Finite graphs for infinite functions.

$a_0 + a_1 x + a_2 x^2 + a_3 x^3$ finitely many terms - polynomials

Series $a_0 + a_1 x + a_2 x^2 + \cdots$

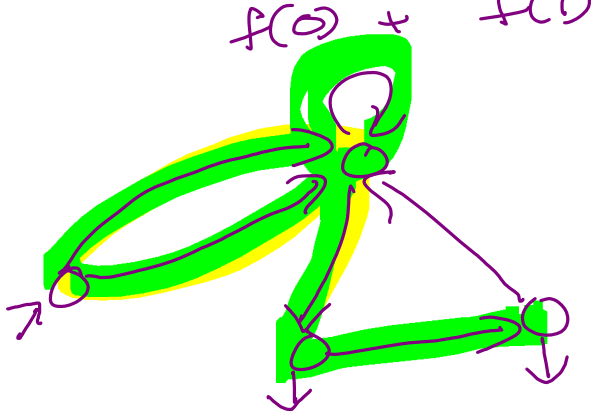
$1 + x + x^2 + x^3 + \cdots$

$3 + 4x + 5x^2 + 6x^3 + \cdots$

$f : \mathbb{N} \to \mathbb{Z}$

$i \to a_i$

$f(0) + f(1)x + f(2)x^2 + \cdots$

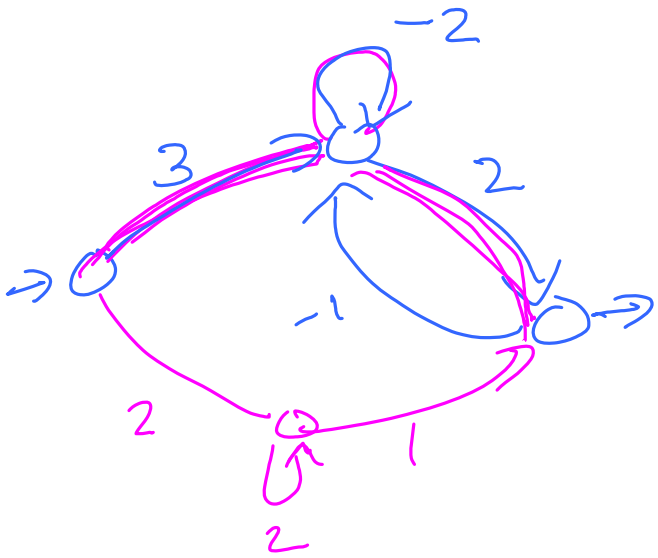


directed
not simple
initial vertices ($\to\circ$)
final vertices ($\circ\to$)

