GROUP THEORY: MY DEVELOPMENT IN THE SUBJECT

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Abstract. I have long been fascinated by group theory. I was initially interested only in pure group theory, but I am now fascinated by representation theory as well. Group theory is one of the areas where I would like to pursue research. Here, I describe my progress in group theory so far.

1. First brushes with group theory

1.1. An old book by Shanti Narayan. *Time period: 2002 - 03*

When in eleventh standard, I chanced upon an old algebra book by Shanti Narayan. The rudimentary introduction to groups was my first exposure to these algebraic structures that I have explored so much in the last few years.

1.2. The first Olympiad experience. *Time period: May-June 2003*

At the **International Mathematical Olympiad Training Camp**(camp name) (IMOTC)[1] that I attended in twelfth standard, I learnt of the following applications related to group theory:

- "Transformation geometry": The camp was my first systematic introduction to transformations. The transformations (automorphisms) of any structure naturally form a group. Rotations, translations etc. thus form "subgroups" of the group of transformations that preserve distances in the plane (called "isometries"). The Olympiad teacher Shailesh Shirali quoted the following words of Felix Klein: "Geometry is the study of (groups of) transformations and the properties of objects invariant under those transformations".
- "Functional equations": The functional equation $f(x + y) = f(x) + f(y)$ with $f : \mathbb{R} \to \mathbb{R}$ is basically the assertion that f *preserves* the additive structure on $\mathbb{R}$. A function between groups that preserves the group structure is called a "homomorphism". I realized that solving this functional equation was equivalent to finding all homomorphisms $\mathbb{R} \to \mathbb{R}$ as an additive group. Similarly, solving the pair of functional equations $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ was equivalent to finding all ring homomorphisms from $\mathbb{R}$ to itself. This understanding did not help me with too many functional equations but it felt good to have achieved it.
- "Number theory": Arithmetic modulo n is equivalent to working with the cyclic group of order n. I had realized this long ago, but it was during the camp that I started reformulating my understanding of number theory and basic results of arithmetic in group-theoretic terms, and the subject came to grow in elegance. In particular, the fact that Fermat's little thoeorem and Euler's theorem were trivial consequences of Lagrange's theorem increased my appreciation of group theory significantly.

1.3. Mathworld. *Time period: January – April 2004*

Towards the end of twelfth standard, I started becoming more curious about groups and subgroups. I did not have any standard algebra textbook apart from the book by Shanti Narayan so I used Mathworld (`http://mathworld.wolfram.com/`) as my primary reading resource. I read about lots of subgroup properties as well as about specific subgroups of interest (such as the center and commutator). I made the observation that any subgroup defined in a purely group-theoretic fashion must be isomorphism-invariant and hence normal.

1.4. Dummit and Foote. *Time period: beginning of May 2004*

The book *Abstract Algebra*(book name) by Dummit and Foote was a gem that I found while browsing the Tata Book House during the KVPY camp. I started reading it immediately and discovered the concept of "characteristic subgroup" which made many ideas become clearer. In particular, I realized that any subgroup of a group which is the "only one" of its kind must be isomorphism-invariant and

[1]The IMOTC is a selection-cum-training camp for the International Mathematical Olympiad Learn more at `http://en.wikipedia.org/wiki/IMOTC`.

hence must be, not just normal, but in fact characteristic. Tidbits that I continued picking up from Dummit and Foote fuelled my interest in group theory.

One sequence of sentences in Dummit and Foote sent me thinking :

"The property of characteristicity is transitive. A characteristic subgroup of a normal subgroup is normal. A normal subgroup of a normal subgroup may not be normal. Thus characteristic subgroups can be thought of as *strongly normal* subgroups."

I was to revisit these ideas later in more detail.

1.5. **The next training camp.** *Time period: beginning of June 2004*

During my second IMO Training Camp, I used the extensive library at HBCSE to learn more about groups and about algebraic structures related to groups, such as semigroups, monoids, quasigroups, algebra loops etc. I felt very nice on reading these because prior to reading them, I had myself explored what would happen if certain axioms of the group structure were removed. And on reading, I discovered further directions of exploration.

2. College life begins

2.1. **On joining CMI.** *Time period: August – September 2004*

In August 2004, I joined **Chennai Mathematical Institute**(place name)[2] for the B.Sc. (Hons) Mathematics and Computer Science programme.

