PROBLEM SET 9

ANALYSIS II

Problem 1. Suppose f is a complex linear map from $\mathbb{C}^2$ to $\mathbb{C}^2$. What can we say about the corresponding map from $\mathbb{R}^4$ to $\mathbb{R}^4$? Is it $\mathbb{R}$-linear? Can we characterize it further?

Problem 2. For each of the following complex power series, show that the disk of convergence is the unit disk, and analyze the behavior on the unit circle. For part (b) you can first try drawing the sequence of partial sums for a few specific values of θ.

(a) $\sum_{n=0}^{\infty} z^n$

(b) $\sum_{n=0}^{\infty} \frac{z^n}{n}$

(c) $\sum_{n=0}^{\infty} \frac{z^n}{n^2}$

Problem 3. Write out the function $z \mapsto z^2$ as a map from $\mathbb{R}^2 \to \mathbb{R}^2$, and examine its Jacobian derivative.

Problem 4. Write out the function $z \mapsto e^z = e^x e^{iy}$ as a map from $\mathbb{R}^2 \to \mathbb{R}^2$, and examine its Jacobian derivative.

Problem 5. Show that if

$$f'(a) := \lim_{\Delta z \to 0} \frac{f(a + \Delta z) - f(a)}{\Delta z}$$

exists and equals $\alpha + i\beta$, then

$$Df(a) = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}.$$

(Hint: if the complex derivative $f'(a)$ exists, then it doesn't matter how Δz approaches 0. Make it approach zero along the real axis and along the imaginary axis and compare results.)

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