ACTS
CMI Chennai
10-2-2015

Reasoning with reflexive-transitive path logics

Diego Figueira
CNRS, LaBRI

data word

b	c	a	b	c	c	a	a	b	c	b
3	1	5	1	1	1	4	4	5	1	4

data word

b	c	a	b	c	c	a	a	b	c	b
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$\in (\mathbf{A} \times \mathbf{D})^*$

D: infinite domain

A: finite alphabet

data word

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$\in (\mathbf{A} \times \mathbf{D})^*$

- **D** → infinite domain
- **A** → finite alphabet

Reasoning with logics on data words:

high complexity
or
limited expressive power

data word

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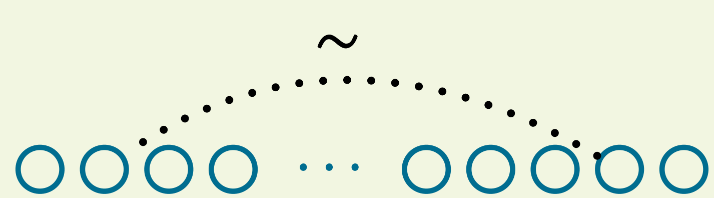
$\in (\mathbf{A} \times \mathbf{D})^*$

infinite domain
finite alphabet

Reasoning with logics on data words: **high complexity** or **limited expressive power**

Things become ugly as soon as:



Logics for data words

Logic	SAT	
$FO^2(<,+1,\sim)$	~ PN-reach	[Bojańczyk & al.]
$FO^2(<,\sim)$	NExpTime-c	[Bojańczyk & al.]
$LTL^{\downarrow}(F, U, X)$	Decidable, non-PR hard	[Demri, Lazić]
$LTL^{\downarrow}(F)$	Decidable, non-PR hard	[F, Segoufin]
$LTL^{\downarrow}(F, F^{-1})$	Undecidable	[F, Segoufin]
BasicDataLTL	~ PN-reach	[Kara & al]
LRV	2ExpSpace-c	[Demri, F, Praveen]
LRV + P	~ PN-reach	[Demri, F, Praveen]

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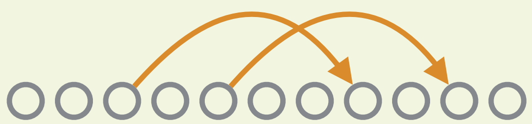
Path logics

XPath on data words

path expressions

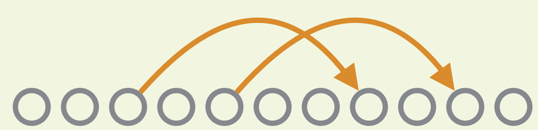


node expressions

Path logics

XPath on data words

path expressions



$$\alpha,\beta ::= \varepsilon \mid \alpha\beta \mid \alpha[\phi] \mid o \qquad o \in \{ \longrightarrow, \longrightarrow^{+}, \longrightarrow^{*}, \longleftarrow, {}^{+}\!\longleftarrow, {}^{*}\!\longleftarrow \}$$

node expressions

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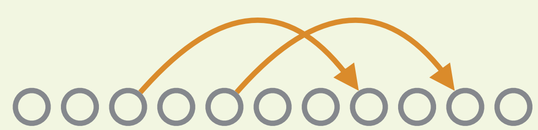

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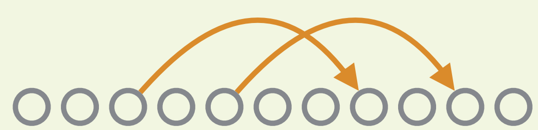
node expressions

$$\phi,\psi ::= \mathbf{a} \mid \neg\phi \mid \phi\wedge\psi \mid \langle\alpha=\beta\rangle \mid \langle\alpha\neq\beta\rangle \mid \alpha? \qquad \mathbf{a}\in\mathbf{A}$$

Path logics

XPath on data words

path expressions



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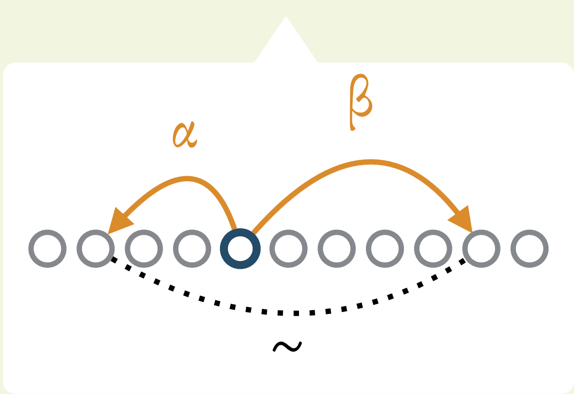

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eg: $\longrightarrow^*[\mathbf{b}]\longrightarrow[\mathbf{c}]$

Path logics

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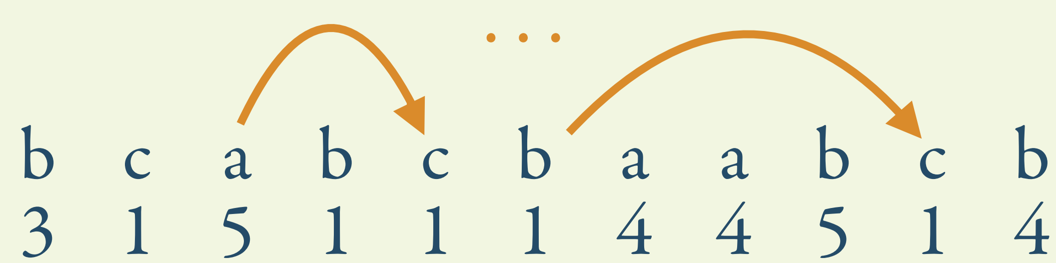

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Path logics

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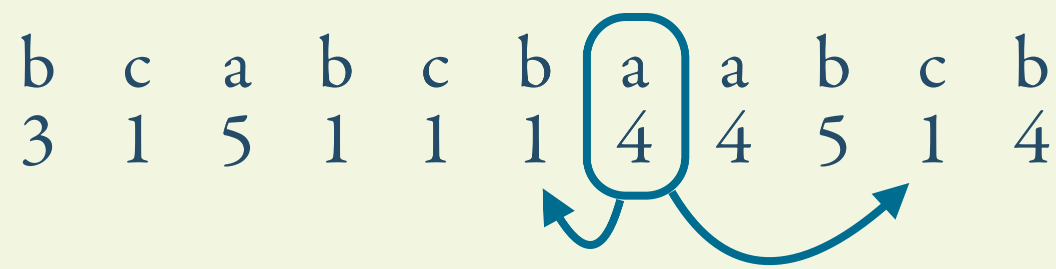
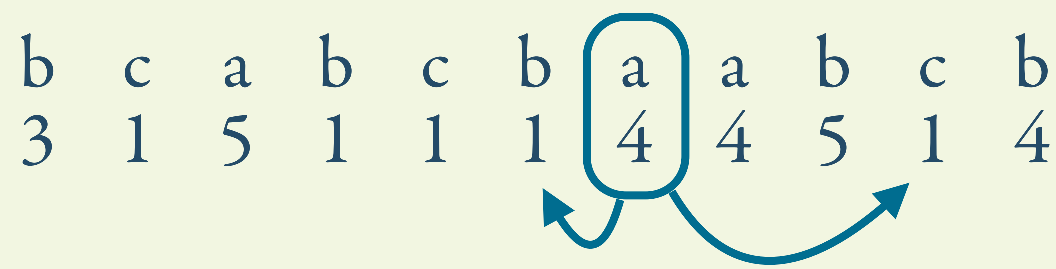


eg: $\langle [\mathbf{b}]\longleftarrow \; = \; \longrightarrow^{*}[\mathbf{b}]\longrightarrow[\mathbf{c}] \rangle$

Path logics

XPath on data words



eg: $\longrightarrow^*[\mathbf{a} \wedge \langle [\mathbf{b}]\longleftarrow \; = \; \longrightarrow^*[\mathbf{b}] \longrightarrow[\mathbf{c}] \rangle]$?

Path logics

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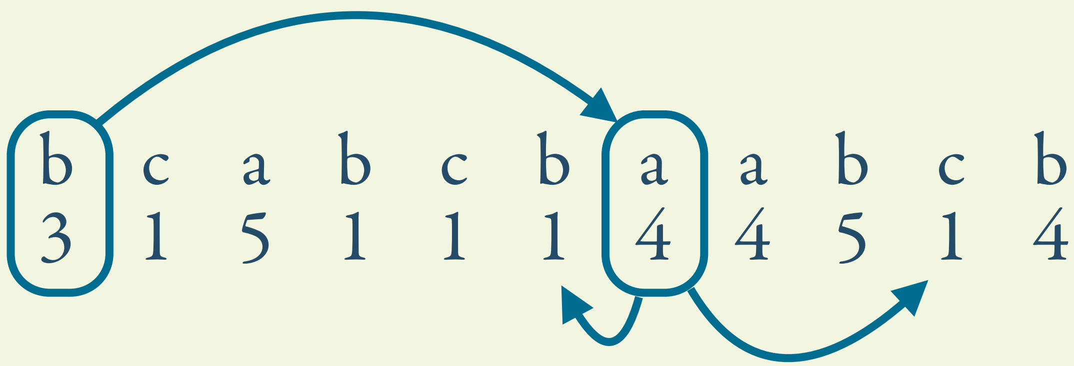

eg: $\longrightarrow^*[\mathbf{a} \wedge \langle [\mathbf{b}]\longleftarrow \;=\; \longrightarrow^*[\mathbf{b}]\longrightarrow[\mathbf{c}] \rangle]$?

Satisfiability of XPath on data words

XPath($\longrightarrow^{+}$, $^{+}\longleftarrow$): undecidable ♣

♠ [Demri, Lazić, 2006]

♣ [F., Segoufin, 2009]

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XPath($\longrightarrow^{+}$, ${}^{+}\!\longleftarrow$): undecidable ♣

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XPath($\longrightarrow^{+}$): decidable, non-PR ♠♣

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In particular, any fragment with $\longrightarrow^{+}$ or $^{+}\!\!\longleftarrow$ is **undecidable** or has a **non-PR complexity**

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why?

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why?