The **Algebra I**(course name) course, taught by Professor K.R. Nagarajan, was in the area of group theory and matrix groups. I was already familiar with a lot of group theory and had read a bit in matrix groups. So I devoted my energies to learning more in the subject. I came to use Wikipedia(`http://en.wikipedia.org`). I learnt of other subgroup properties.

I read about "fully characteristic subgroups" and "strictly characteristic subgroups" (also called "distinguished subgroups") on Wikipedia. I observed that they display the same relation as characteristic and normal subgroups: every fully characteristic subgroup of a strictly characteristic subgroup is strictly characteristic, but every strictly characteristic subgroup of a strictly characteristic subgroup need not be strictly characteristic. This set me thinking.

Which subgroup properties are "transitive"? For a subgroup property which is not transitive, what is the weakest possible "transiter" for it? Is characteristic the transiter for normal? Transiters could be viewed both ways – what is the transiter for normality the other way? That is, what is the property π such that every normal subgroup of a subgroup with property π is normal in the whole group? Rather, what is the *weakest* property π satisfying the above?

I discussed this with Professor Nagarajan who encouraged me to read further and suggested that I acquaint myself with "free groups". I later also talked to Professor Balaji who encouraged me to think along these lines and proved a willing audience. I presented some of my ideas to him.

2.2. **The problem of extensible automorphisms.** *Time period: October – November 2004*

It was around October to November that I formulated the problem of extensible automorphisms. The problem is:

> An automorphism of a group is termed **extensible**(defined) if given any embedding of the group in a bigger group, the automorphism lifts to an automorphism of the bigger group. Clearly, every inner automorphism of a group is extensible. Is the converse true? That is, is every extensible automorphism of a group inner?

Initially, I could make no progress. I took the problem to Professor Balaji and also explained how I had come across the problem. The approach he suggested to me was to embed it in a linear group which is "complete" in the sense that its only automorphisms are inner, and then show that every automorphism of the bigger group that keeps the smaller group within itself restricts to an inner automorphism of the subgroup.

More information on the problem is available at:
`http://www.cmi.ac.in/~vipul/unsolvedproblems/extensibleautomorphisms/`

2.3. **Professor Ramanan and Algebra II.** *Time period: January – February 2005*

Professor Ramanan taught the **Algebra II**(course name) course. I enjoyed this course a lot. Although I had studied some topics before, the clarity with which Professor Ramanan presented the topics made the ideas attain a new meaning and I began to understand the importance of group theory in the overall scheme of mathematics. I also discussed the problem of extensible automorphisms with Professor

[2]Check the website: `http://www.cmi.ac.in`

Ramanan. He initially suggested an approach similar to Professor Balaji. He also gave a concrete plan and himself found a counterexample to one of the possible routes he had determined for solving the problem.

By discussing the problem with him, I also learnt that there are class automorphisms of finite groups (automorphisms preserving conjugacy classes) that are not inner. I saw an explanation of this in terms of cohomology. I had read about the use of cohomology in the theory of finite groups earlier, but this concrete use increased my appreciation of the subject.

2.4. **Other books I read.** *Time period: March – April 2004*

I read parts of the book on "Finite group theory" by Michael Aschbacher. I also skimmed through some pages of volumes on the classification of finite simple groups. I had read about the "classification of finite simple groups" in popular math books even before learning what a group is, and was quite fascinated with it. Unfortunately, I did not know any professor in or around CMI involved with the classification project.

2.5. **Representation theory of finite groups.** *Time period: March – April 2005*

During the last month of the **Algebra II**(course name) course, Professor Ramanan had to leave for an international conference. Professor Ramanan wanted at least one nontrivial topic to be covered in the course, so he asked Professors K.N. Raghavan and Amritanshu Prasad from the **Institute of Mathematical Sciences**(place name) (IMSc)(`http://www.imsc.res.in`) to cover the equivalent of Chapter 9 from Artin's text, on the representation theory of finte groups.

Professor Raghavan's style of teaching was amazing. His emphasis on the statement of the theorem, and the long time he spent over interpreting it, made me see the beauty of the orthogonality theorems of representation theory. Further, he alluded to close connectinos with the study of compact and Lie groups, which made me more curious about these areas.

Dr. Amritanshu Prasad also gave a hands-on feel for computing the irreducible representations by writing the character table for A_5, the smallest non-Abelian simple group.

