Chennai Mathematical Institute | Due Date: –/Oct/2018

Assignment-2

Theoretical Foundations of Computer Science

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Problem1. We have seen that the hypercube graph Q_n has a perfect matching, counting problem is the following. How many number of perfect matchings are there in Q_n.

Problem2. We have seen combinatorial proofs in the class at several places, one was, the proof of the principle of inclusion exclusion formula. Give a combinatorial proof for the following :

a. $\binom{n}{k} = \frac{n}{k}.\binom{n-1}{k-1,}$ $k \neq 0$.

b. $(n-k).\binom{n}{k} = n\ \binom{n-1}{k}$

Problem3. Prove the following identities, algebraically or combinatorially :

a. $\Sigma_{k\leq n}\ \binom{m+k}{k} = \binom{m+n+1}{n}$.

b. $\Sigma_{0\leq k\leq n}\ \binom{k}{m} = \binom{n+1}{m+1}$, $m, n \geq 0$.

c. $\Sigma_k\ \binom{n}{k}.\binom{s}{n-k} = \binom{n+s}{n}$.

b. $\Sigma_k\ \binom{l}{m+k}.\binom{s}{n+k} = \binom{l+s}{l-m+n}$.

Problem4. How many relations are there, defined on a set $A = \{1, 2, \ldots, n\}$, which are of the type :
(a). Reflexive (b) Symmetric (c) antisymmetric (d) asymmetric (e) irreflexive
(f) reflexive and symmetric (g) neither reflexive nor irreflexive.

Problem5. Prove that the number of ways to choose k points, from a collection of m points, and arranged them in a circular fashion, so that no two of them are consecutive, are $\frac{m}{m-k}.\binom{m-k}{k}$.

Problem6. Considet the set $A = \{1, 2, \ldots, n\} = [n]$, find the number of k-tuples $(A_1, A_2, \ldots, A_k)$, where $A_i \subseteq A$ and k, n are positive integers, such that

a. $A_1 \cap A_2 \cap \ldots \cap A_k = \Phi$.

b. $A_1 \subseteq A_2 \subseteq \ldots \subseteq A_k$.

Problem7. Find the number of subsets A_i of $A = [n]$, such that A_i contains no two consecutive elements of A.

Problem8. Find the number of n-tuples $(a_1, a_2, \ldots, a_n)$ where each $a_i \in \{0, 1\}$, such that, $a_1 \leq a_2 \geq a_3 \leq \ldots$
.

Problem9. Find the unique sequence of real numbers with $a_0 = 1, a_2, a_3, \ldots$ such that, $\Sigma_{k=0}^{n} a_k a_{n-k} = 1\ \forall n \in \mathbb{N}$.

Problem10. Remember the postman, from your childhood memory, he used to carry postcards, with envelopes. Here is the postman from my hometown, carrying n-square-envelopes of different sizes. I unpretentiously asked the following counting problem : In how many different ways, can they be arranged by inclusion. Obviously the postman is confused, and needs your help. Could you help him in counting? Here is a small instance : for $n = 2$, let the envelpoes are labelled by A, B and A is larger than B, and $I \in J$ denotes that the envelope I is inside the envelope J, then there are following two ways to arrange the envelopes by inclusion, namely : **1**. Φ, keep them seperately, **2**. $B \in A$.

The Following seven problems are from the book by Robert-Tesman :

Problem11. From exercise 5.4 : Problem no. 18,19,20.

Problem12. From exercise 5.5 : Problem no.9.

Problem13. From exercise 6.2 : Problem no. 17.

Problem14. From exercise 6.2 : Problem no. 19.

Problem15. From exercise 6.2 : Problem no. 29.

Problem16. From exercise 6.2 : Problem no. 30.

Problem17. From exercise 6.1 : Problem no. 32

Problem18. Consider a $n \times n$-grid. If you are only allowed to either move upward[i.e. from (i, j) to $(i, j+1)$] or rightward [i.e. from (i, j) to $(i+1, j)$] in one step, then how many number of paths are there from the point $(0, 0)$ to (n, n). How many paths are there from $(0, 0)$ to (n, n), which do not cross the diagonal points [i.e. does not go beyond points (i, i)].