Design and Analysis of Algorithms
Hints to the endsem questions

1. Both these statements are false. These are questions 22.3.7 and 22.3.8 from CLRS which ask for a counter-example.
2. For the sake of simplicity, assume the elements to be all distinct. Individual medians of X and Y can be found in $O(1)$ time. If $median(X) < median(Y)$, first half part of X and second half of Y can be discarded, as the combined median can not be one among them. This can continue only for $\log n$ iterations.
3. Take log of the reciprocals of probabilities and execute Dijkstra's algorithm.
4. Select any vertex as source, assign capacity 1 to all edges and solve the flow problems for each of the remaining vertices as sinks. The graph is k-edge connected iff the flow is at least k in all cases.
5. Construct a flow network with a source s connected to L and sink t connected to R. A cut could be of 3 types: s in one part and every other vertex in the other part. This cuts n edges. Similarly t in one part and rest in the other. Otherwise s, $A \subset L$, $B \subset R$ in one part and rest in the other is a generic cut. Let $B = N(A)$. Then this cut involves $n - |A| + |B|$ edges - those between s and $L \setminus A$ and those between B and t. If $|A| > |N(A)|$ then this is strictly smaller than n. Other direction can be proved similarly.
6. This has a dynamic programming algorithm of time complexity $O(2^n)$. The NP-completeness reduction is from subset sum. Let all the values in the set sum up to m and target be t such that $m - t \geq t$. Then add $m - 2t$ to the set to get an instance of balanced partition.
7. A dynamic programming algorithm runs in $O(t)$ time. The algorithm is similar to that of 0/1 knapsack which we have seen in the class.