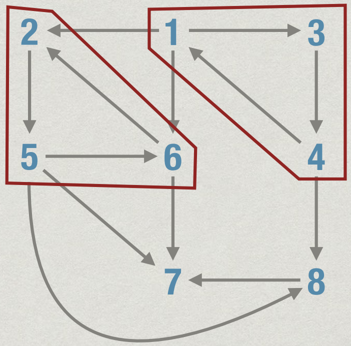
[Dasgupta, Papadimitriou,Vazirani]

Computing SCCs



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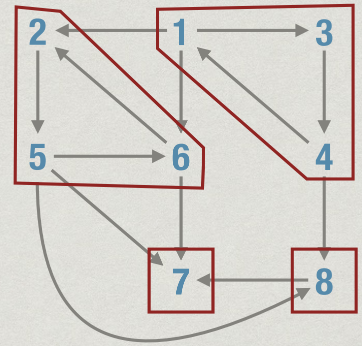
[Dasgupta, Papadimitriou,Vazirani]

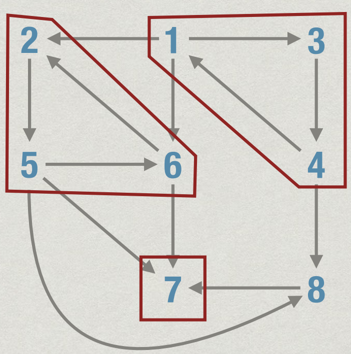
Computing SCCs



- DFS numbering (pre and post) can be used to compute SCCs

[Dasgupta, Papadimitriou,Vazirani]

Computing SCCs



- DFS numbering (pre and post) can be used to compute SCCs

[Dasgupta, Papadimitriou,Vazirani]

Other properties

- A number of other structural properties can be inferred from DFS numbering
- Articulation points (vertices)
  - Removing such a vertex disconnects the graph
- Bridges (edges)
  - Removing such an edge disconnects the graph