

MATHEMATICAL JOURNEYS : THE WHAT AND THE WHY

VIPUL NAIK

ABSTRACT. This article contains a statement of the mission, vision and specific goals of the **Mathematical Journeys** series. It further describes the current status and how learners can make effective use of **Mathematical Journeys**.

1. MISSION AND VISION

1.1. **Mission statement.** The mission statement outlined below is *long term*.

The mission of **Mathematical Journeys** is to provide a comprehensive coverage of undergraduate topics in mathematics and allied disciplines that students and educators across the world can use to complement and supplement their course material, develop their mathematical knowledge, perspective and understanding, and get a “feel” for the relevant areas.

1.2. **The vision.** The vision statement outlined below is *medium term*. I do not see this vision as being realized immediately, but I hope that the wheels have been set in motion for its realization.

The vision of **Mathematical Journeys** is to:

- Make accessible to all, via the Internet or via print or in both ways, a collection of mathematical journeys that help complement the undergraduate education of students.
- Provide students an opportunity to develop a “feel” for the various activities that constitute a life in mathematics – reading, writing, discussing, teaching, listening, learning, via the **Mathematical Journeys** articles.
- Provide instructors at the school level a way of enriching their instruction and guiding gifted students to more advanced topics.
- Providing instructors at college level a supplementary text to their instruction that can be used for homework reading, problems, and as a guideline to students.

1.3. **The target segments.** The primary target segments are:

- College students (undergraduate level)
- College level instructors

The secondary target segments are:

- People wanting to develop competency or interest in particular areas of mathematics for work in other disciplines
- School level instructors seeking ways of stimulating the minds of gifted students

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©Vipul Naik, B.Sc. (Hons) Math and C.S., Chennai Mathematical Institute.

2. THE APPROACH

2.1. **Why journey?** The word “journey” has been chosen carefully for describing the articles covering elementary mathematics topics. Here are some important “journey” like aspects:

- I have written these articles along the lines in which I myself have explored the subjects, or, in hindsight, feel is the best way to explore the subjects. The articles thus represent the “road I have travelled” or the “journey I have taken”.
- The style of the articles is largely exploratory, like an expedition guide taking the expedition forward. The path that the guide (in this case I myself) chooses is exciting, fraught with perils, but on the whole *safe*. Journeyers are encouraged to stray from the path I have sketched, so long as they do not lose sight of it.

2.2. **Choice of person.** *Mathematical Journeys* differs from other articles in mathematics because of the emphasis on the personal touch. Most mathematical articles make use only of “we” (the first person plural) in order to appear impartial, impersonal and universal.

The *Mathematical Journeys*, on the other hand, reflect *my experiences*, and are often coloured with *my opinions*. So the articles shall include three kinds of pronouns:

- The singular first person “I” shall refer to me, the journey guide. This shall be used when expressing personal opinions.
- The singular second person “you” shall refer to the journeyers reading and following the article. This shall be used when asking questions, particularly related to personal opinion.
- The plural first person “we” shall be used for most other purposes.

2.3. **Centrality of definitions.** The fundamental idea that drives all the *Mathematical Journeys* is that the mind has a number of ways of internally representing information and good instruction exploits all of them. Sometimes, *history* and a *genetic perspective* might be useful, sometimes *examples* and *counter examples* might be needed. *Problems* and *riders* could really help hammer in understanding for certain subjects, while for others, it may be *real world applications*. Some learners prefer a *visual representation*, others may like to work things out *with their hands*. Some people might revel in *symbols*, other may prefer *words*.

Much of today’s “good instructional design” tips talk about catering to more and more representational systems, by peppering the instruction with history, examples, problems, analogies, real world applications, diagrams and so on. But they miss out on a very important learning methodology – a *definition centric* one, where learners get to appreciate and understand a concept simply by *staring at* and *parsing* the definition, understanding its *ramifications*. and using the tools of *creative variation*.