3. Over the summer

3.1. **Summer programme in groups, representations and algebras.** *Time period: May – June 2005*

More details can be found in the file:

`http://www.cmi.ac.in/~vipul/academics/summercamps.pdf`

During the summer, I attended a summer programme on **Groups, Representations and Algebras**(course name) conducted at the Institute of Mathematical Sciences which is a sister institute to the Chennai Mathematical Institute. I had been to the IMSc library before but had not visited it on a regular basis earlier. I used this increased exposure to the institute to read books across a wide range of subjects, particularly in group theory and representation theory.

Professor V.S. Sunder began with a lecture series on modules for C^* algebras. He showed how the group algebra is a special case of a C^* algebra, and thus, reduced the proof of decomposition of the regular representation of the group algebra to a result in C^* algebras.

Professor Raghavan gave proofs of the Jacobson density theorem and Wedderburn-Artin theorem. Although neither of these was directly connected with groups, Professor Raghavan indicated some relations by looking at the special case when the ring under consideration was a group algebra.

Dr. Anritanshu Prasad did in detail the representation theory of $GL_2(F_p)$. This concrete example of computing representations of specific groups helped give me a feel of representation theory and also made me appreciate the immense machinery involved.

3.2. **Correspondence with Marty Isaacs.** *Time period: June 2005*

I came across some books by I.M. Isaacs on characters and representations. I still had Professor Ramanan's approach for the problem of extensible automorphisms in mind and wanted to figure out how to make it work. So I sent a mail to Professor Isaacs. He suggested a somewhat different line of attack, which settled the case for cyclic groups. I generalized it to Abelian groups, and managed to take it somewhat further.

The email correspondence with Professor Isaacs was illuminative and educative for me. It made me realize that even for hard problems, it is always important to begin by settling the simplest cases. Moreover, it often makes sense to look at easier variants of problems, and see if counterexamples to the easier variants can give counterexamples to the tougher problems.

More on the content of what was proved can be found on:
`http://www.cmi.ac.in/~vipul/unsolvedproblems/extensibleautomorphisms`

4. Further reading and work

4.1. Discussion with Pranab Sardar. *Time period: September – November 2005*

Later, I discovered that Pranab Sardar, a research scholar who had independently come to hear of the extensible automorphisms problem from some people I had told, had settled the case for Abelian groups. He also gave a generalization which was in a somewhat different direction from my generalization. I was able to combine the two generalizations to get an even greater generalization which settled a large number of cases.

I found that my exploration of subgroups and their properties helped me to reach the generalizations more quickly than I would have simply by using the standard notations. Thus encouraged, I developed my formalism more and explored the theory of group and subgroup properties within this formalism.

4.2. More on subgroup properties. On a whim I decided to search the Internet for "paranormal subgroup" and was surprised to find that such a term actually existed. In fact, I looked at a number of properties of subgroups. I had a look at Wikipedia and also added entries there for subgroup properties that I had found elsewhere.

I read many articles, such as those on "conjugate-permutable subgroups", on the page by Professor Tuval Foguel of the Auburn University of Montgomery:
`http://sciences.aum.edu/~tfoguel`
My explorations led me to the following question:

> For a subgroup $H \leq G$, define the **normalizer series**(defined) of H as the ascending chain H_i wher e$H_0 = H$ and $H_i = N_G(H_{i-1})$ for $i \geq 1$. This normalizer series can also be transfinitely extended. Define the **hypernormalizer**(defined) of H as the limit of the normalizer series. Call a subgroup **hypernormalized**(defined) if its hypernormalizer is the whole group.
>
> For finite groups, the normalizer series of any subgroup stabilizes after a finite length. Thus, for finite groups, every hypernormalized subgroup is subnormal. Is the converse true? Is every subnormal subgroup of a finite group hypernormalized?

I sent the question to Professor Tuval Foguel, and also asked him for references on related aspects of group theory. Professor Foguel found a counterexample, namely, a subgroup of order 2 generated by the double transposition, sitting inside S_4. He also sent me links to material in the literature, particularly papers by Camina, which discussed the class of groups for which every subnormal subgroup is hypernormalized. [3]

Professor Foguel also suggested the book *A Course in Group Theory*(book name) by Derek J.S. Robinson, who works at the University of Illinois Urbana-Champain (UIUC). I located the book at the Institute of Mathematical Sciences library and also corresponded with Professor Robinson on some other related problems in group theory.

