Assignment 1 : Complexity Theory*

To be submitted by Feb 19th

1. Let S_1, S_2, f be space-constructible with $S_2(n) \geq log(n)$ and $f(n) \geq n$. Show that NSPACE($S_1(n)$) $\subseteq$ SPACE($S_2(n)$) implies NSPACE($S_1(f(n))$) $\subseteq$ SPACE($S_2(f(n))$).
2. For any real number $a > 0$ and natural number $k > 1$, show that DTIME(n^k) $\subset$ DTIME ($n^k log^a(n)$). (Note that one consequence of this result is that within P, there can be no "complexity gaps" of size $(logn)^{\Omega(1)}$.)
3. **PRIMES** is in **NP** $\cap$ **co-NP**.
4. Show that $\mathbf{P} = \mathbf{NP} \implies \mathbf{EXP} = \mathbf{NEXP}$.
5. Prove that the problem of deciding whether a polynomial with integer coefficients has an integer solution is **NP** - hard.
6. Show that 2-**SAT** is in **NL**.
7. Show that $\mathbf{NP} \neq \mathbf{SPACE}(n)$.
8. L is a sparse set if there is a polynomial p such that $|L \cap \{0,1\}^n| \leq p(n)\ \forall\ n$. Prove that if a sparse set is **NP**-complete, then $\mathbf{P} = \mathbf{NP}$.
9. Suppose we pick a random language C by choosing every string to be in C with probability $\frac{1}{2}$. Prove that, with high probability, $\mathbf{P^C} \neq \mathbf{NP^C}$.
10. Show that there is a language $B \in \mathbf{EXP}$ such that $\mathbf{NP^B} \neq \mathbf{P^B}$.
11. Suppose $L_1, L_2 \in \mathbf{NP} \cap \mathbf{co} - \mathbf{NP}$. Then show that $L_1 \oplus L_2$ is in $\mathbf{NP} \cap \mathbf{co} - \mathbf{NP}$, where $L_1 \oplus L_2 = \{x : x$ is in exactly one of $L_1, L_2\ \}$.

*Questions collected by Abhiroop Sanyal