Graphs

$0 \mapsto 0$
$1 \mapsto 0$
$2 \mapsto 2$
$3 \mapsto 4$

$i \mapsto$ no. of paths of length i from src/initial to final



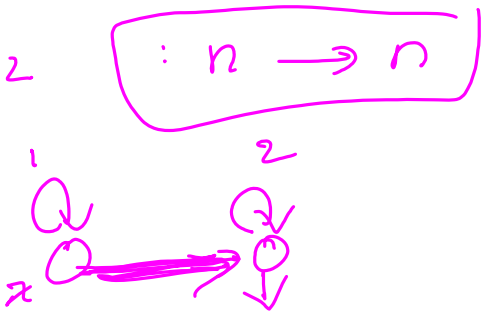
$i \to$

$2 \mapsto 6$

$3 \mapsto (3 \cdot -2 \cdot 2 + 2 \cdot 2 \cdot 1)$

1. $f_1 : n \longmapsto 1$

2. $f_2 : n \longrightarrow n$

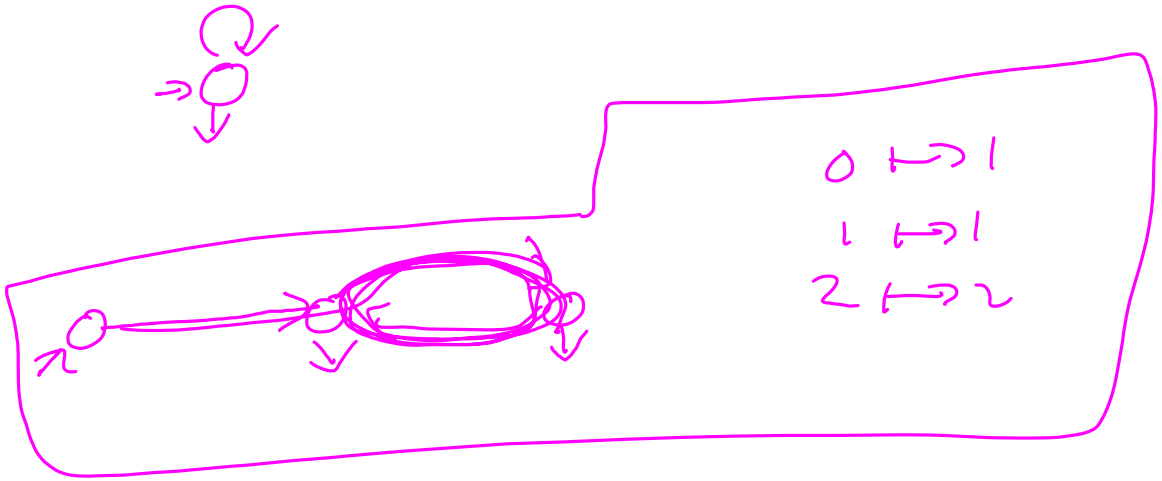


$0 \mapsto 0$

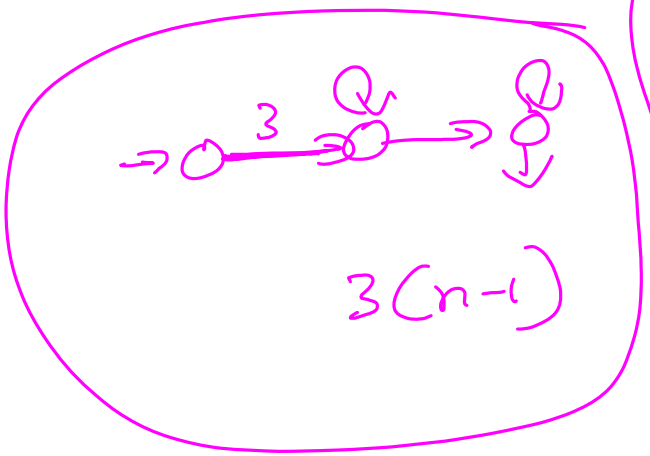
$1 \mapsto 1$

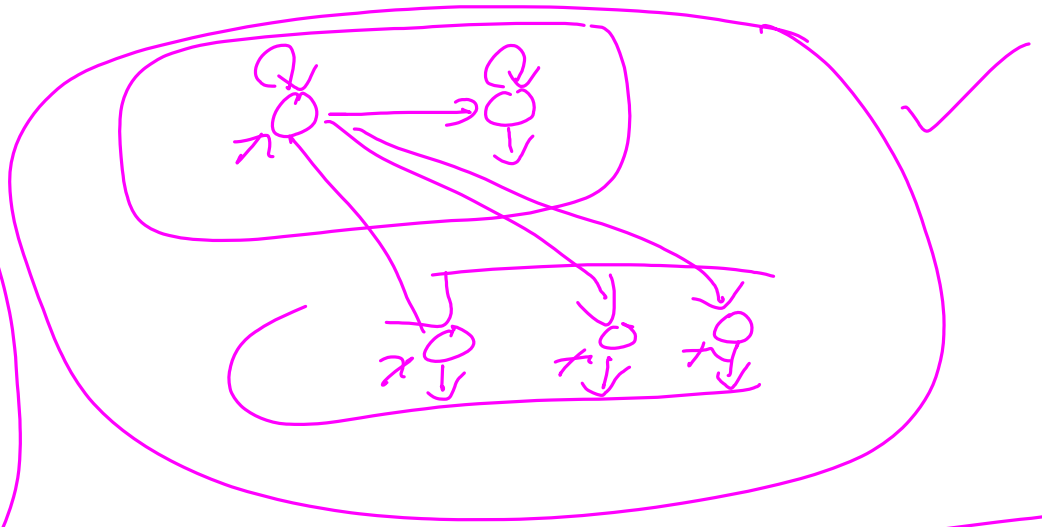
$2 \mapsto 2$

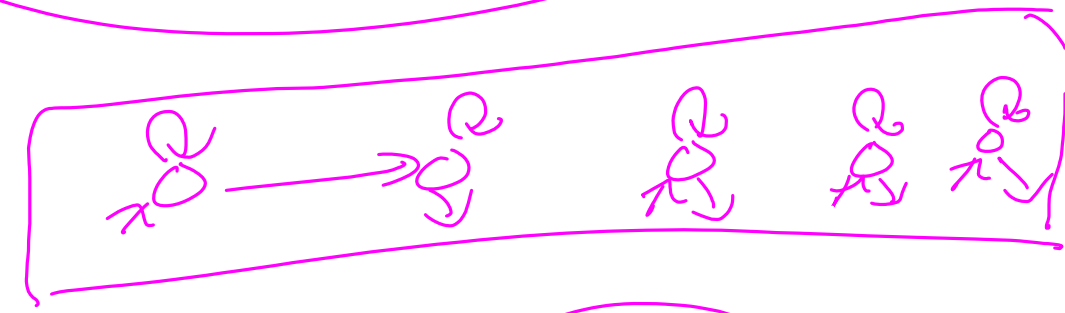