4.3. Course in representation theory. *Time period: August-November 2006*

In the fifth semester, I attended a course on **Representation Theory of Finite Groups**(course name) taught by Professor Senthamarai Kannan. In the course, the following topics have been covered:

- Basics of representation theory: Representations over $\mathbb{C}$, culminating in the fact that the irreducible characters form an orthonormal basis for the space of class functions and that the character determines the representation.
- The numerical constraints on degrees of irreducible representations. In particular, the degree of an irreducible representation divides the order of the inner automorphism group.
- Brauer's induction theorem
- Rationality questions and orthonormality theorems over non-algebraically closed fields.

Further material covered in student seminars includes Artin's induction theorem, and results on monomial characters.

I enjoyed a lot of the course content, and I am sure it will serve as a good foundation for the more detailed study of representation theory that I plan to undertake.

A fuller course outline is available at:

[3]Interestingly, the standard literature also used the term "hypernormalizer" which I had myself concocted. Mathematical terminology is so predictable!

http://www.cmi.ac.in/~vipul/courses/reptheoryoffinitegroups/
A write-up based on which I gave my talk is available at:
http://www.cmi.ac.in/~vipul/writeupsandpresentations/artinstheoremforseminar.pdf

4.4. **Course in algebra and computation.** *Time period:January – April 2007*
In my final semester of undergraduate study at CMI, I am credited a course titled **Algebra and Computation**(course name) taught by Professor Arvind. The first half of the course has focussed on computational problems in group theory, as well as the conversion of problems such as Graph Isomorphism to group theory, and the construction of algorithms to solve these problems based on this conversion.
Lecture notes for these lectures are available at:
http://www.cmi.ac.in/~ramprasad/lecturenotes/

4.5. **Summer work in Paris.** *Time period:May – June 2007*
As part of an exchange programme between CMI and the Ecole Normale Superieure, Paris [4] I am spending the months of May and June in the ENS, Paris. In the ENS, I am studying the topic of the Grassmannian and Schubert varieties under the guidance of Professor Olivier Schiffmann. This is not itself group theory, but it is closely linked with the representation theories of the symmetric group and the general linear group, in particular with Young tableaux. The representation theory of the symmetric group is something I have glimpsed often and which I have long wanted to study properly. This summer I finally have the opportunity to pursue the study.
If I am able to reach a sufficient point of closure with Schubert varieties, I will probably shift over to studying quantum groups, which are an exciting generalization (or rather, deformation) of the group algebra associated with a group.

5. Attempts at posing and solving problems

5.1. **Extensible automorphisms.** As I mentioned earlier, I first posed the extensible automorphisms problem during my first semester. The problem statement:

> An automorphism of a group is termed **extensible**(defined) if given any embedding of the group in a bigger group, the automorphism lifts to an automorphism of the bigger group. Clearly, every inner automorphism of a group is extensible. Is the converse true? That is, is every extensible automorphism of a group inner?

I had done sporadic work on this problem during the first four semesters. In the fifth semester, I decided to put in a full-fledged systematic effort and to organize the work I had done so far. When the work reaches a logical point, I shall put it up at:
http://www.cmi.ac.in/~vipul/unsolvedproblems/extensibleautomorphisms/
I have also given a series of two student talks on the work I have done on extensible automorphisms, which can be found at:

- http://www.cmi.ac.in/~vipul/studenttalks/ccrdpresentation.pdf
- http://www.cmi.ac.in/~vipul/studenttalks/semidirectproductsandextautos.pdf

5.2. **The direct product problem.** *Time period: June - September 2006*
This problem occured to me. The problem statement was:

> Under what conditions on groups G, H and K is it true that:

$$G \times H \cong G \times K \implies H \cong K$$

I kept attempting this question off and on for two months before finally proving the affiramtive for finite groups. Later, I discovered that the proof technique I had used was quite general and could be used for a large class of algebraic objects. I also found the "dual" result where direct product was replaced by free product.
It in fact turns out that for direct products, a much stronger property than cancellation – namely unique factorization holds. This is the content of the Remak-Schmidt theorem.
I am still trying to explore what weaker hypotheses can be put on G, H and K for which the result continues to hold, and whether direct product can be replaced by various other notions of product.
I gave a student talk on the subject. Slides for the talk are available at:
http://www.cmi.ac.in/~vipul/studenttalks/directproductsslides.pdf

[4] http://www.dma.ens.fr

5.3. **The potentially characteristic subgroup problem.** While trying to solve the problem of extensible automorphisms, I naturally encountered the following question:

> A subgroup G of a group H is termed **potentially characteristic**(defined) if there is a group K containing H such that G is characteristic in K. Clearly, every potentially characteristic subgroup is normal. Is the converse true? That is, is every normal subgroup potentially characteristic?

