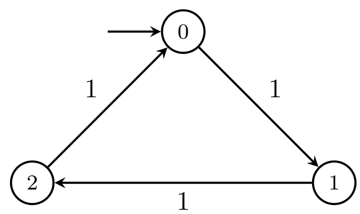
1. Show that in any open interval of the real line having length $1/[n(n-1)]$ there is atmost one rational of the form p/q where $1 \leq q \leq n$.

2. Consider a mean-payoff game with n vertices and integer weights coming from the set $\{-W, \ldots, W\}$. Prove the following statements about the value $v(a)$ of a vertex a:

   i) $-W \leq v(a) \leq W$

   ii) $v(a)$ is a rational number of the form p/q where $1 \leq q \leq n$

3. What is the value of the following game (starting at vertex 0)? All vertices belong to Maximizer.



   Run the value iteration algorithm of [ZP96] on the above game.

4. Consider a mean-payoff game whose graph is a cycle: vertices are $\{1, 2, \ldots, n\}$; there is an edge $i \to i+1 \mod n$ with weight 1; all vertices belong to the Maximizer. Run the value iteration algorithm on this game.

5. Consider a mean-payoff game whose graph is a cycle: vertices are $\{1, 2, \ldots, n\}$; there is an edge $i \to i+1 \mod n$; the edge from $1 \to 2$ has weight 1 and the other edges have weight 0; all vertices belong to the Maximizer. Run the value iteration algorithm on this game.

6. Consider Figure 1 in page 348 of [ZP96]. Show that the value iteration method arising out of the equations in Theorem 2.1 needs at least $\Omega(n^3 W)$ iterations to come within $1/[2n(n-1)]$ distance of the actual value.

7. Understand Theorem 2.4 of [ZP96].

8. From a parity game $G = (V_0, V_1, E)$ with n vertices, define a mean-payoff game as follows. The graph for the mean-payoff game remains the same. Player 0 and 1 become respectively the Minimizer and Maximizer in the mean-payoff game. Let $p(u)$ be the priority of a vertex u. Add the weight $-(-n)^{p(u)}$ to all outgoing edges of u in the mean-payoff game.

   Player 1 wins the parity game from a vertex a if she can force a play where the maximum priority occuring infinitely often is odd. Show that Player 1 wins the parity game at a vertex a iff the value $v(a) > 0$ in the mean-payoff game.

References

[ZP96] Uri Zwick and Mike Paterson. The complexity of mean payoff games on graphs. *Theoretical Computer Science*, 158(1):343 – 359, 1996.