nC_1



3. $f \quad n \mapsto n+3$







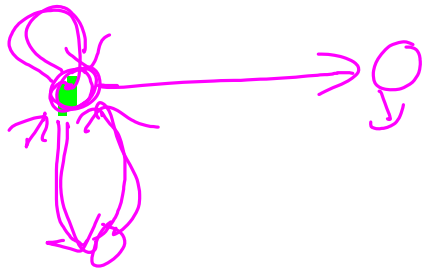
$f_1 \quad + \quad f_2$

4. $f : n \to 3$

$f(0) \rightarrow 0$

$f(1) \rightarrow 1$

$f(n) \rightarrow f(n-1) + f(n-2)$ $\quad \forall n \geq 2$



f g h

$f(i) = g(i-2) + h(i-1)$

$h(i) = 2h(i-1) + fh(i-3)$

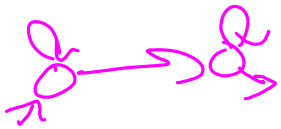
$g(i) = i/2$

$n \mapsto 2^{(2^n)}$,

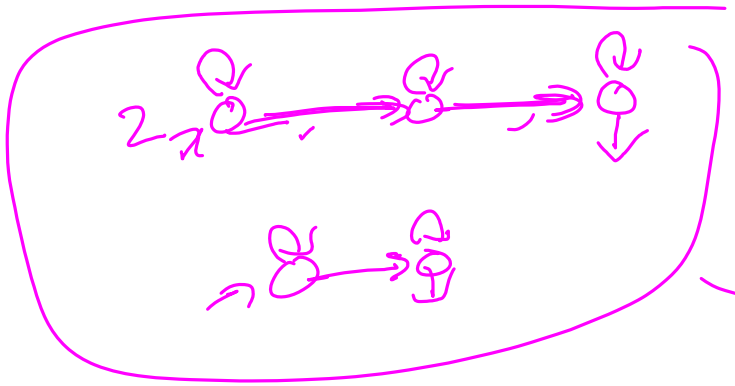
$n \mapsto 2^n$

$n \mapsto 2^{(n^2)}$

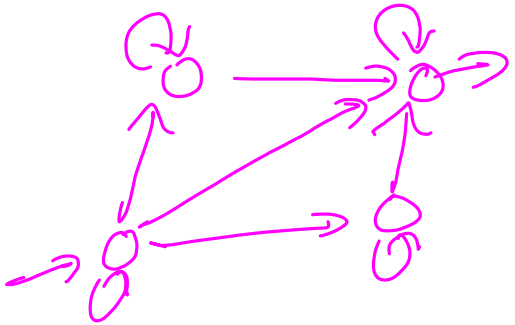
$n \mapsto n$