SAT-XPath($\longrightarrow$,$\longrightarrow^+$)

$\rightsquigarrow$

SAT-XPath($\longrightarrow^+$)

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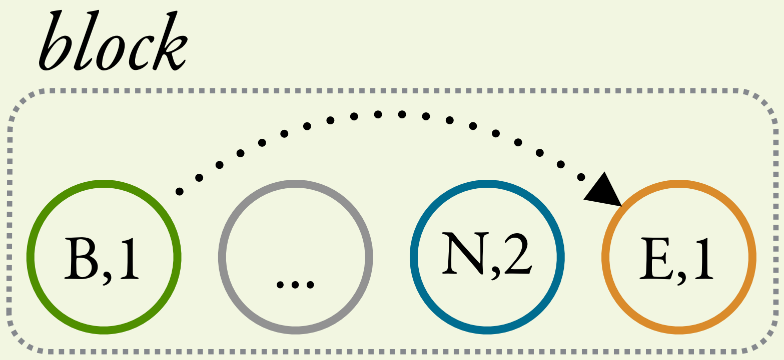
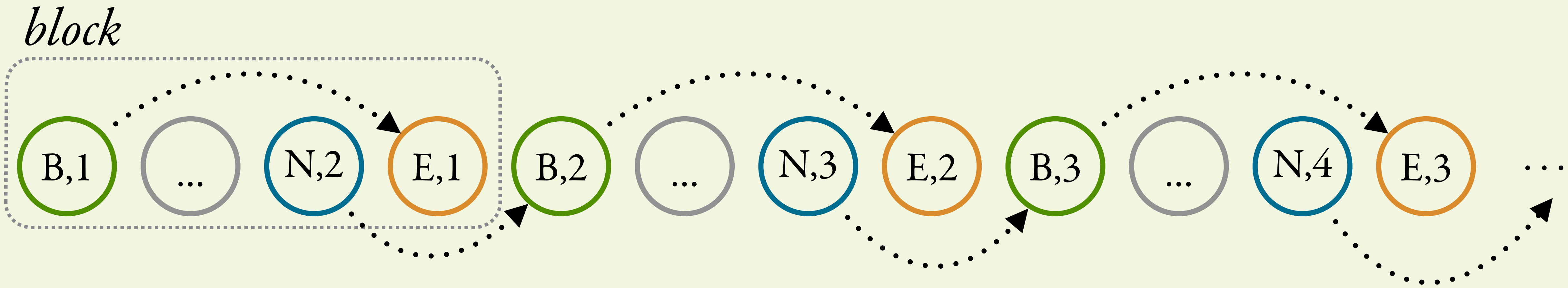
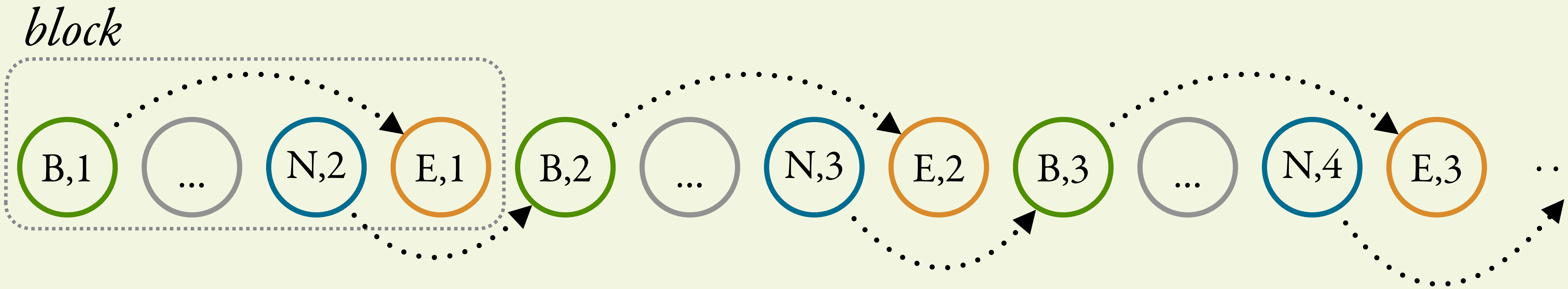
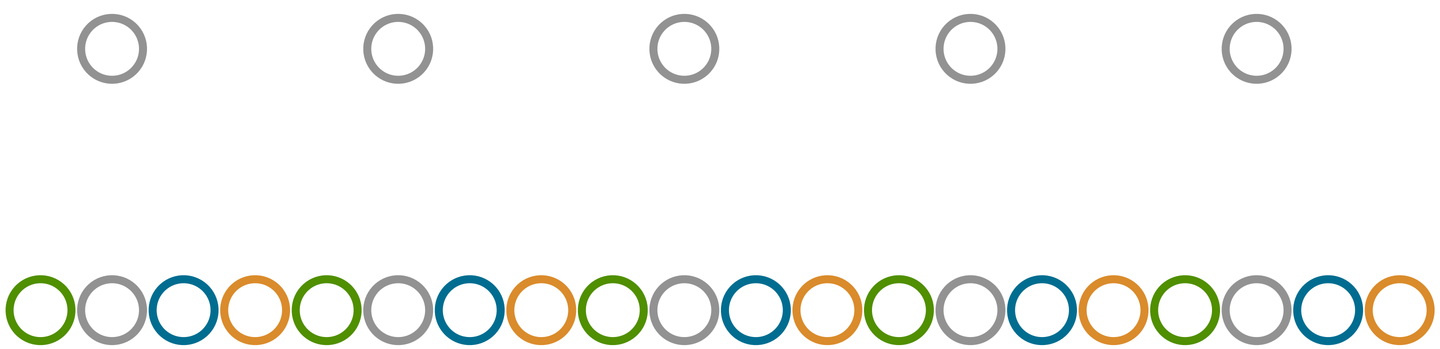

block
B,1
...
N,2
E,1



block
B,1
...
N,2
E,1
B,2
...
N,3
E,2
B,3
...
N,4
E,3

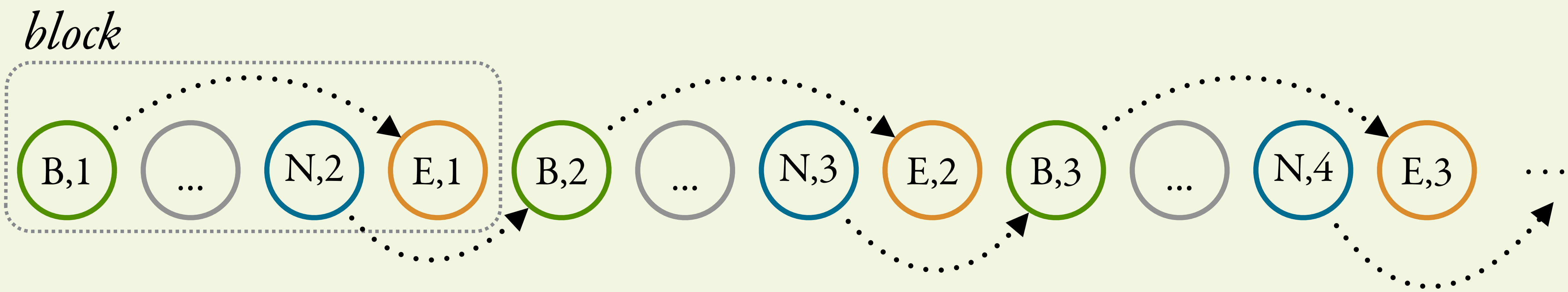




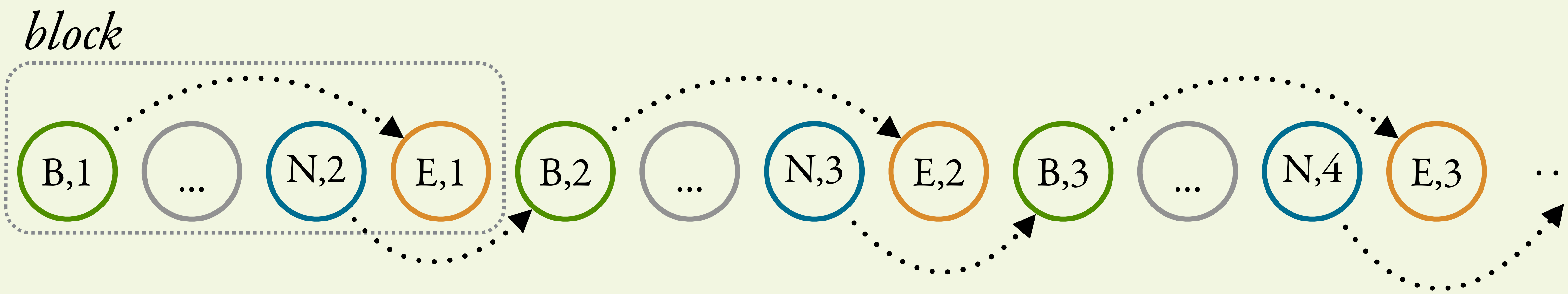
$\models \phi \in \mathrm{XPath}(\longrightarrow, \longrightarrow+)$

$\Updownarrow$

$\models \phi' \in \mathrm{XPath}(\longrightarrow+)$

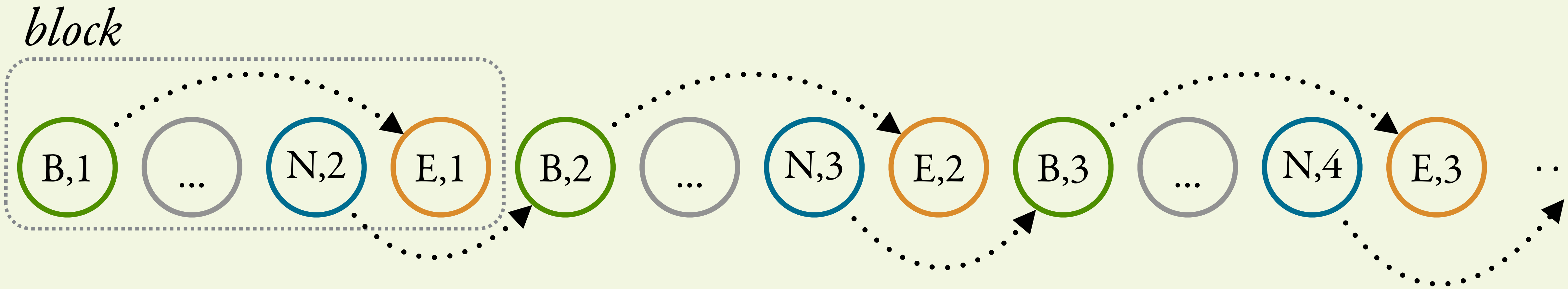


No more than one B,x , N,y , E,x per block



No more than one (B,x) , (N,y) , (E,x) per block

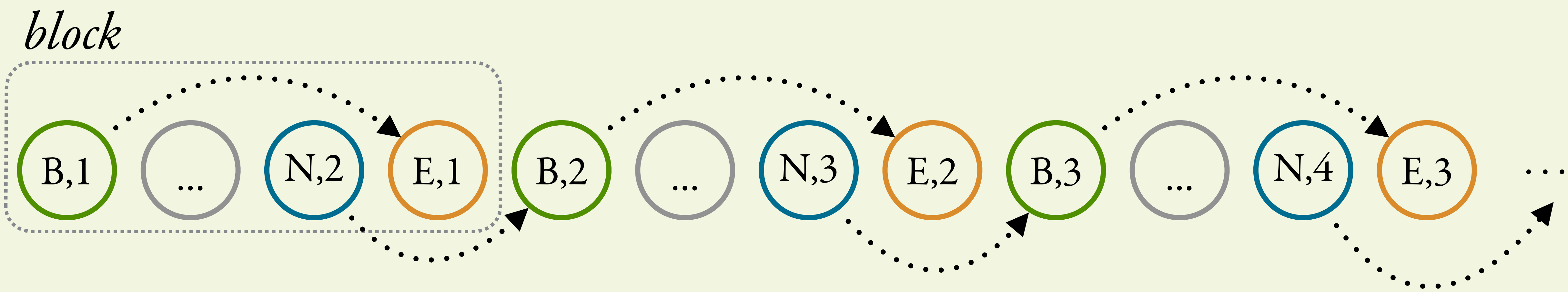
All (B,x) have different data (resp (N,y) , (E,x))



No more than one B,*x* , N,*y* , E,*x* per block

All B,*x* have different data (resp N,*y* , E,*x*)

N,*y* points to next block

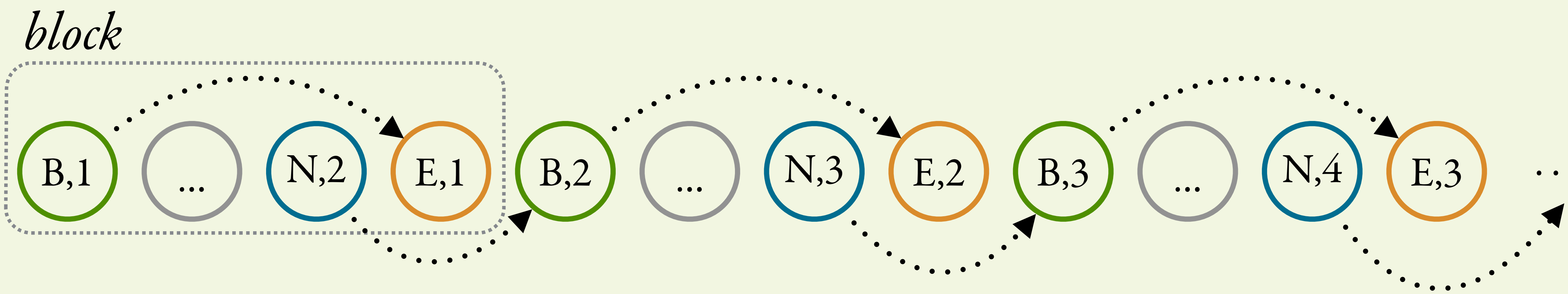


No more than one B,x , N,y , E,x per block

All B,x have different data (resp N,y , E,x)

