Examples of zone graphs

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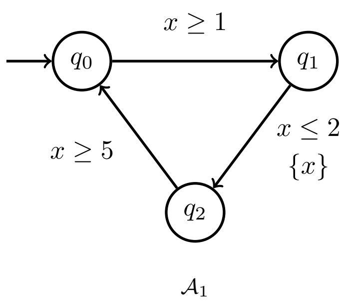
1 Zone graphs without $Closure_M$

In the lecture notes on "Reachability of timed automata using zones", we had given two algorithms: Algorithm 1.1 and Algorithm 1.3. In Algorithm 1.1, a new node (q, Z) is not explored if there exists an already visited node (q, Z') such that $Z \subseteq Z'$. This algorithm would work only on few timed automata. We give some examples. In Figures 1.1, 1.2, and 1.3, Algorithm 1.1 would terminate and the resulting graph computed is shown alongside. Figure 1.4 shows an automaton for which Algorithm 1.1 would not terminate.



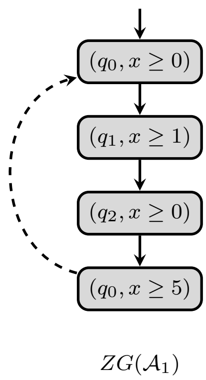


Figure 1.1: The dashed edge shows that $(q_0, x \geq 5)$ is a node that is covered by $(q_0, x \geq 0)$. This is because $x \geq 5 \subseteq x \geq 0$. So the node $(q_0, x \geq 5)$ will not be added to the Passed list by Algorithm 1.1

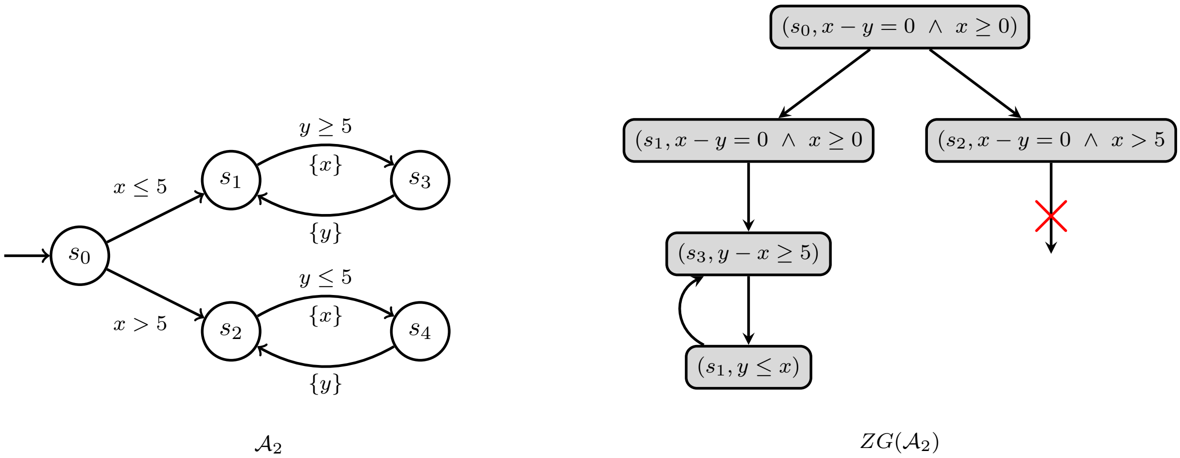


Figure 1.2: The red cross shows that the transition with guard $y \leq 5$ is not enabled from the zone $x - y \geq 0 \ \wedge \ x > 5$. Such a transition is called a *disabled* transition/edge.

