## Unit-10: Algorithms for CTL

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Chennai Mathematical Institute

NPTEL-course

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# Module 2: EX, EU and EG

## CTL model-checking problem

Given transition system M and a CTL formula  $\phi$ , find all states of M that satisfy  $\phi$ 

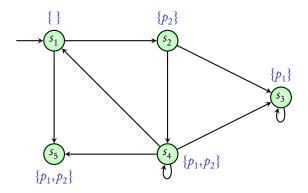
## CTL model-checking problem

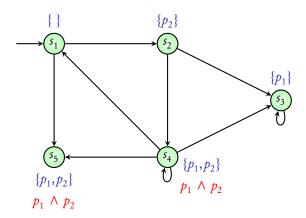
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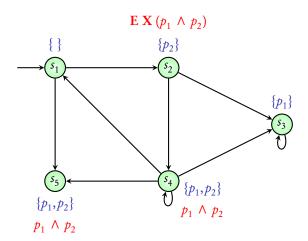
In this unit: Special case when  $\phi$  is either E X, E U or E G

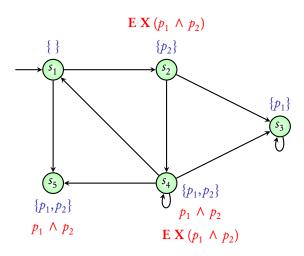
### Part 1:

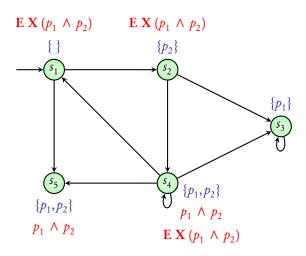
## Algorithm for E X

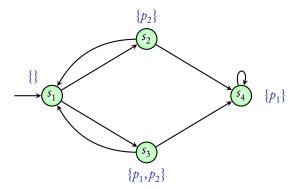


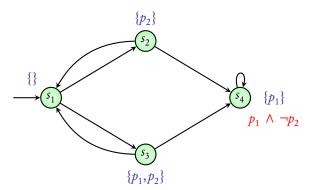


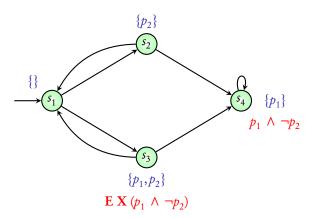


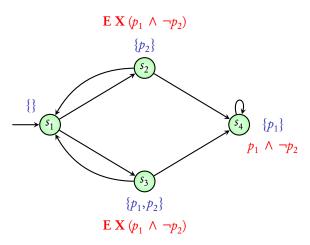


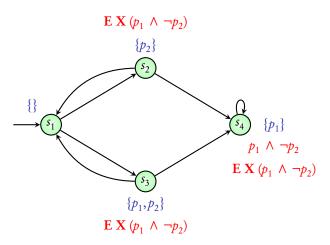




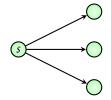






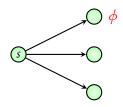


## Algorithm for E X $\phi$



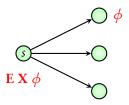
### Algorithm for E X $\phi$

Suppose states satisfying  $\phi$  have been labelled



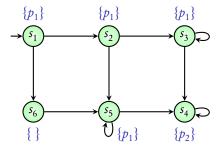
### Algorithm for E X $\phi$

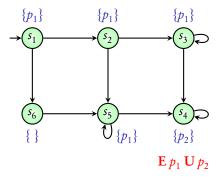
Suppose states satisfying  $\phi$  have been labelled

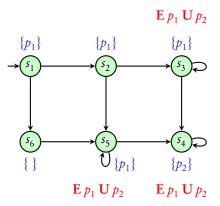


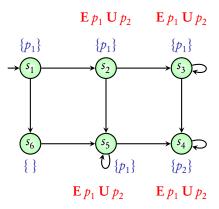
State s is labelled with E X  $\phi$  if there exists a successor which is labelled  $\phi$ 

