

Theoretical Foundations of Computer Science (Test 1)

Time: 1 hr 15 min

Marks: 30

Questions:

1. Let G be a graph with no 3-cycles, and let each vertex in G have degree at least k . What is the minimum number of vertices in G ? Can you give an example of such a graph with minimum possible vertices where there is no 3-cycle and degree of each vertex is exactly k ?
2. Let G be a simple graph such that degree of each vertex is at least 3. Prove that G has a cycle of even length. Also prove that G has a cycle with a chord.
3. Let G be a tournament with in-degree of each vertex at least 1. Prove that G has at least 3 kings.
4. Prove or disprove: Every finite tree has at most one perfect matching.

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