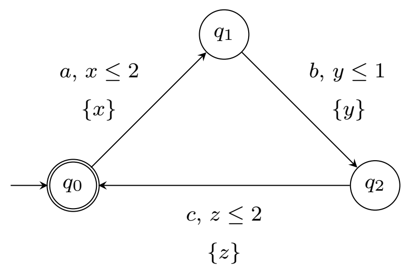
1. Give a timed automaton over $\Sigma = \{a, b\}$ that accepts all timed words.

2. Let $\mathcal{A} = (\{q\}, \{a, b\}, \{x\}, T, \{q\}, \{q\})$ be a timed automaton with a single state $q$ and a single clock $x$. Note that $q$ is also an accepting state. Let $T$ be the set of transitions. Give an instance of $T$ that makes $\mathcal{A}$ reject at least one timed word.

3. What is the timed word accepted by the following accepting run of some timed automaton with two clocks $x$ and $y$?

$$\begin{array}{ccccccccc} q_0 & & q_0 & & q_1 & & q_1 & & q_F \\ x:0 & \xrightarrow{\delta_0} & x:0.6 & \xrightarrow{a} & x:0.6 & \xrightarrow{\delta_1} & x:1.9 & \xrightarrow{a} & x:0 \\ y:0 & & y:0.6 & & y:0 & & y:1.3 & & y:1.3 \end{array}$$

4. Let $\mathcal{B}$ be the following timed automaton:




Consider the timed word $s = (abcabc, 0.5, 1, 1.5, 1.8, 1.9, 3)$.

a) Does $\mathcal{B}$ accept $s$? If so, write down the accepting run of $\mathcal{B}$ on $s$.

b) For a timed word $(w, \tau)$ we define the *time span* of $(w, \tau)$ to be the time at which the last letter occurs, i.e., if $|w| = n$, then time span of $(w, \tau)$ is $\tau_n$.
For every $k \in \mathbb{N}$, give a timed word in $\mathcal{L}(\mathcal{B})$ that has length greater than $k$ and whose time span is lesser than 1.

5. Are the following timed languages timed regular? Justify.

i. $L_1 = \{ (a^k, \tau) \mid \tau_{i+2} - \tau_i \leq 1 \text{ for all } i \leq k-2\}$

ii. $L_2 = \{ (\,(ab)^k, \tau\,) \mid \tau_{2i+2} - \tau_{2i+1} < \tau_{2i} - \tau_{2i-1} \text{ for each } i \geq 1\}$

iii. $L_3 = \{ (\,(ab)^k, \tau\,) \mid \tau_{2i} = i \text{ and } \tau_{2i+2} - \tau_{2i+1} < \tau_{2i} - \tau_{2i-1}\}$

iv. $L_4 = \{ (w, \tau) \mid w \in (a+b)^*, \ \exists i \text{ s.t. } w_i = a \text{ and } \forall j \text{ we have } \tau_j \neq \tau_i + 1\}$