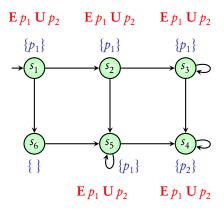
## Part 2: Algorithm for E U

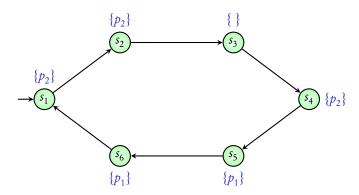


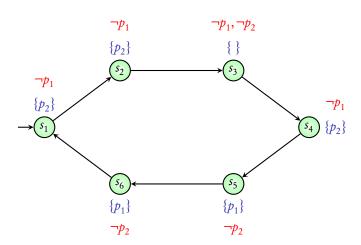


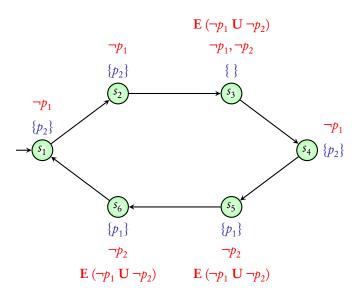


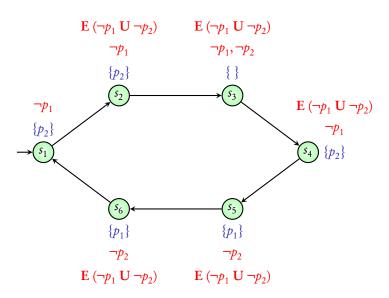


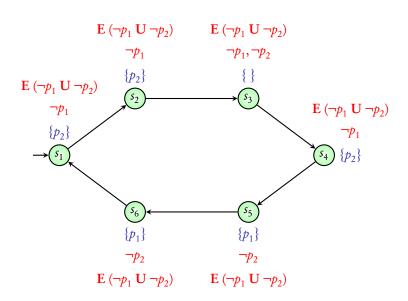




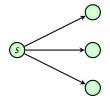






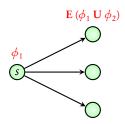


### Algorithm for E ( $\phi_1 \cup \phi_2$ )



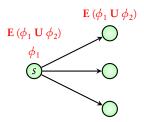
- ► If any state is labelled with  $\phi_2$ , label it with  $\mathbf{E}$  ( $\phi_1$   $\mathbf{U}$   $\phi_2$ )
- ► Repeat: Label any state with  $\mathbf{E}$  ( $\phi_1$   $\mathbf{U}$   $\phi_2$ ) if it is labelled with  $\phi_1$  and at least one successor is labelled with  $\mathbf{E}$  ( $\phi_1$   $\mathbf{U}$   $\phi_2$ ) until no change

### Algorithm for E ( $\phi_1 U \phi_2$ )



- If any state is labelled with  $\phi_2$ , label it with  $\mathbf{E} (\phi_1 \mathbf{U} \phi_2)$
- ▶ *Repeat*:
  Label any state with  $\mathbf{E}$  ( $\phi_1 \mathbf{U} \phi_2$ ) if it is labelled with  $\phi_1$  and at least one successor is labelled with  $\mathbf{E}$  ( $\phi_1 \mathbf{U} \phi_2$ ) *until no change*

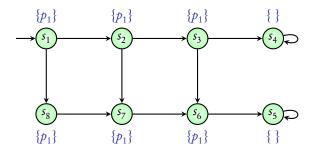
### Algorithm for E ( $\phi_1 U \phi_2$ )



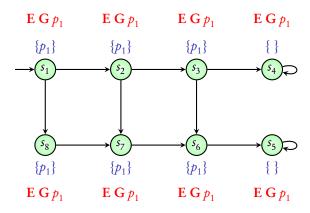
- If any state is labelled with  $\phi_2$ , label it with  $\mathbf{E} (\phi_1 \mathbf{U} \phi_2)$
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  Label any state with  $\mathbf{E}$  ( $\phi_1 \mathbf{U} \phi_2$ ) if it is labelled with  $\phi_1$  and at least one successor is labelled with  $\mathbf{E}$  ( $\phi_1 \mathbf{U} \phi_2$ ) *until no change*

## Part 3: Algorithm for E G

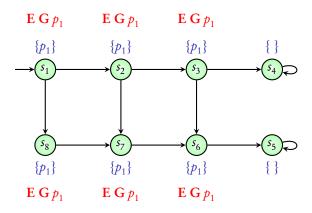
### $\mathbf{E} \mathbf{G} p_1$

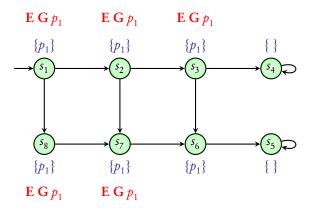


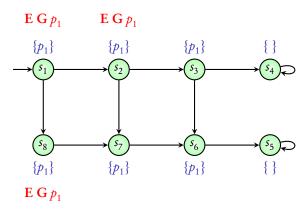
 $\mathbf{E} \mathbf{G} p_1$ 

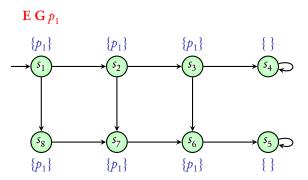


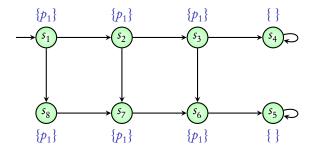
### $\mathbf{E} \mathbf{G} p_1$

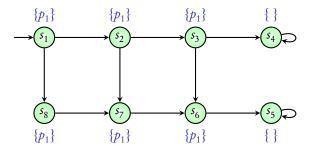




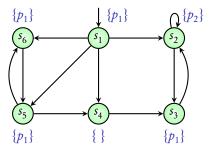


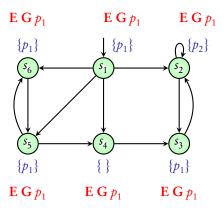


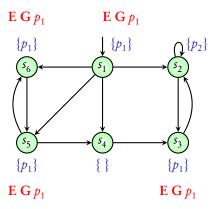


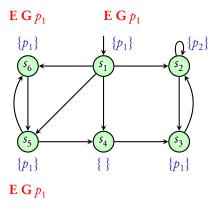


No state of the above transition system satisfies **E**  $G p_1$ 









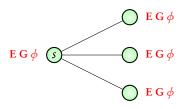
▶ Label all states with **E G**  $\phi$ 

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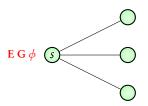
#### ► Repeat:

- ▶ Label all states with **E G**  $\phi$
- If any state is **not** labelled with  $\phi$ , **delete** the label **E G**  $\phi$



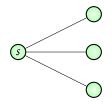
#### ► Repeat:

- ▶ Label all states with **E G**  $\phi$
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#### ► Repeat:

- ▶ Label all states with **E G**  $\phi$
- If any state is **not** labelled with  $\phi$ , **delete** the label **E G**  $\phi$



#### ► Repeat:

# Summary

Algorithms

EX, EU, EG