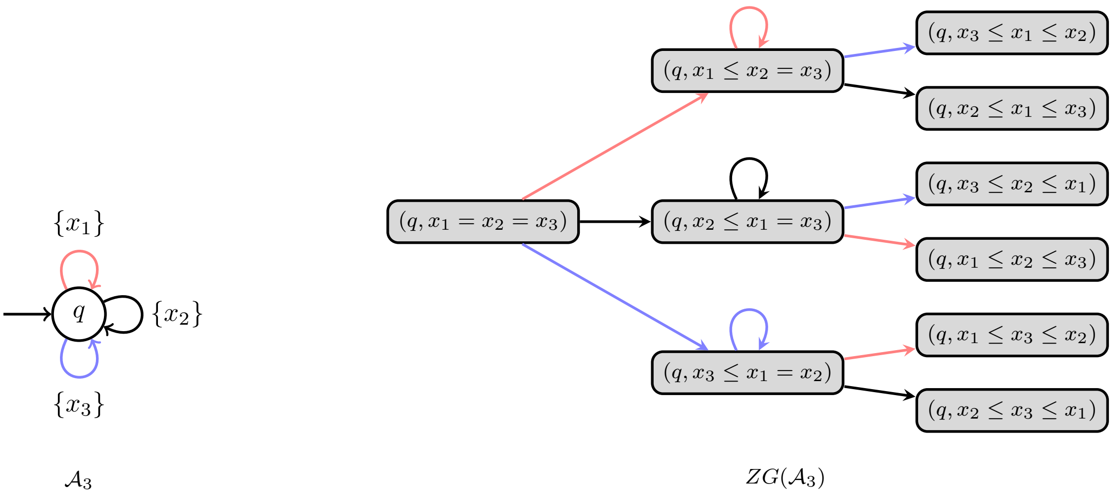


Figure 1.3: Zone graph contains all possible orderings of the three clocks. Transitions out of the nodes in the third column of $ZG(\mathcal{A}_3)$ have not been shown for clarity. However, note that the transitions out of these nodes will lead to already existing nodes. As none of these nodes are included inside the other, all these nodes will be explored by Algorithm 1.1 that uses the mere zone inclusion for stopping.

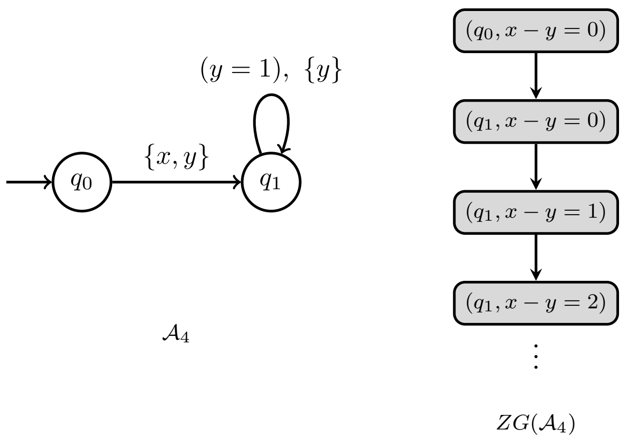
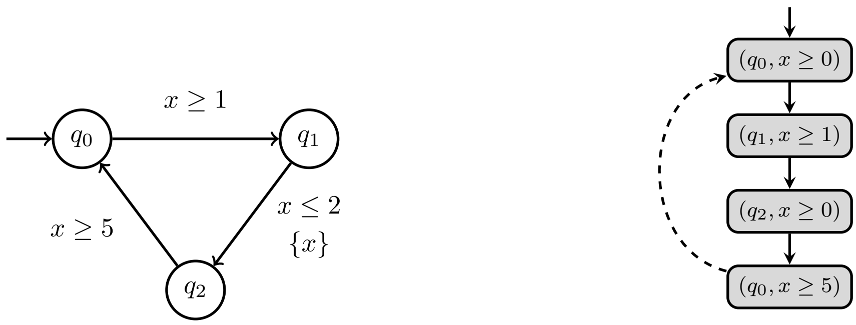