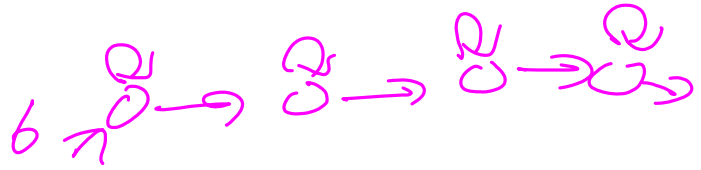
$n \mapsto n^2$



${}^nC_2$ $\frac{n(n-1)}{2}$

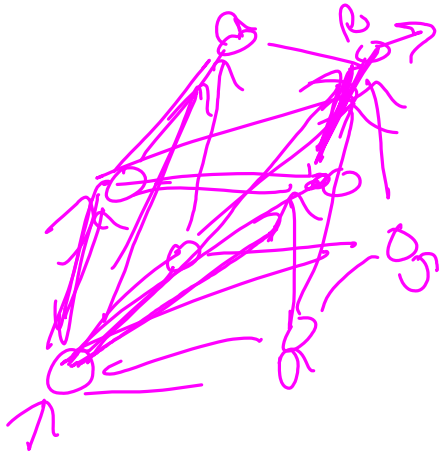


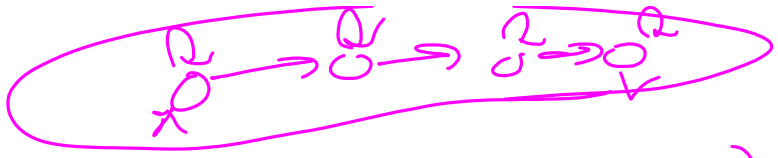
$\frac{n(n+1)n-2)}{6}$



$n \mapsto n^3$

$n^3 - 3n^2 + 2n$



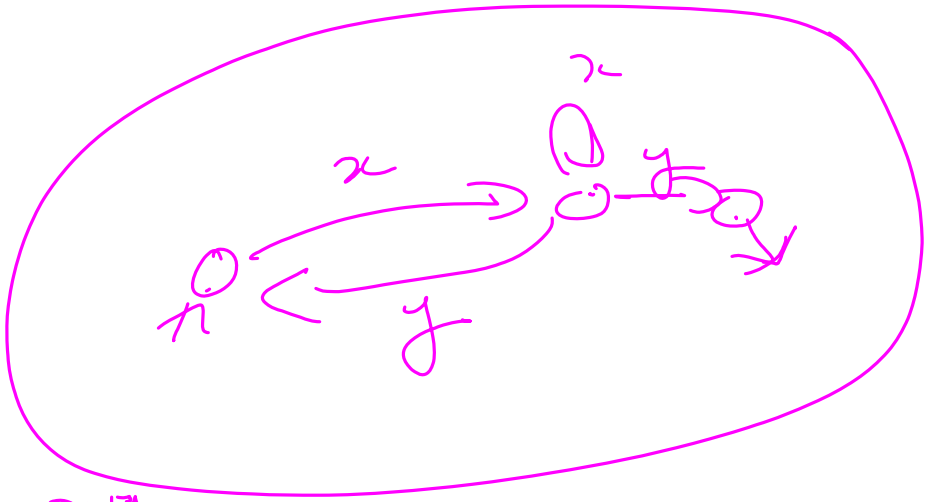


$$\left(6 \cdot \frac{n(n-1)(n-2)}{6} + 6\,\frac{n(n-1)}{2} + n \right) = n^3$$

$$x + 5x^2 + x^3$$

$xyx \notin xxy$

Weighted automata



$xy\,xy$

Language.

finite state automata

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