This becomes particularly important in mathematics, where everything lies inside the definition. For this reason, all the *Mathematical Journeys* articles pay maximum importance to definitions, and give a lot of stress to understanding the definitions. Problems and examples are introduced only to enhance the appreciation of the definition.

Of course, the journeys do not neglect other ways of grasping ideas. However, they always *start out* from the definition, and gradually move towards examples, history, and other motivational approaches.

More on learning strategies can be found in my *Assimilate Knowledge Better* series. These are based on the theoretical foundation of property theory, that I have developed.

2.4. **Organization of text.** Some tools used to facilitate the learning process, in particular processes using a property theoretic learning strategy, are the following:

- Whenever new terms have been defined, the word is put in **bold highlighting** and the word (defined) is written immediately afterward. This is particularly useful for learners interested in identifying definitions, creating glossaries, and so on, because many new terms are introduced throughout the journey.
- Main definitions in the text are given in separate paragraphs begun with the bold word **Definition** and alternative definitions begun with **Alternative Definition**.
- Other environments used to convey important information are the **upshot**, the **key point**, the **observation**, the **theorem**, the **claim**, the **lemma** and so on. Some of these are numbered, some are unnumbered. Some are italicized while others are in straight print.
- Formal proofs begin with the italicized word *proof* and end with a hollow square on the right.

2.5. **Section organization.** Each article is divided into numbered sections, and each section is divided into numbered subsections. Cross referencing is done primarily based on the section and subsection number.

The primary criteria for splitting into sections are logical structure, size and palatability.

Here are some features common across the journeys:

- The **introductory section** sets the pace for the journey. It describes the specific tools and learning strategies employed in the journey. It also sets up the **prerequisites** for the journey, as well as an **explicit promise** of what learners are expected to gain at the end of it.
- Some sections have an **end section review**, others have a **beginning section overview**. Sometimes, some sections have a **mid journey progress review** where the current state of progress with respect to the explicit promise is discussed.

2.6. **Problems.** There are two kinds of problem environments:

- The **CONCEPT TESTERS** are given at the end of some subsections. There may be some subsections that do not have **CONCEPT TESTERS**. New terms may be introduced in the **CONCEPT TESTERS** (as usual, new definitions shall be indicated for easy referencing). All the problems in this are based directly on the material covered in the subsection and an inability to solve these problems indicates a need to go back and revise the subsection or previous subsections.
- The **POINTS TO PONDER** are given at the end of some subsections. Questions here are relatively more open ended and many of them may not be solvable using the ideas presented so far, but they can definitely be *mulled over*. New terms introduced in the **POINTS TO PONDER** are usually not referred to in the main text (if they are, they may be redefined or referred back). Thus, during a first reading, it may not be necessary to attempt all problems in the **POINTS TO PONDER**. However, attempting them is a very important step to becoming mature journeyers.

Some other environments that I do not use very frequently are the **TRY IT OUT!!**, **IN THE REAL WORLD** and **PLEASE EXPLAIN**.

2.7. Summary information. For the convenience of learners, each article ends with an index of important terms. The index has been prepared in a *property theoretic fashion*. In particular, what this means is that if I want to locate the term “normal subgroup”, I can look at “normal subgroup” as well as “normal” under “subgroup”.

Simply scanning the index can give the learner an idea of what terms representing the dominant contexts of the article, because such terms are likely to have the maximum indented entries under them.

The following principles are followed:

- Definitions are always indexed
- The first use of a term (except perhaps in an introductory overview) is always indexed, even if it is without a definition.

Other summary information is not to be found within the journey. I will prepare glossaries and summary pages for an entire series of journeys on a topic together.

2.8. Solutions and more problems. Solutions to the CONCEPT TESTERS, as well as more practice problems, will probably be prepared separately and not along with the main journey. This depends on the response and feedback I receive from willing journeyers.

