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Chennai Mathematical Institute

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Tools of the Mathematical Trade

Chennai Mathematical Institute is starting a series of lectures aimed at graduate students and interested researchers. The aim of these lectures, to be given by active researchers, is to expose different mathematical techniques that are important in current mathematical research. The first in this series will be on the subject

Categories and Sheaves

by

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Dates: January 26 - February 5, 2009.

Limited travel support and local hospitality is available for interested participants. Those interested should apply with a short CV to cat@cmi.ac.in before **December 15, 2008**.

It is intended to follow-up this series of lectures with a lecture series on Categories and Motives.

Abstract

Sheaves play an important role in geometry. The study of cohomology of algebraic varieties led Grothendieck to define more general notions of topologies and sheaves. They were first used to define étale cohomology which was a key-ingredient in the proof of the Weil conjectures. These topologies also appear in more recent theories like crystalline cohomology or Voevodsky's theory of motives.

In this series of lectures, I shall define Grothendieck topologies and explain the associated notion of sheaves. It will use some category theory as the sections of a sheaf will no longer be parametrised by the open subsets of a space but by objects in a category. I will introduce the étale topology on schemes. As we shall see, this can be done using only rings i.e. without depending on the language of schemes. Then, I will explain how homological algebra can be used to define sheaf cohomology. This definition will include the definition of étale cohomology and we will also see that group cohomology can be interpreted as a special case of sheaf cohomology associated to a Grothendieck topology. Moreover, the Galois cohomology of a field, which is the cohomology of its profinite Galois group, can also be interpreted as a significant special case of étale cohomology. I also hope to prove some results in the cohomology of manifolds which have analogues in étale cohomology.