N,y points to next block

For every B,x there is an E,x with same datum



No more than one (B,x), (N,y), (E,x) per block

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(N,y) points to next block

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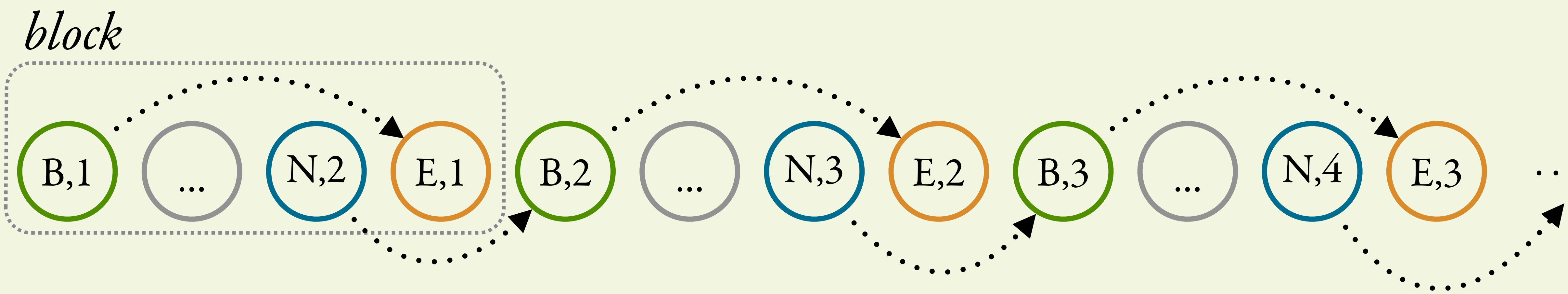
Order in the block: (B,x) (...) (N,y) (E,x)



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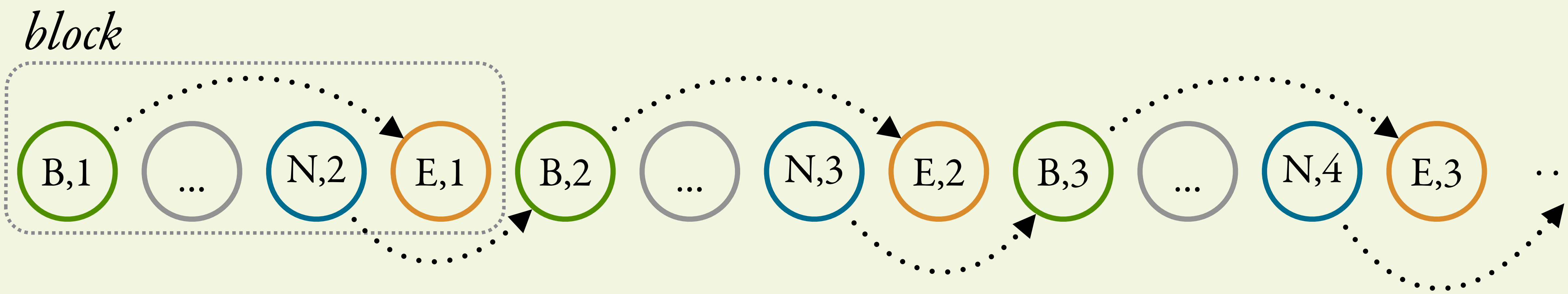
$$\text{next-block}(\phi) := \langle \varepsilon = \longrightarrow^{*}[N \wedge \langle \varepsilon = \longrightarrow^{*}[B \wedge \phi] \rangle]\longrightarrow^{*}[E]\rangle$$



$\longrightarrow^+$ $\longrightarrow^*$

✔ No more than one (B,x), (N,y), (E,x) per block

✔ All (B,x) have different data (resp (N,y), (E,x))

✔ (N,y) points to next block

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✔ Order in the block: (B,x) (...) (N,y) (E,x)

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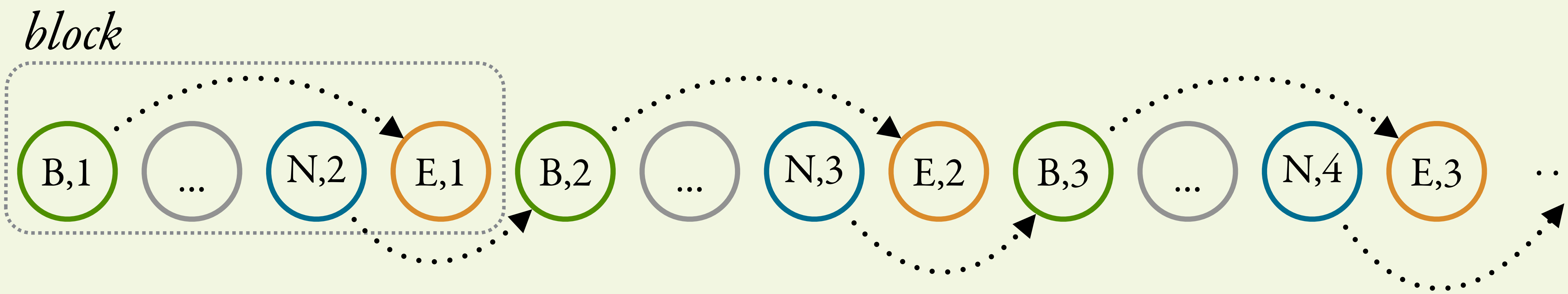

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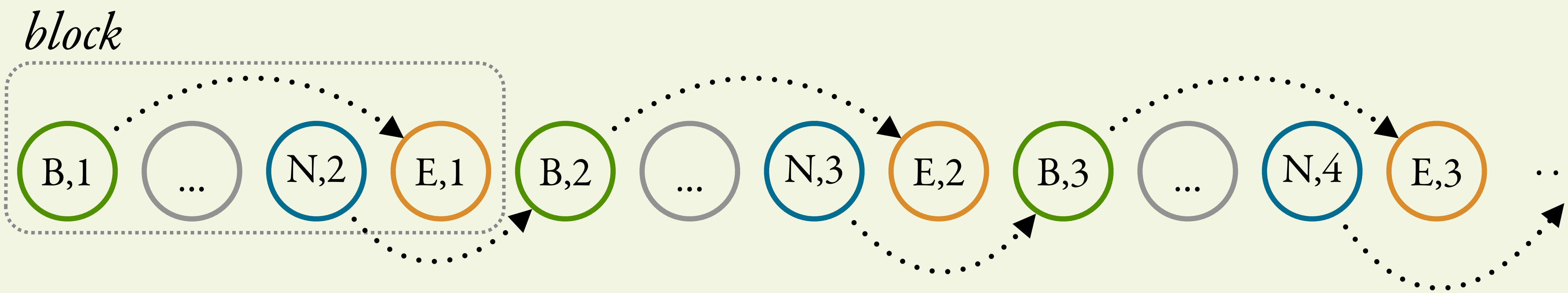

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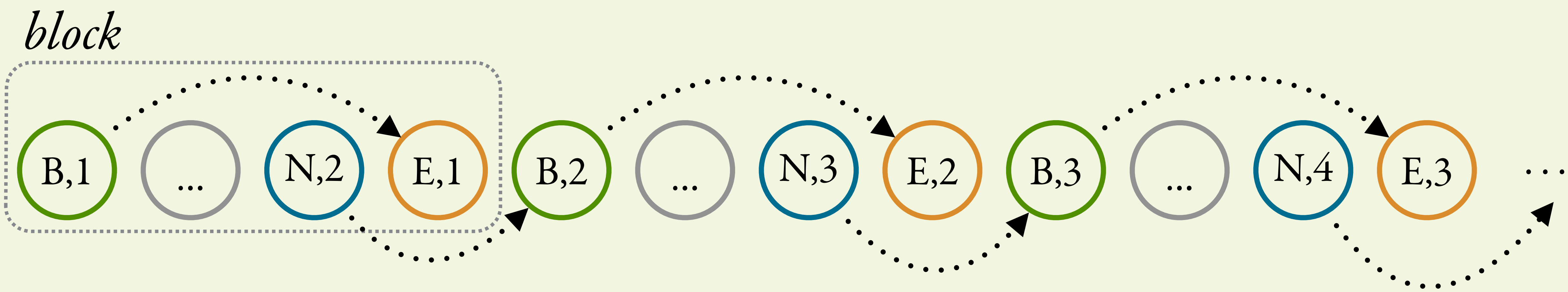

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Satisfiability of XPath on data words

XPath($\longrightarrow^+$, $^+\!\longleftarrow$): undecidable ♣

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XPath($\longrightarrow$, $\longrightarrow^*$,$^*\!\longleftarrow$): undecidable ♣

XPath($\longrightarrow^+$): decidable, non-PR ♠ ♣

In particular, any fragment with $\longrightarrow^+$ or $^+\!\longleftarrow$ is **undecidable** or has a **non-PR complexity**

What about XPath($\longrightarrow^*$)?

♠ [Demri, Lazić, 2006]

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XPath($\longrightarrow^*$) is **decidable** in 2ExpSpace ♦

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why?

SAT-XPath($\longrightarrow^*$,$^*\longleftarrow$) decidable in 2ExpSpace

Proof idea:

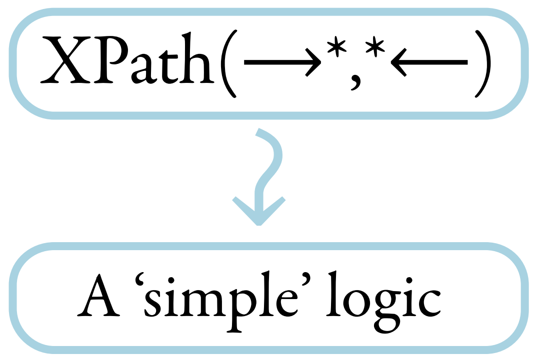
XPath($\longrightarrow^*$,$^*\longleftarrow$)

⇓

A 'simple' logic ...in *'Scott Normal Form'*

SAT-XPath($\longrightarrow^*$,$^*\longleftarrow$) decidable in 2ExpSpace

Proof idea:



...in *'Scott Normal Form'*

- There is a position labelled '**a**'.