2.9. Formats. My primary intention is to put up these journeys as PDF files. The advantage of PDF files is that they can be viewed on almost all systems with a GUI, as the Acrobat Reader software is freely available. Further, the PDF files are hyperlinked, so that learners can click from one part of the file to another part that it refers to.

The PDF file is also easy to print from. Some people may prefer the PS format for printing. They are welcome to contact me for the PS format.

The journeys are being prepared in L^AT_EX. I do not intend to put up the source text of the journeys on the Web. However, those interested in studying or modifying them somewhat for their private use are welcome to contact me for the source.

3. LARGE SCALE ORGANIZATION OF JOURNEYS

3.1. The journey subdivision for a topic. Usually, a huge and important topic in mathematics is covered across a series of journeys. There is no hard and fast approach to subdivision, and size and complexity considerations, as well as the need to have some parts quickly and readily available. Generally, the following is done:

- The **First Journey** usually provides introductory definitions and motivations of the topic. It is usually short, and often omits out stuff that may be taught in the beginning in other courses. The aim is to let the journeyers get really thorough with those concepts and ideas that shall be used most frequently. Simply reading a **First Journey** and stopping is akin to stopping midway while climbing a mountain. It is an effort half done without reaping the rewards!
- The **Journey Continues** or **Real Journeys** provide the bulk of theory and content, as well as delve more into the history, motivations, current status and flavour of the subject. These rely thoroughly on the approach developed in the **First Journey**. These are often split across multiple parts, depending on the nature of the topic. A completion of these is sufficient for attaining competencies for a regular course. Thus, journeyers can stop after completing these.
- The **In The Thick** or **Delve Deep** use the foundation developed so far to study particular links to more advanced topics. Some of these may coincide with additional topics covered by instructors. Each of the topics is covered independently.

3.2. The order of journey preparation. Ideally, of course, it would be great if all the journeys were already prepared and available. However, I need *time* and *energy* to prepare the journeys, and there are *conflicting demands on them*. Further, as I am still a student and learning many of the subjects on which I would like **Mathematical Journeys**, I can only begin a very small segment of the task right now.

As of now, preparing the journeys is not a time bound activity for me. I am preparing the journeys based on my *current competencies*, the *feedback constraints* and the *personal utility* I derive from journey preparation. More in the next section.

4. OVERALL STRATEGY

4.1. Lofty missions to grim realities. The lofty worldwide mission, diluted somewhat in the vision statement, is still a far cry from the current situation under which I am operating. But I definitely feel that the vision is attainable, even if not in the immediate future.

Here are the steps in which I plan to expand **Mathematical Journeys**:

- **Preparation:** The first step is to prepare a sizeable number of journeys. This in itself could take a lot of time. I don't know. The only series of journeys that I think I will finish in the near future is that on group theory.
- **Publicity and Distribution:** Publicity is in two forms – through the website, and through word of mouth. This is where *you* can help. If you enjoy the journeys that I put up, please make it a point to tell all your friends who might be interested. Do not be worried about copyright hassles – distribute copies freely, even sell them if you please. You could help these journeys reach their intended audience.
- **Feedback:** I hope that all those of you who read and benefit even in small measure from the journeys write to me. Even a mail just telling me whether the journey was useful or not would be valuable feedback to me. Of course, specifics will be even more welcome.
- **Supplementation:** Based on the feedback and my own constraints, I plan to write supplementary material to the journeys, such as solutions and discussions, summaries, and further practice problems.

4.2. How you can help. If you find the idea of **Mathematical Journeys** and the specific journeys put up on the website useful, *spread word* and *provide me feedback*. Your moral support as well as your intellectual support is valuable for the purpose.

The strategic steps planned so far do not require any financial resources, but it is possible that if **Mathematical Journeys** become more popular, financial resources will be needed – for instance, to make the journeys available in print form for people who do not have ready machine access or high printer costs. Please keep checking the page at regular intervals to find out.