Computational Complexity
Assignment 1

February 13, 2013

Marks: 50

1. Prove that P$\neq$SPACE(n). **7**

2. The language 2SAT is defined as follows:

   2SAT=$\{\phi : \phi$ is a satisfiable CNF formula where each clause has exactly two literals$\}$.

   Prove that 2SATis NL-complete. **7**

   (Hint: Reduce 2SATto directed s-t connectivity. For each clause $x \vee y$, draw a directed edge from $\bar{x}$ to y and from $\bar{y}$ to x. Show that the reduction is correct and can be done in logspace.)

3. Prove that $\Sigma_k^p = \Pi_k^p$ implies PH$= \Sigma_k^p$. **7**

4. *Cook reduction:* A language L is polynomial-time *Cook reducible* (also known as *Turing reducible*) to a language L' if there is a polynomial-time TM M that, given an oracle for deciding L', can decide L.

   Show that Cook reducibility is transitive. Show that the usual notion of reductions (also called *Karp reduction* or *many-one reduction*) is a special case of Cook reduction. **7**

5. Define UCYCLE $= \{G|G$ is an undirected graph with a simple cycle $\}$. Show that UCYCLE $\in$ L. **7**

   (Hint: You may assume that, for each vertex v, you are given a function $next_v$ which gives a cyclic ordering of neighbours of v. Thus, if u is a neighbour of v, then $next_v(u)$ is the neighbour of v which is next to u in that cyclic ordering.)

6. Define CYCLE $= \{G|G$ is a directed graph that has a directed cycle$\}$. Prove that CYCLE is NL-complete. **7**

   (Hint: You may consider the notion of a layered graph $G^{(k)}$ corresponding to the given graph G. The graph $G^{(k)}$ has vertex set $V \times [k]$. If $(u, v) \in E(G)$, then $((u, i), (v, i + 1)) \in E(G^{(k)})$ for $1 \leq i < k$.)

7. For a list S of integers, let mode(S) be the element that appears the maximum number of times in S. Show that there is a deterministic logspace algorithm that prints 'no' if mode(S) appears at most $\lfloor n/2 \rfloor$ times, and prints 'yes' and mode(S) if mode(S) appears at least $\lfloor n/2 \rfloor + 1$ times. Show that this can be done *simultaneously* in logspace and linear time. **8**