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Proof idea:

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A 'simple' logic ...in *'Scott Normal Form'*

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- For every position labelled '**a**', a BC of:

$$\langle\ [\phi_n]^*\!\longleftarrow \cdots {}^*\!\longleftarrow[\phi_1]^*\!\longleftarrow[\phi_0] \ = \ \longrightarrow^*[\psi_0]\longrightarrow^*[\psi_1]\longrightarrow^* \cdots \longrightarrow^*[\psi_m]\ \rangle$$

$\phi_i, \psi_i \in \mathrm{BC}(\mathbf{A})$

$d \in \mathbf{D}$

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Proof idea:

XPath($\longrightarrow^*$,$^*\longleftarrow$)

$\downarrow$

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$$\langle\ d\ =\ \longrightarrow^*[\psi_0]\longrightarrow^*[\psi_1]\longrightarrow^* \cdots \longrightarrow^*[\psi_m]\ \rangle$$

$$\vdots$$

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$d \in \mathbf{D}$

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Proof idea:

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↓

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$$\langle\, [\phi_n]^*\!\longleftarrow \cdots {}^*\!\longleftarrow [\phi_1]^*\!\longleftarrow [\phi_0] \overset{\neq}{=} \longrightarrow^*[\psi_0]\longrightarrow^*[\psi_1]\longrightarrow^* \cdots \longrightarrow^*[\psi_m]\,\rangle$$

$$\langle\, [\phi_n]\longrightarrow^* \cdots \longrightarrow^*[\phi_1]\longrightarrow^*[\phi_0] \overset{\neq}{=} \longrightarrow^*[\psi_0]\longrightarrow^*[\psi_1]\longrightarrow^* \cdots \longrightarrow^*[\psi_m]\,\rangle$$

$$\langle\, d \overset{\neq}{=} \longrightarrow^*[\psi_0]\longrightarrow^*[\psi_1]\longrightarrow^* \cdots \longrightarrow^*[\psi_m]\,\rangle$$

$$\vdots$$

$\phi_i, \psi_i \in \mathrm{BC}(\mathbf{A})$

$d \in \mathbf{D}$

SAT-XPath($\longrightarrow^*$,$^*\!\longleftarrow$) decidable in 2ExpSpace

Proof idea:

XPath($\longrightarrow^*$,$^*\!\longleftarrow$)

A 'simple' logic ...in *'Scott Normal Form'*

- There is a position labelled '**a**'.
- For every position labelled '**a**', a BC of:

"$\overleftarrow{\phi_n \bullet \cdots \bullet \phi_1 \bullet \phi_0} = \overrightarrow{\psi_0 \bullet \psi_1 \bullet \cdots \bullet \psi_m}$"

$$\langle\ [\phi_n]^*\!\longleftarrow \cdots {}^*\!\longleftarrow[\phi_1]^*\!\longleftarrow[\phi_0] \ \overset{\neq}{=}\ \longrightarrow^*[\psi_0]\longrightarrow^*[\psi_1]\longrightarrow^* \cdots \longrightarrow^*[\psi_m]\ \rangle$$

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$$\vdots$$

$\phi_i, \psi_i \in \mathrm{BC}(\mathbf{A})$

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ϕ

there is only one dv under a **c**

for every **a**, there is a **b** accessible via a **c** with the same dv

there is a position labeled **c**

subpaths of ϕ = {**c** • **b**, **c**, **b**}

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w

a	b	b	a	b	c	a	b	c	c	a	a	b	c	b	$\models \phi$
1	2	4	4	3	1	5	1	1	1	4	4	5	1	4	

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w

a	b	b	b	a	a	b	c	a	b	c	c	a	a	a	a	b	c	b	$\models \phi$
1	2	4	9	4	9	3	1	5	1	1	1	4	9	4	9	5	1	4	

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						i			j					

$\models \phi$

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1	2	4	4	3	1	[5	1	1	1]	4	4	5	1	4	
						i			j						

π

profile
- internal — reachable data
  - from the left [-->]
  - from the right [<--]
- external — reachable data
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abs(w,i,j) = π =

[-->] : { (1,c), (1,b) }

[<--] : { (1,c), (1,c•b), (1,b) }

[ ]--> : { (5,b), (4,b), (4,c•b), (1,c) }

<--[ ] : { (1,c), (3,c•b), (4,c•b), (4,b), (3,b) (2,b) }

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1	2	4	4	3	1	5	1	1	1	4	4	5	1	4

i j $\models \phi$

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$\leq$

ϕ

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there is a position labeled **c**

subpaths of ϕ = {**c • b**, **c**, **b**}

w

a	b	b	b	a	a	b	c	[a	b	c	c]	a	a	a	a	b	c	b		
1	2	4	9	4	9	3	1	[5	1	1	1]	4	9	4	9	5	1	4		
								i			j									

$\models \phi$

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<--[ ] : { (1,c), (3,c•b), (4,c•b), (4,b), (3,b) (2,b) } ∪ { (9,c•b), (9,b) }

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$\leq$

Induces a **wqo** ≤ on the profiles

→ every $\pi_1, \pi_2, \pi_3, \ldots$ has some $i<j$ with $\pi_i \leq \pi_j$

Atomic profiles

a	b	b	a	b	c	[a]	b	c	c	a	a	b	c	b
1	2	4	4	3	1	[5]	1	1	1	4	4	5	1	4

$\pi = \mathrm{abs}(\boldsymbol{w}, i, i{+}1)$ (compatible with ϕ)

Atomic$_\phi$ = all ϕ-abstractions of positions within data words

Concatenation of profiles

a	b	b	a	b	c	a	b	c	c	a	a	b	c	b
1	2	4	4	3	1	5	1	1	1	4	4	5	1	4

Atomic profiles

a	b	b	a	b	c	[a]	b	c	c	a	a	b	c	b
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Concatenation of profiles

a	b	b	[a	b	c]	a	b	c	c	a	a	b	c	b
1	2	4	[4	3	1]	5	1	1	1	4	4	5	1	4

π_1

Atomic profiles

a	b	b	a	b	c	[a]	b	c	c	a	a	b	c	b
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π_1 π_2

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$\pi_3 \;=\; \pi_1 \cdot \pi_2$

Atomic profiles

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$$\pi_3 = \pi_1 \cdot \pi_2$$

a) a profile is an abstraction of a model ⇔ it is derivable (from atomic profiles) and complete

Der

[ ]--> : ∅

<--[ ] : ∅

Atomic profiles

a	b	b	a	b	c	[a]	b	c	c	a	a	b	c	b
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$a{+}b$: SAT(ϕ) ⇔ There is complete $\pi \in$ **Der** so that $\pi \models \phi$

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$a{+}b$: SAT(ϕ) ⇔ There is complete $\pi \in$ **Der** so that $\pi \models \phi$

How to compute **Der***?*

1 By property before: $\mathbf{Der} = \uparrow \mathbf{Der}$

$$\{ \pi \mid \pi' \leq \pi, \pi' \in \mathbf{Der} \}$$

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2 MIN(**Atomic**) is finite and computable

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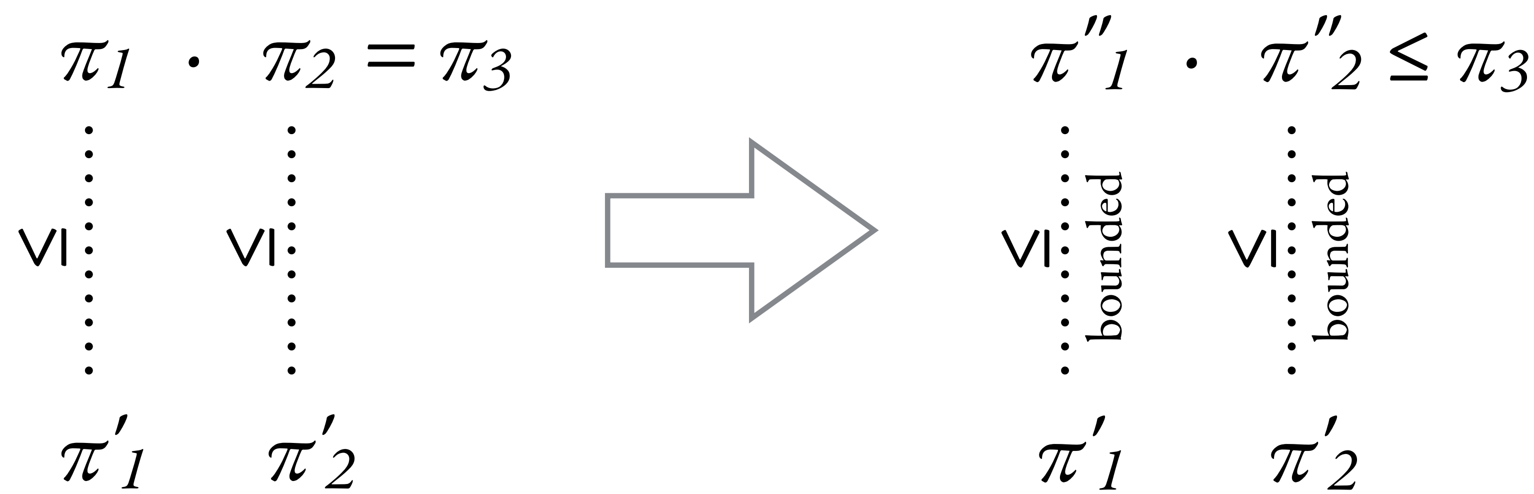

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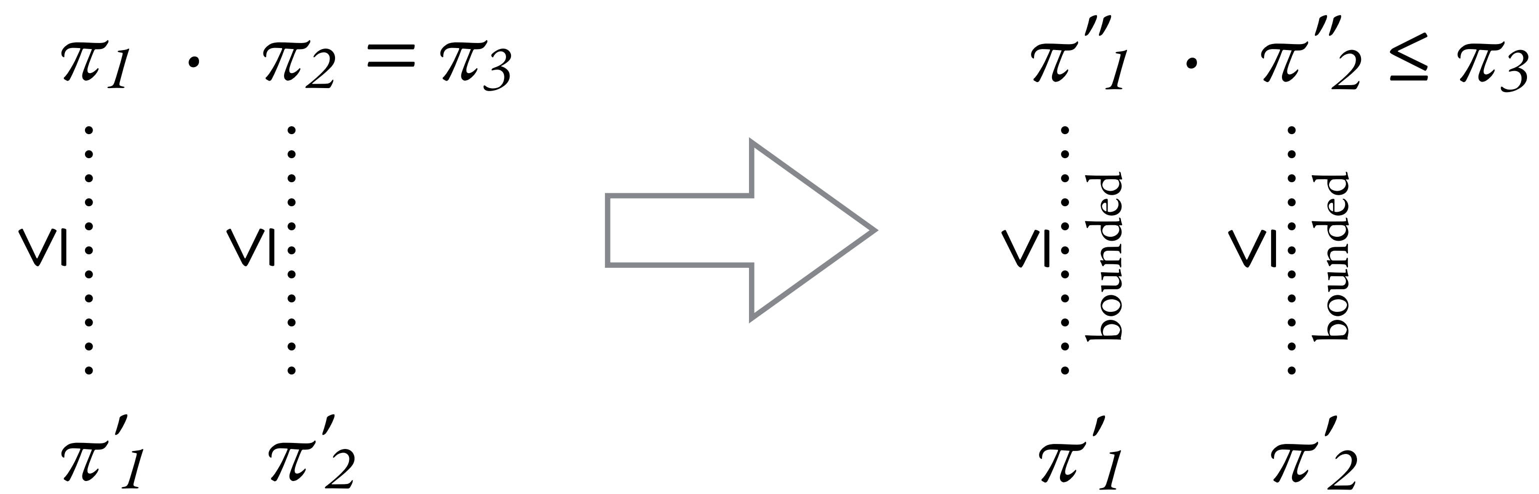

1+2 =

```
R0 = MIN( Atomic ) ;
while (Ri ≠ Ri+1) :
    Ri+1 = MIN(Ri ∪ ↑Ri · ↑Ri)

return( Ri );
```

computes MIN(Der)

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3

$$\pi_1 \cdot \pi_2 = \pi_3 \quad \Rightarrow \quad \pi''_1 \cdot \pi''_2 \leq \pi_3$$

$\pi_1 \geq \pi'_1$, $\pi_2 \geq \pi'_2$ ⇒ $\pi''_1 \geq \pi'_1$ (bounded), $\pi''_2 \geq \pi'_2$ (bounded)

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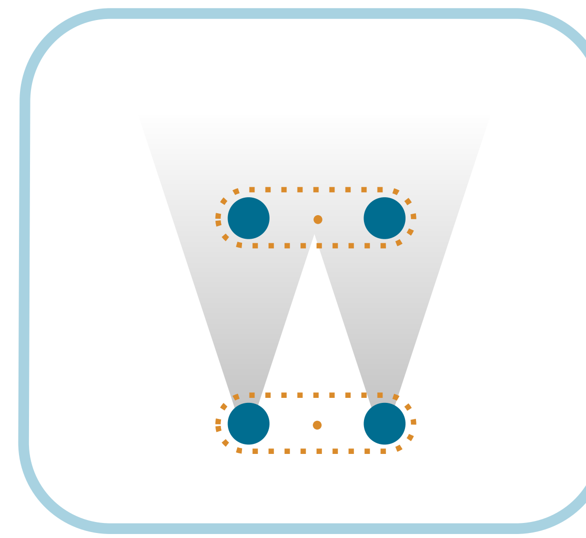
$$\pi_1 \cdot \pi_2 = \pi_3 \quad (\pi_1 \geq \pi'_1,\ \pi_2 \geq \pi'_2) \quad \Rightarrow \quad \pi''_1 \cdot \pi''_2 \leq \pi_3 \quad (\pi''_1 \geq \pi'_1 \text{ bounded},\ \pi''_2 \geq \pi'_2 \text{ bounded})$$