I have sent this problem to some people, who all say that they haven't seen the problem before, and that it is probably quite hard. I have some ideas on the problem and am working on those ideas.

6. Work on the group theory wiki

6.1. **What gave me the idea.** Some time towards the end of November, I decided to clean up all the work I had done on extensible automorphisms and put up a single cogent place where all the definitions and ideas I was using for extensible automorphisms could be put up.

I started off with a wiki specifically for the extensible automorphisms problem. But then, somehow, I got inspired into making this into a much larger wiki, touching on the many aspects of group theory that I had been explored. I also hoped that this wiki could capture the many ideas about organizing the wiki.

The wiki was initially hosted on editthis.info, but due to their using an older version of MediaWiki, as well as certain other technical problems, I moved the wiki over to wiki-site. The new address is:
`http://groupprops.wiki-site.com`

6.2. **How I went about it.** During June-July 2006, I had spent considerable time putting many new terms and definitions on Wikipedia. This effort paid off quite well. But I was not satisfied with the organizational flexibility that I had there, and felt that things would be much better in a wiki where I had fuller control. However, at the time, I felt it would be very hard to create my own wiki.

Some time around October, I came across external wiki hosting sites such as editthis.info. This was in the context of a wiki page being used for CMI shifting concerns, and the starting off of a wiki for CMI Spark (CMI's social work group). The ease with which these wikis were created and managed inspired me to start off on my own.

Initially I just imitated the things I had done on Wikipedia. However, I was soon reminded of my original motivation – which was to organize things in a way that seemed good to me. So, even as I was adding content, I started developing templates, formats, and organizational principles.

6.3. **The slow but steady expansion.** I have now tried to make the wiki grow in a number of directions, and I soon plan to "open it up" for collaborative work. For now, I am trying to make it have:

- Basic definitions in group theory
- Stuff related to finite groups and the classification of finite simple groups
- Representation theory
- Computational aspects
- Extensible Automorphisms Problem
- Nexus with model theory

7. What excites me about group theory

7.1. **It is a vast area.** Both group theory as a subject and as a tool have become so diversified that I think that by studying group theory, I'll be in touch with a whole lot of mathematics.

Just about every branch of mathematics involves group theory with a prefix: *algebraic* group theory, *Lie* group theory, *arithmetic* group theory, *geometric* group theory, *combinatorial* group theory.

Some people have told me that, akin to "Euclidean geometry", pure group theory is largely a dead subject. But I think that we have just begun to see some of the exciting results in group theory. Moreover, I think I can contribute to research within the area, both by working on existing important problems and by exploring new research directions.

7.2. **Classification problems: simple groups.** In 1980, the problem of classification of finite simple groups was completed. However, the proof was scattered across a vast number of journals and attempts have been going on for now to fix the gaps and put the proof together. Even though the proof is now believed to be *essentially correct*, I think there's a long way to go before we can truly claim to have *understood* the finite simple groups.

I have read about attempts to recast our curent understanding of the simple groups in terms of finite geometries, buildings. I was recently reading a book on *Finite Geometries*(book name) by Dembowski. I also read a book on *Finite Group Theory*(book name) by Michael Aschbacher which discusses some of the preliminary steps needed for the classification of finite simple groups.

I would be really interested in working on whatever steps are being taken currently to simplify and improve our understanding of the finite simple groups.

7.3. **Classification problems: nilpotent groups.** Since every nilpotent group is a direct product of its Sylow subgroups, and every p-group is nilpotent, the classification problem for nilpotent groups is equivalent to the classification problem for p-groups. As far as I have gathered, the classification problem for p-groups is still in its initial stages.

I plan to read about the classification of p-groups over the coming days. I definitely want to contribute towards this area if it fits in with my other competencies.

