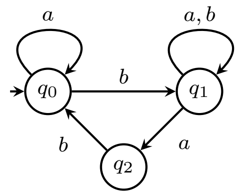
1. Does the following diagram represent a DFA?



2. Construct DFA for the following languages:

   a. $\emptyset$

   b. $\{\epsilon\}$

   c. The set of strings $x \in \{0,1\}^*$ such that $\#0(x)$ is even and $\#1(x)$ is a multiple of 3 ($\#0(x)$ and $\#1(x)$ denote the number of 0s and 1s in string x)

   d. The set of strings over $\{0,1\}^*$ that do not contain 001 as a substring.

   e. The set of strings in $\{0,1\}^*$ such that every consecutive block of three letters contains at least two 0s.

3. Give DFAs accepting the following languages over the alphabet $\{0,1\}$:

   a. The set of all strings that when interpreted as a binary integer, is a multiple of 3. For example, 0, 11, 1001 are in the language, whereas 100, 0001 and 11001 are not.

   b. The set of all strings that, when interpreted *in reverse* as a binary integer, is divisible by 3. For instance, 0, 0011, 1001 are in the language, and 1, 01, 101 are not.

4. Let $M = (Q, \Sigma, \delta, s, F)$ be an automaton. We defined the extended transition function $\widehat{\delta}$ as follows:

$$\begin{aligned} \widehat{\delta}(q,\epsilon) &= q && \text{for all states } q \in Q \\ \widehat{\delta}(q,xa) &= \delta(\widehat{\delta}(q,x),a) && \text{for all strings } x \in \Sigma^* \text{ and symbols } a \in \Sigma \end{aligned}$$

   Prove the following:

   a. For all states q and for all strings $x, y \in \Sigma^*$

$$\widehat{\delta}(q,xy) = \widehat{\delta}(\widehat{\delta}(q,x),y)$$

   b. For all states q, for all string y and for all input symbols a:

$$\widehat{\delta}(q,ay) = \widehat{\delta}(\delta(q,a),y)$$

5. Let $A = (Q, \Sigma, \delta, q_0, \{q_f\})$ be a DFA and suppose that for all $a \in \Sigma$, we have $\delta(q_0,a) = \delta(q_f,a)$.

   a. Show that for all $w \neq \varepsilon$, we have $\widehat{\delta}(q_0,w) = \widehat{\delta}(q_f,w)$.

   b. Show that if x is a nonempty string in $L(A)$, then for all $k > 0$, x^k (i.e., x written k times) is also in $L(A)$.