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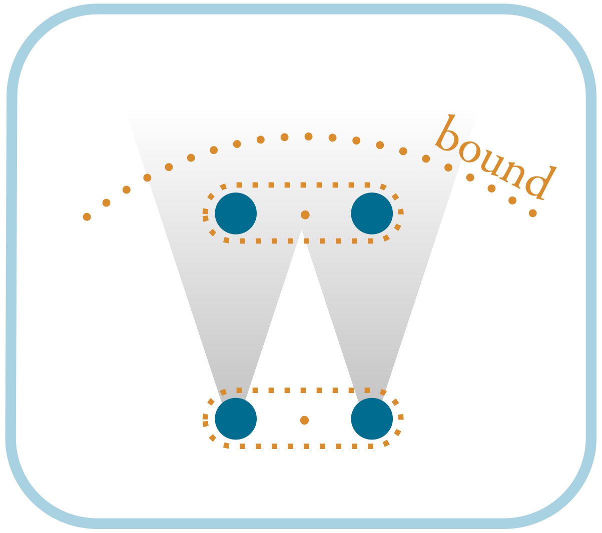
$$\pi_1 \cdot \pi_2 = \pi_3 \qquad \pi'_1 \leq \pi_1 \quad \pi'_2 \leq \pi_2$$

⇒

$$\pi''_1 \cdot \pi''_2 \leq \pi_3 \qquad \pi'_1 \leq \pi''_1 \text{ (bounded)} \quad \pi'_2 \leq \pi''_2 \text{ (bounded)}$$



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$$\pi_1 \cdot \pi_2 = \pi_3$$

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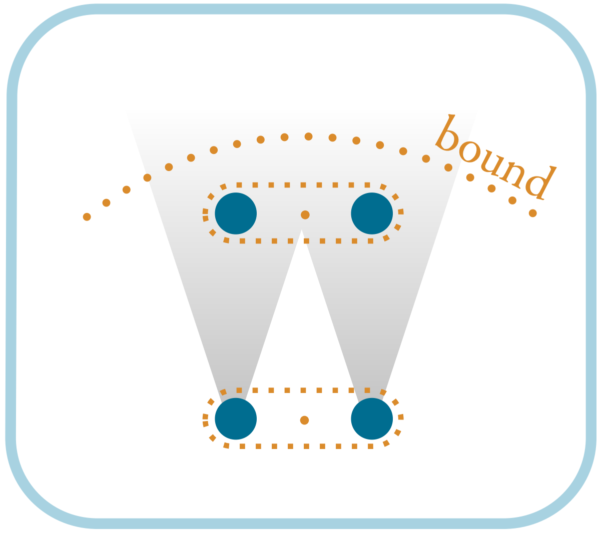
⇒

$$\pi''_1 \cdot \pi''_2 \leq \pi_3$$

$\pi'_1 \leq \pi''_1$ (bounded), $\pi'_2 \leq \pi''_2$ (bounded)



1+2+3=

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    R_{i+1} = MIN($R_i \cup \uparrow R_i \cdot \uparrow R_i$)
        = MIN($R_i \cup \uparrow_{bounded} R_i \cdot \uparrow_{bounded} R_i$);
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Complexity? In principle, **non-primitive recursive.**

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Observation: MIN(↓abs(w,i,$|w|$)) determines whether $w \models \phi$

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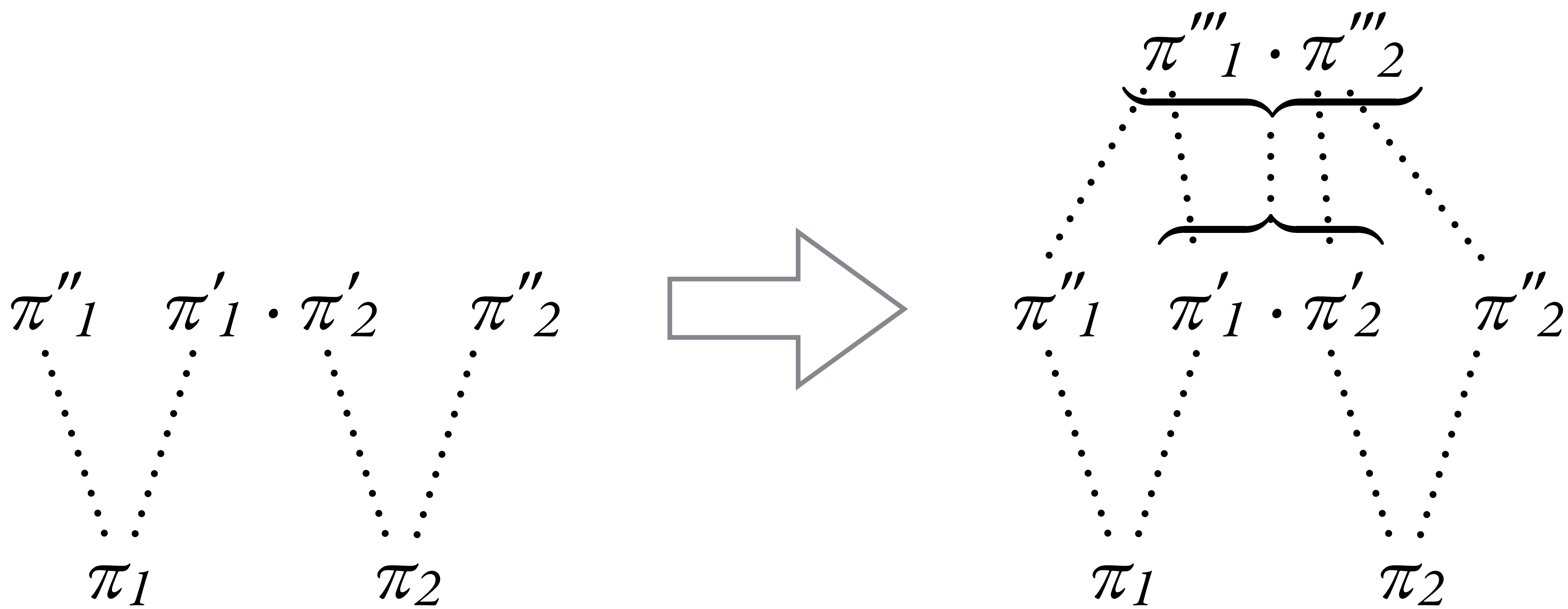

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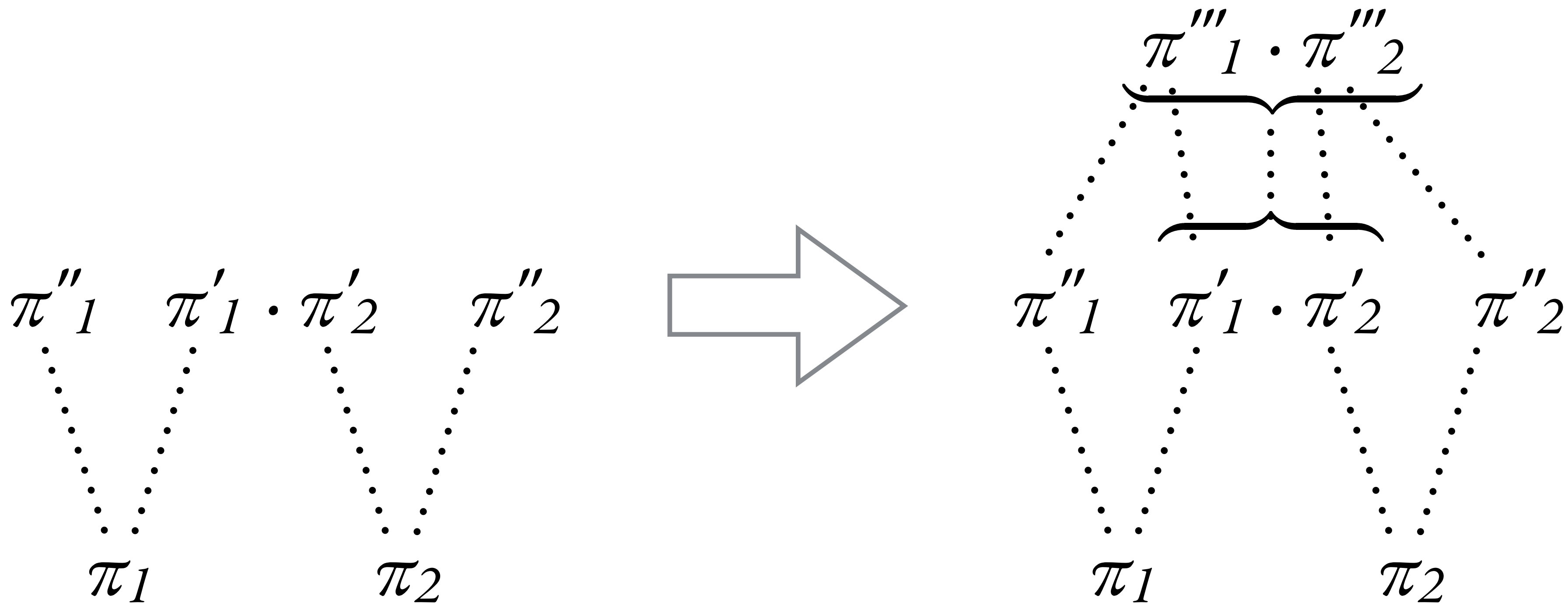

1+2+3+4 =

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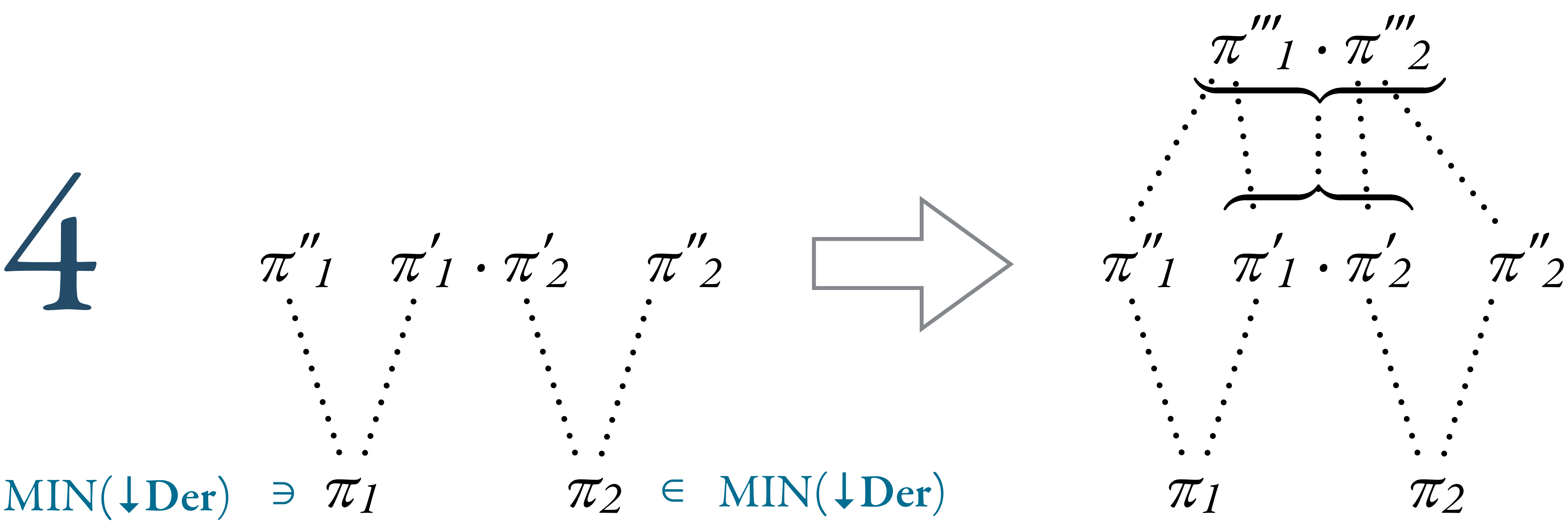