7.4. **Other classification problems.** The classification of *all* finite groups may be a very hard question, but we can definitely try to classify finite groups that are quite close to being simple. Having classified simple groups, the next question to explore is: can we classify the groups whose composition series has length k for small values of k?

I do not know what efforts have been made in this direction so far, but I would be glad to learn more about the subject.

7.5. **Study of subgroup and automorphism properties.** I was quite surprised to discover that the extensible automorphisms conjecture (section 2.2) has not been documented anywhere in the literature so far. Similarly, I was also surprised at not being able to find any references to an equivalent of the potentially characteristic subgroup problem (section 5.3).

I think that our current understanding of subgroup properties, and of automorphism groups, is not too good. I feel that one possible reason is that we are so used to studying properties of groups in the context of their actions, that we have not explored the intrinsic structure of groups to the same extent.

I have some ideas on exploring the theory of properties of groups, subgroups and automorphisms. In fact, it was these ideas that led me to the questions about extensible automorphisms and potentially characteristic subgroups. These ideas have also helped me understand a lot of the literature on properties of subgroups.

Currently I am working on a wiki completely devoted to the study of groups, subgroups and automorphisms. The wiki address is:

`http://editthis.info/groupprops`

8. Areas related to group theory

8.1. **Representation theory.** I have encountered representation theory in two distinct ways. One is, as a subject in its own right. I first studied representation theory in **Algebra II**(course name). In my fifth semester, I studied a course by Kannan on the representation theory of finite groups.

I have also encountered representation theory in my attempt to solve problems arising from pure group theory. Though I formulated the automorphisms problem (section 2.2) without any knowledge of representation theory, the solution approaches so far have also made use of group actions and representations. Interestingly, the representation theory I encountered in this manner made a much deeper impression on me.

In particular, the partial solution to the extensible automorphisms problem (which I worked out based on guidance from Dr. Isaacs of Wisconsin-Madison) required me to use both the fact that characters form a basis for the space of class functions, and the fact that the character determines the representation. This helped me understand the deeper meaning of both these results.

8.2. **Representation theory of the symmetric group and general linear group.** I have been fascinated by the close relation between the combinatorics of the symmetric group and its representation theory. During a summer camp in the first year, I learnt that the representations of the general linear group are closely related to those of the symmetric group, and that both are closely related to the combinatorial construction of Young tableaux.

Currently, I am studying more about these at the Ecole Normale Superieure, under the guidance of Professor Olivier Schiffmann. I have also been exploring various theories and formalisms for studying the matrix groups.

8.3. **Local representation theory.** I have of late become interested in the representation theory of a group where the characteristic of the field divides the order of the group. One of my friends was studying about these in a course called **Topic in Representation**(course name) by Dr. Amritanshu Prasad, that I was unfortunately unable to take. Some of the results he told me fuelled my curiosity in the subject.

I am currently trying to read a book called "Local representation theory" by Jon L. Alperin, which takes a module-theoretic approach and covers important results like Green's indecomposability theorem, the notion of vertex and source, the notion of Brauer correspondence, and so on. I have prepared notes on important results from this book, and I plan to study the notes this winter.

8.4. **Generating sets and Cayley graphs.** In my first semester, I read a proof of the fact that every subgroup of a free group is free. The prof used notions of generating sets and Schreier systems. I started reading more about these topics, and was particularly influenced by many of Gromov's ideas, even though I lacked the mathematical maturity to understand many of them.

I downloaded many papers from the Internet on these topics.

During my second year, I lost touch with many of the aspects of group theory that I had been exploring. Recently, I have started reviving that interest. In a course on computational complexity, the instructor discussed the problem of uniform sampling from groups given by their generating sets. This was done as an application of expanders. This motivated me to review many of the papers in the area of geometric/combinatorial group theory that I had downloaded earlier.

8.5. **Permutation groups and Galois groups.** During my second year, I read a book on permutation groups by P. J. Cameron. Later, I also read some interesting material relating permutation actions and Galois groups. Unfotrunately, I have not been able to study these topics in detail. I hope to perform a detailed study soon.

In the **Algebra IV**(course name) course, I learnt about criteria for subgroups of the symmetric group to be solvable. The techniques used involve permutation group theory. I was also glancing through a paper by a computer science professor on polynomial-time algorithms for testing properties of Galois groups.

