Hints/answers and marking scheme to Assignment 1

14/09/2012

1. Both $O(\log n)$. Correct answer for sorted, reverse sorted: 1.5 marks each.

2. DFS + listing of tree edges (as they are obvious from the DFS): 3 marks Other types of edges: 1 mark each. (DFS tree has no cross edges).

3. Prim's algorithm with a as root node adds edges in the order $(a,b),(b,c),(c,e),(d,a),(d,g),(e,f)$. Cost: 25. Correct answer: 5 marks.

4. Let T,T' be two MSTs in G. Consider $T \oplus T'$ and choose the smallest weight edge e from it. Let $e \in T$ but $e \notin T$. Then add e to T'. The resulting cycle has another edge $f \in T'$, $f \notin T$, $w(f) > w(e)$ by the above assumption. Deleting f from T' and adding e reduces its weight, contradicting that T' is an MST. Correct proof: 6 marks

5. MST remains MST as relative ordering among edges remains the same A shortest path may not remain a shortest path. For example, if there are two paths from u to v of weights $3+11 = 14$ and $5+10 = 15$, they become $9 + 121 = 130$ and $25 + 100 = 125$ after squaring. Thus the relative order among paths does not remain the same.

   Correct answers: 1 mark each, Correct explanation/counter example: 1 mark each

6. Do a topological sort in linear time. Go over the vertices in the topological order, maintain an array of counts $count[i]$ for number of s to i paths. For each vertex i, $count[i]$ = sum of counts of its in-neighbours. Due to topological ordering, their counts are available while processing i.

   Correct algorithm: 6 marks