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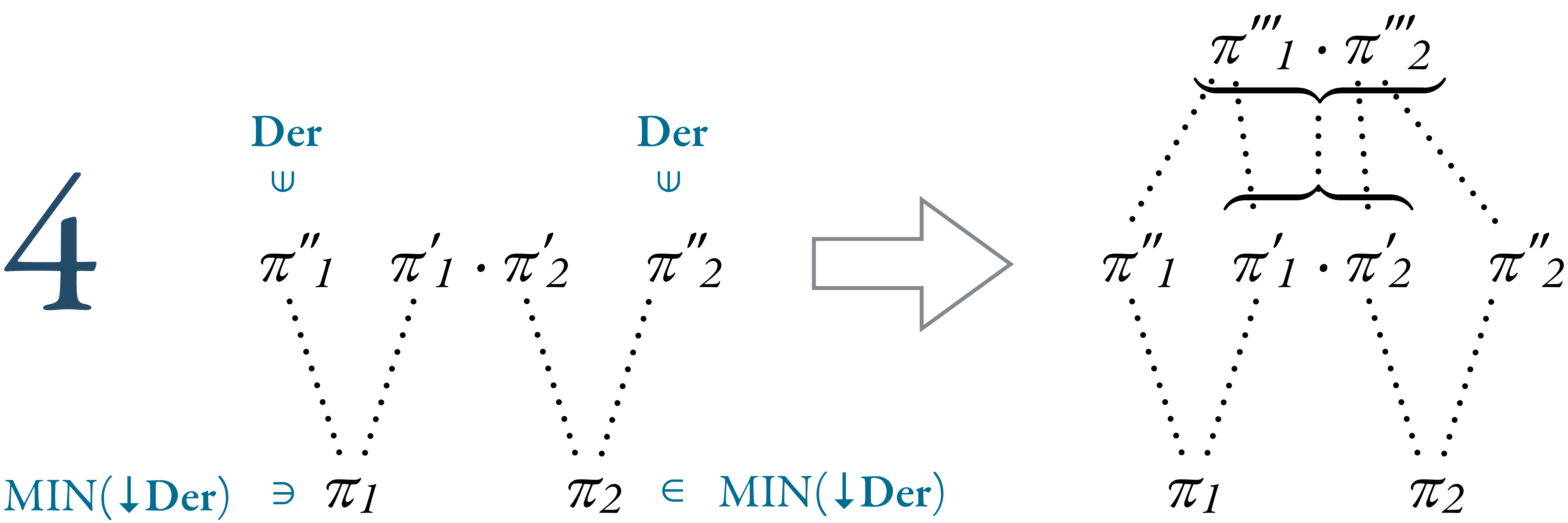

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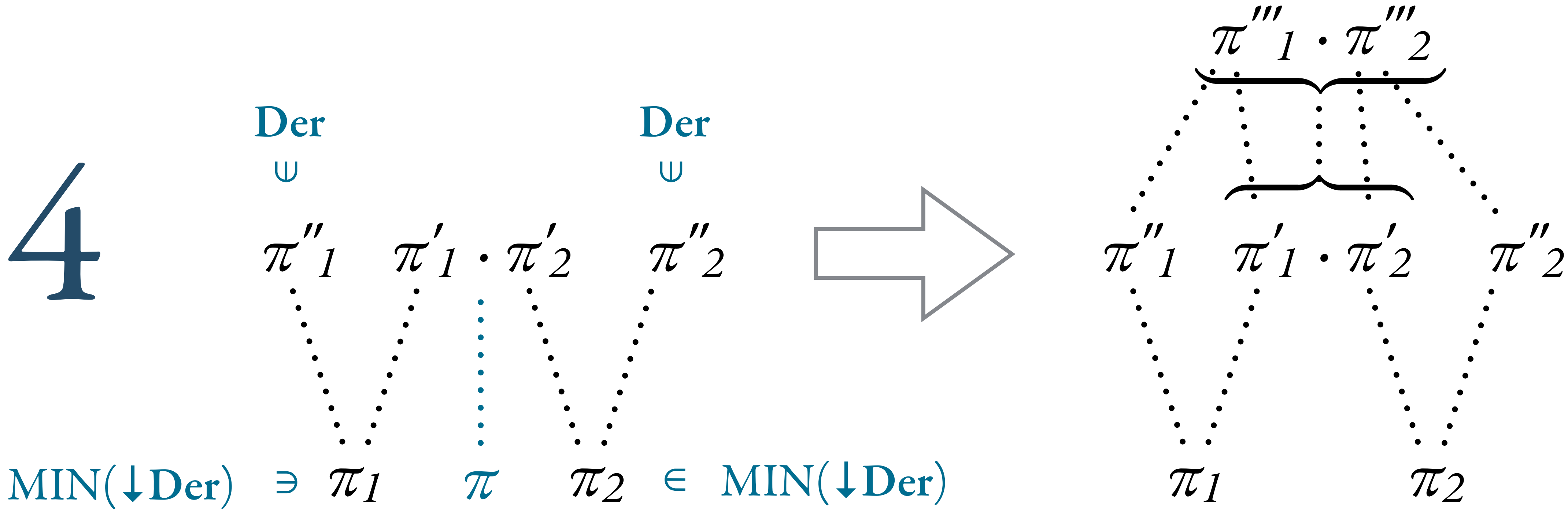

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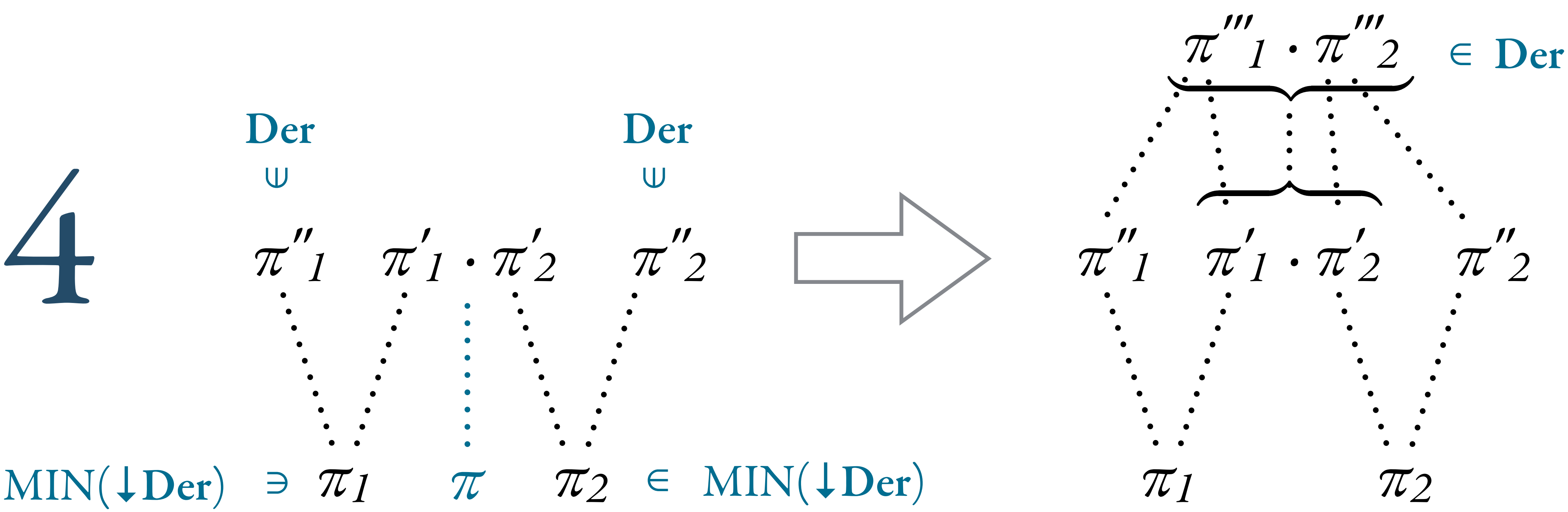
4



1+2+3+4 =

```
R0 = MIN( Atomic ) ;
while (Ri ≠ Ri+1) :
    Ri+1 = MIN↓(Ri ∪ ↑boundedRi · ↑boundedRi);
return( Ri );
```

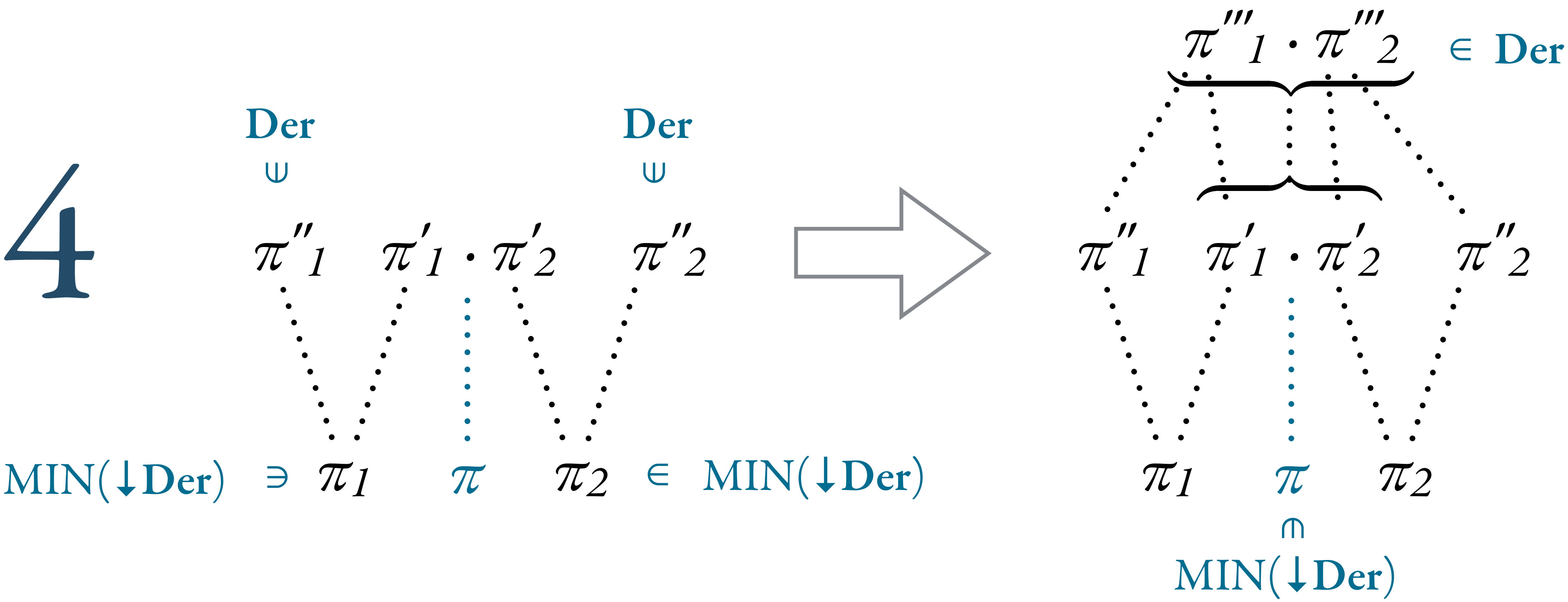
computes MIN(↓Der)

Complexity? In principle, **non-primitive recursive.**

Observation: $\mathrm{MIN}(\downarrow \mathrm{abs}(w, i, |w|))$ determines whether $w \vDash \phi$

$\Rightarrow$ no need to compute MIN(**Der**), it suffices to compute MIN(↓**Der**).

