Design and Analysis of Algorithms
Assignment 2

September 13, 2012

Marks: 30 | Due date: 21st September

1. There are n files $F_1, \ldots, F_n$ with lengths $\ell_1, \ldots, \ell_n$, and access probabilities $p_1, \ldots, p_n$ given as input. The problem is to order these files on a tape so as to minimize the expected access time.

   For instance, if the files are placed in the order $F_{s(1)}, \ldots, F_{s(n)}$, then the access time for file F_i is $\sum_{j=1}^{i} \ell_{s(j)}$. Thus the access is sequential, and to access ith file on the tape, you need to spend as much time as the sum of lengths of all the files which precede it. The expected access time in this case is $\sum_{i=1}^{n} p_{s(i)} \sum_{j=1}^{i} \ell_{s(j)}$.

   For each of the algorithms below, either give a proof that the algorithm is incorrect, or a proof that it is correct: (9 marks)

   (a) Order the files from shortest to longest on the tape. That is $\ell_i < \ell_j$ implies that $s(i) < s(j)$.

   (b) Order the files from most likely to least likely to be accessed. That is $p_i < p_j$ implies $s(i) > s(j)$.

   (c) Order the files from smallest ratio of length to access probability to largest ratio of length to access probability. That is $\frac{\ell_i}{p_i} < \frac{\ell_j}{p_j}$ implies $s(i) < s(j)$.

2. Give an efficient algorithm to find the shortest common super-sequence of two strings A and B. Note that C is a super-sequence of A if and only if A is a subsequence of C. (5 marks)

3. Define a most vital edge of a network as an edge whose deletion causes the largest decrease in the maximum s-t flow value. Let f be an arbitrary maximum s-t-flow. Either prove the following claims or show through counterexamples that they are false: (10 marks)

   (a) A most vital edge is an edge e with the maximum value of $c(e)$.

   (b) A most vital edge is an edge e with the maximum value of $f(e)$.

   (c) A most vital edge is an edge e with the maximum value of $f(e)$ among edges belonging to some minimum cut.

   (d) An edge that does not belong to some minimum cut cannot be a most vital edge.

   (e) A network might contain several most vital edges.

4. Suppose you are given a directed graph $G = (V, E)$ with a source $s \in V$, and a sink $t \in V$, and numbers $w(u, v)$ for each $(u, v) \in E$. We define a flow, and its value as usual, requiring that all nodes except the source and the sink satisfy flow conservation. However, the given numbers are lower bounds on the edge flows. i.e. we require $f(v, w) \geq w(v, w)\ \forall (v, w) \in E$. There is no upper bound on flow values for edges. (6 marks)

   (a) Give a polynomial time algorithm that finds a feasible flow of minimum possible value.

   (b) Prove an analogue of the max-flow min-cut theorem. (Is min-flow = max-cut here?)