Figure 1.4: Automaton where the normal zone graph is infinite

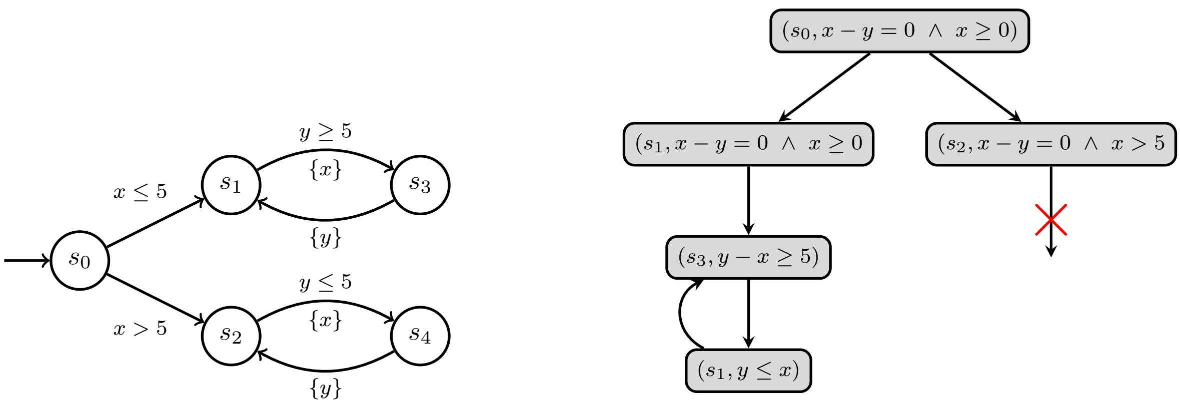
2 Zone graphs with $\mathit{Closure}_M$

We will now show how Algorithm 1.3 executes on the same set of examples. In particular, Algorithm 1.3 will always terminate.



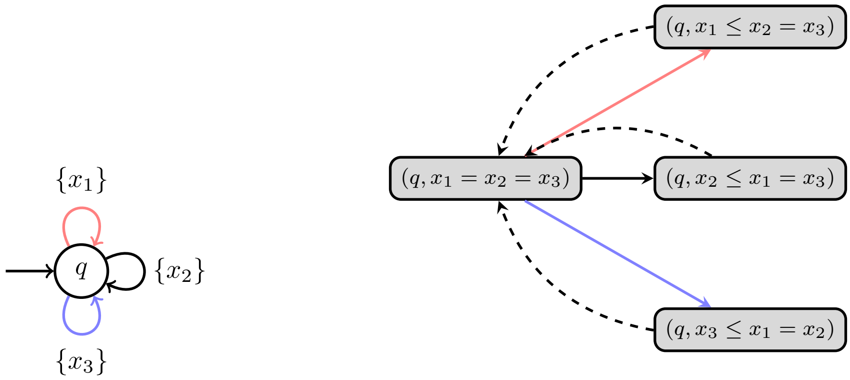
$\mathcal{A}_1$ $ZG^M(\mathcal{A}_1)$

Figure 1.5: Here $M(x) = 5$. We write $ZG^M(\mathcal{A})$ to denote the graph obtained by applying $\mathit{Closure}_M$ as inclusion relation. Note that the same edge would be covered as before. If $x \geq 5 \subseteq x \geq 0$ then $x \geq 5 \subseteq \mathit{Closure}_M(x \geq 0)$ (why?)



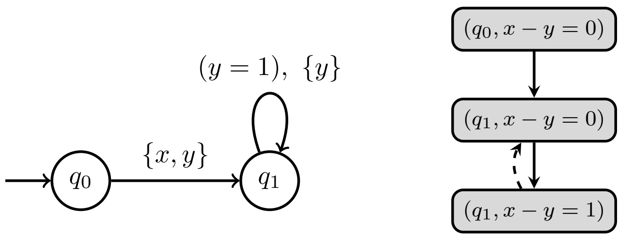
$\mathcal{A}_2$ $ZG^M(\mathcal{A}_2)$

Figure 1.6: Same as before in this case too.



$\mathcal{A}_3$ $ZG(\mathcal{A}_3)$

Figure 1.7: For automaton $\mathcal{A}_3$, $M(x_1) = M(x_2) = M(x_3) = -\infty$ as there is no guard at all. Therefore, the only region is the entire space $\mathbb{R}^3_{\geq 0}$ itself. This means that $Closure_M(x_1 = x_2 = x_3) = \mathbb{R}^3_{\geq 0}$ and hence any other zone would be contained in it. Therefore there will be only one node $(q_0, x_1 = x_2 = x_3)$ in the Passed list computed by Algorithm 1.3



$\mathcal{A}_4$

$ZG^M(\mathcal{A}_4)$

Figure 1.8: For $\mathcal{A}_4$, the maximal bounds function $M(x) = -\infty$ and $M(y) = 1$. Hence we get the inclusion $x - y = 1 \subseteq Closure_M(x - y = 0)$.