4



1+2+3+4 =

R_0 = MIN(**Atomic**) ;
while ($R_i \neq R_{i+1}$) :
  R_{i+1} = MIN↓($R_i \cup \uparrow_{\mathrm{bounded}} R_i \cdot \uparrow_{\mathrm{bounded}} R_i$);
return(R_i);

computes MIN(↓Der)

Complexity?

2^{ϕ} many MIN(**Profiles**) $\Rightarrow$ 2ExpSpace procedure

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Caveat

ϕ

there is only one dv under a **c**

for every **a**, there is a **b** accessible via a **c** with the same dv

there is a position labeled **c**

w

a	b	b	a	b	c	a	b	c	c	a	a	b	c	b
1	2	4	4	3	1	5	1	1	1	4	4	5	1	4

$\models \phi$

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a	b	b	a	b	c	a	b	c	c	c	c	a	a	b	c	b	
1	2	4	4	3	1	5	1	1	9	1	9	4	4	5	1	4	$\not\models \phi$

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for every **a**, there is a **b** accessible via a **c** with the same dv
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w

a	b	b	a	b	c	a	b	[c]	c	c	c	a	a	b	c	b
1	2	4	4	3	1	5	1	[1]	9	1	9	4	4	5	1	4

$\not\models \phi$

Atomic$_{\phi}$

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a	b	b	a	b	c	a	b	[c]	c	c	c	a	a	b	c	b	$\not\models \phi$
1	2	4	4	3	1	5	1	[1]	9	1	9	4	4	5	1	4	

$\not\pitchfork$

Atomic$_{\phi}$

But: There are only polynomially many 'conflicting' data values.
We can treat them as 'constants'.

From words to trees

Satisfiability for XPath($^*\!\!\leftarrow$, $\downarrow_*$,$\rightarrow^*$) is decidable in 2ExpSpace.♦

♦ [F., 2013]

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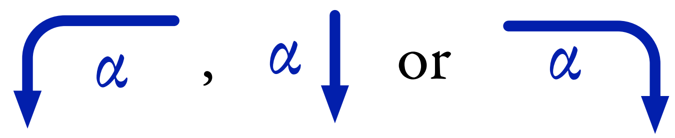
Satisfiability for XPath($^*\!\leftarrow$, $\downarrow_*$, $\rightarrow^*$) is decidable in 2ExpSpace.♦

Composed paths



eg: $\longrightarrow^*[a]\longrightarrow^*[b]\downarrow_*[c]$

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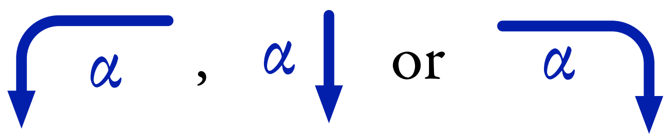

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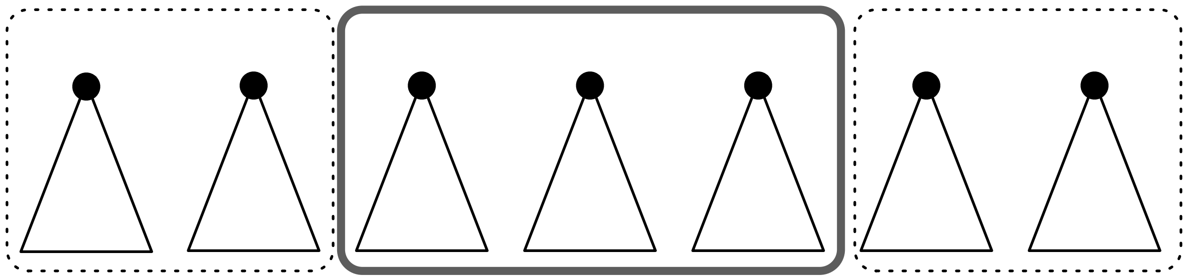
Forest profiles



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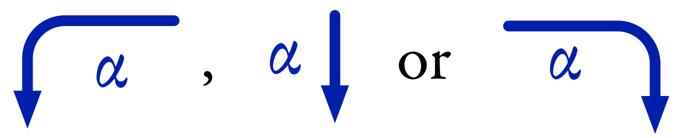
Composed paths 

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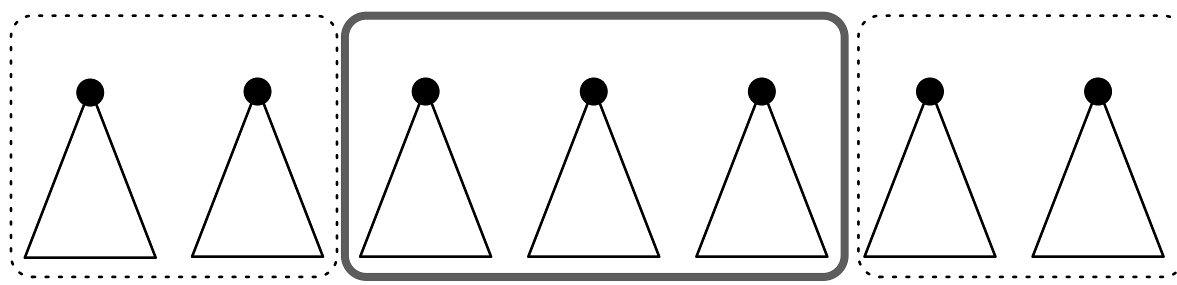
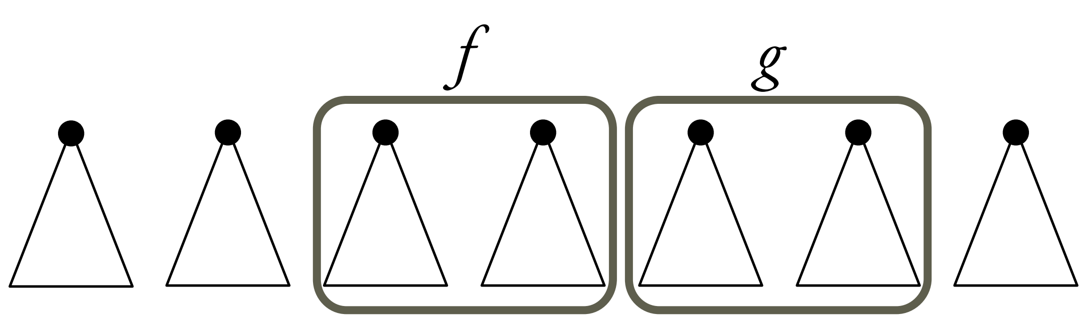


Two-operator algebra



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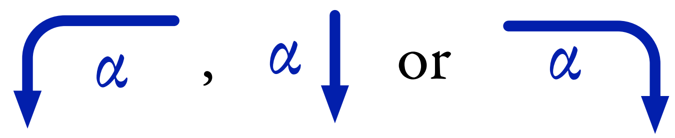

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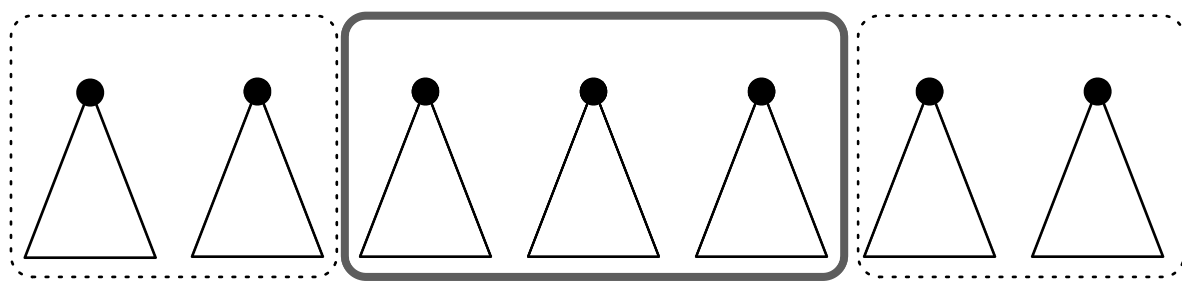

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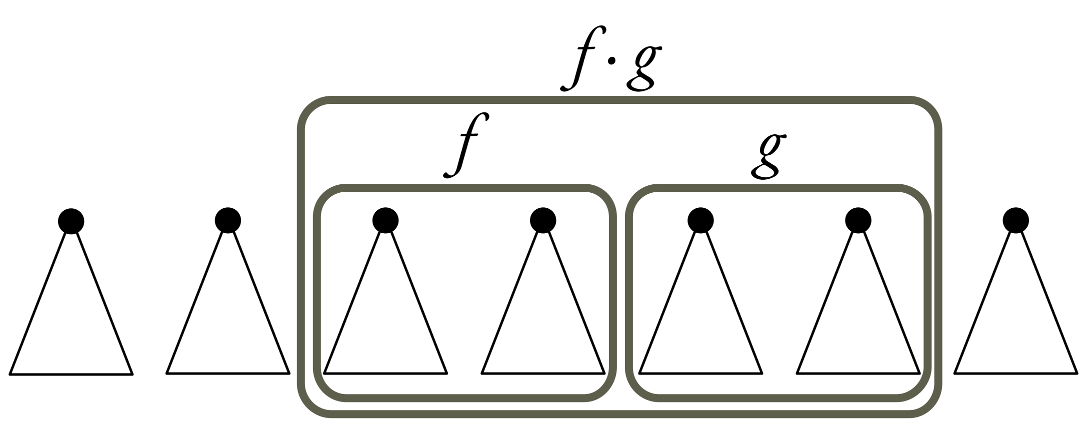

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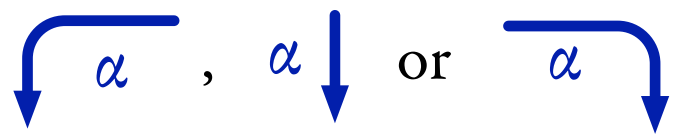

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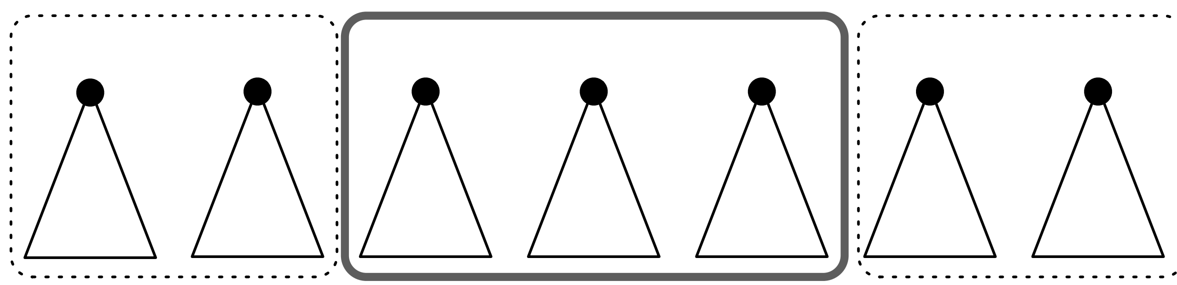
Forest profiles



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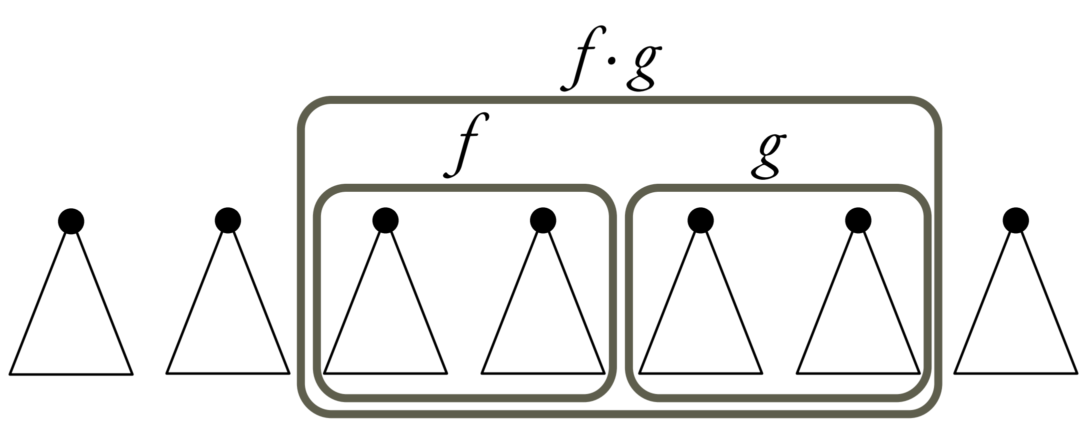