I have heard that the inverse Galois problem is one of the big open problems, and that there are large research groups working on the problem, particularly at the University of Pennsylvania. Although I don't know much about these areas, I would definitely like to learn more about them.

8.6. **Group theory and model theory.** I first got to hear of model theory through interactions with Alexis Saurin, an exchange student from the **Ecole Normale Superieure**(place name)[5]. Alexis and another colleague explained how model theory helps generalize the notion of "algebraic closure". I satisfied my further curiosity by reading *Introduction to Model Theory*(book name) by Bruno Poizat.

I also heard of another book by Poizat that discusses the notion of "stable groups" and "definable groups". Unfortunately, I was not able to get access to this book. In later surfing, I read about the *Borovik program* which aims to prove that every group of finite Morley rank is algebraic. I learnt that Simon Thomas and Gregory Cherlin at the University of Rutgers are working in this area. I am eager to learn more about this area.

Appendix A. Books for learning group theory from

A.1. Introductory books.

(1) *Abstract Algebra*(book name) by Dummit and Foote.

How I used the book: The book by Dummit and Foote was an excellent introduction in basic group theory for me. In addition to understanding the fundamentals, I also started thinking along important lines after reading the theorems and propositions and attempting the exercises. The book is also strong on somewhat advanced group theory (such as the theory of nilpotent and solvable groups). However, it lacks a ring-theory-free introduction to representation theory.

[5]http://www.ens.fr

(2) *Topics in Algebra*(book name) by Israel N. Herstein.
How I used the book: I have never studied from Herstein's book. However, I have come across many important and interesting questions in Herstein's book, usually asked of me by others studying the book. These selected problems from Herstein are challenging and illuminative.
(3) *Basic Algebra*(book name) by P.M. Cohn, and its companion volume *Further Algebra and Applications*(book name) by P.M. Cohn.
How I used the book: I have only studied from *Further Algebra and Applications*(book name), because by the time I came to know of the books, I had already learnt plenty of basic algebra. However, from the little I have perused of the basic algebra book, I think it is a good start.
(4) *Algebra*(book name) by Michael Artin.
How I used the book: Artin was the book in vogue before Dummit and Foote became popular. I have read Artin mainly for getting a rough idea of the motivations and the way the calculations proceed. Artin also has a neat introduction to representation theory. For grasping the basics of group theory, however, I would prefer Dummit and Foote any day.

A.2. **Books on representation theory.**

(1) *Representations of Finite Groups*(book name) by J. P. Serre
How I used the book: I referred to the book for my course on **Representation Theory of Finite Groups**(course name) by Senthamarai Kannan. I had also referred to it earlier, and I am referring to some parts of it for an introduction to modular characters (which is not part of the formal course, but is of interest to me).
(2) *Representation Theory*(book name) by Fulton and Harris
How I used the book: I studied the first few chapters, which give a fair introduction to the representation theory of finite groups. The strength of the book lies in its later chapters on Lie algebras and their representations. I have studied a bit of this but have not really managed an in-depth study. I might refer to this book later if I take a course on Lie algebras next semester.
(3) *Character theory of Finite Groups*(book name) by I.M. Isaacs
How I used the book: I clarified basic ideas on characters and representations of finite groups by reading this book. I also learnt of Isaacs from this book, and this led me to initiate a fruitful correspondence.
(4) *Local representation theory*(book name) by Jon L. Alperin
How I used the book: I am reading it to learn basics of local representation theory. It seems to be well-written and compact.

A.3. **Books on finite groups and classification.**

(1) *Finite group theory*(book name) by Michael Aschbacher.
How I used the book: For me, this book wasa first exposure to the techniques and ideas that formed the foundation for the classification work of finite groups. I came to realize the importance of linear representation theory, finite geometries, and various techniques for analyzing subgroups.

Appendix B. Useful and interesting webpages in group theory

B.1. **Organizational webpages.**

- `http://www.grouptheory.org` is the defining page for group theory.
- `http://zebra.sci.ccny.cuny.edu/web/nygtc/problems/` is a page listing important open problems in group theory

B.2. **Personal webpages.**

- `http://sciences.aum.edu/~tfoguel` is Tuval Foguel's home page.

B.3. **My own wiki.** `http://groupprops.wiki-site.com`