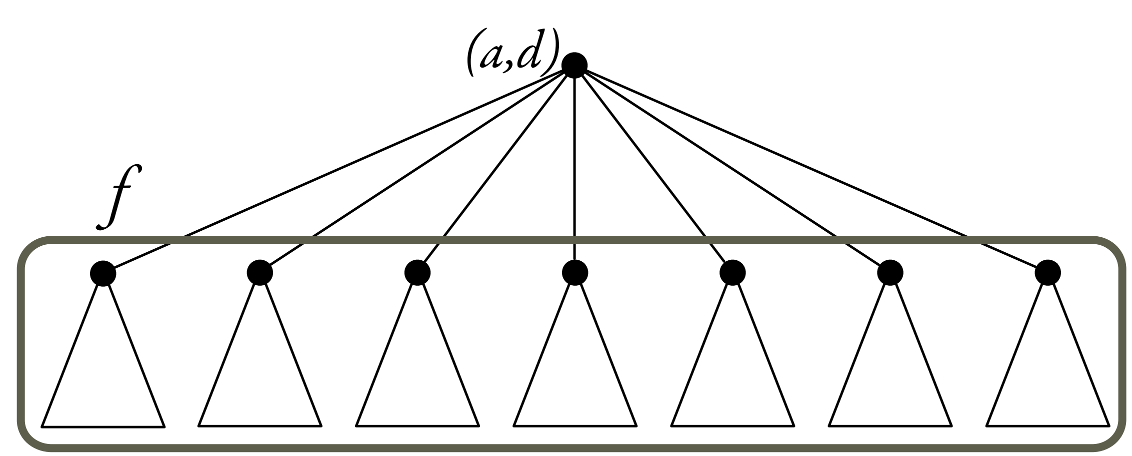

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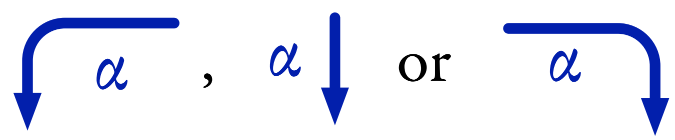
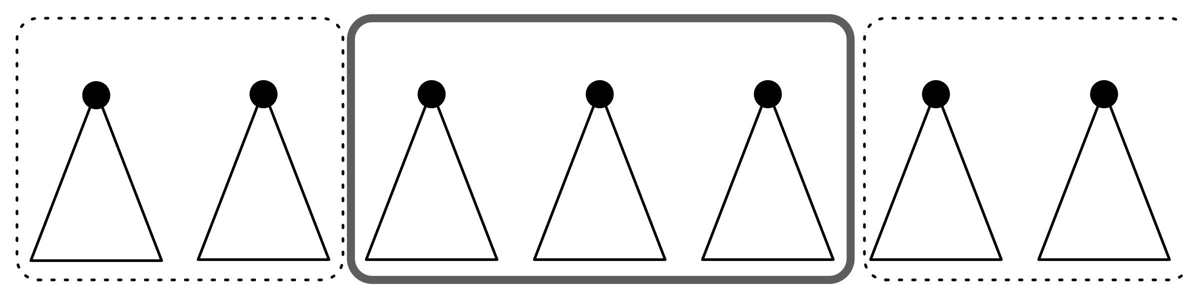
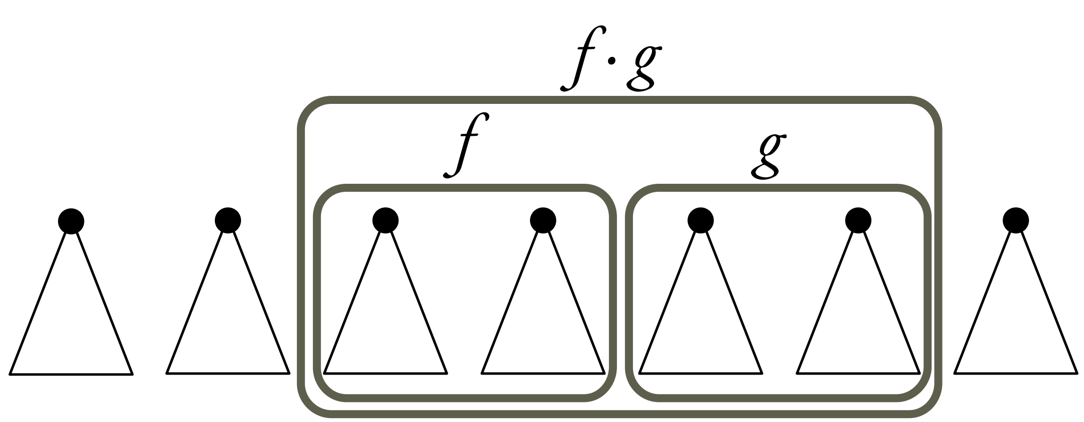


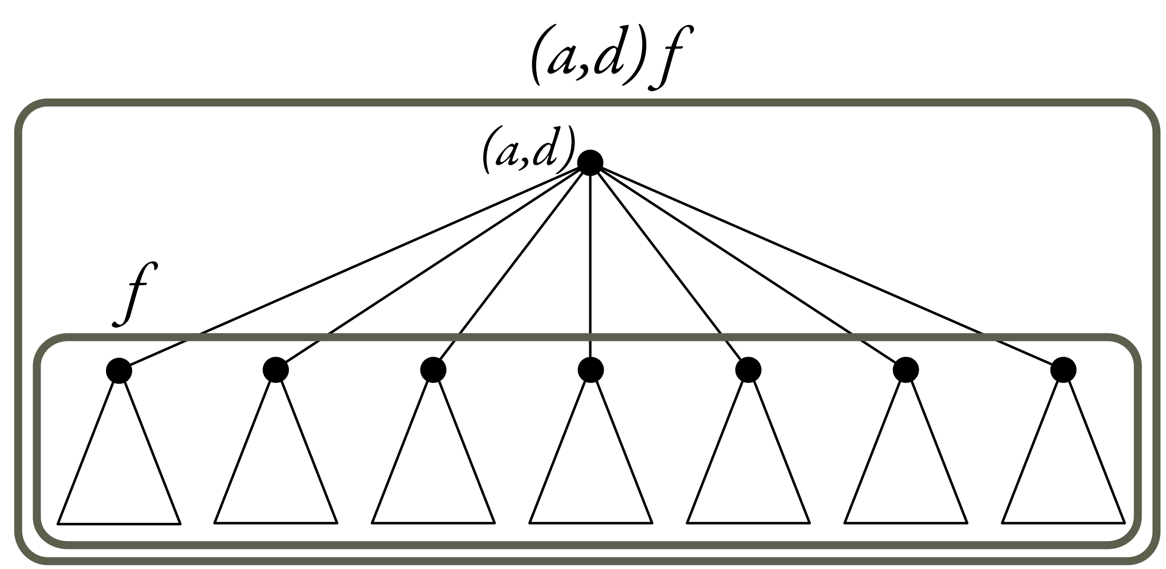
eg: $\longrightarrow^*[a]\longrightarrow^*[b]\downarrow_*[c]$

Forest profiles



Two-operator algebra





♦ [F., 2013]

Final remarks

Changing $\longrightarrow^+$ by $\longrightarrow^+$: undecidable

Adding $\uparrow^*$: non-PR

Adding domain-dependant relations?

Adding $\downarrow$: still decidable?

Complexity: 3ExpSpace (2ExpSpace in normal form)

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Changing $\longrightarrow^+$ by $\longrightarrow^+$: undecidable

Adding $\uparrow^*$: non-PR

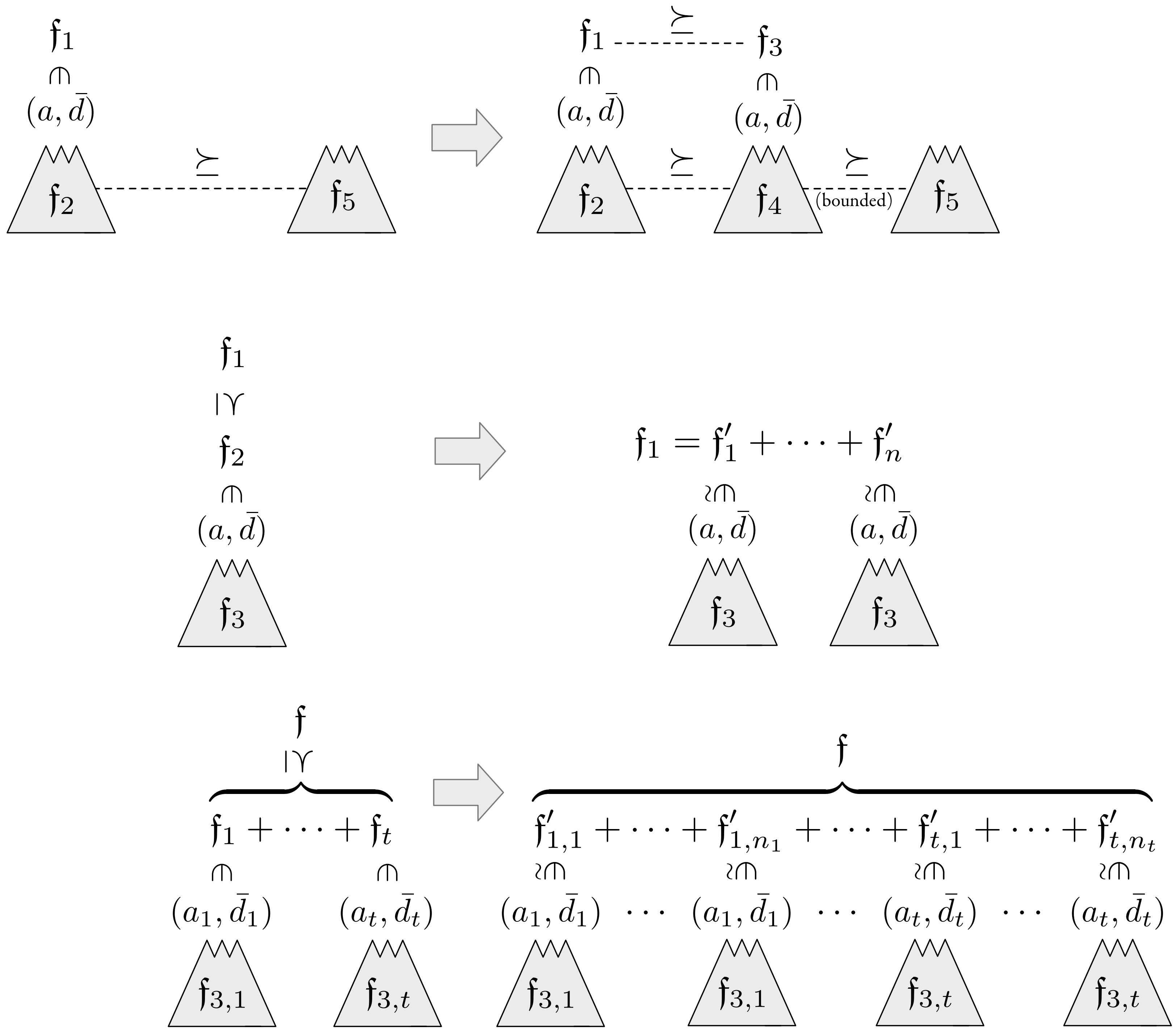
Adding domain-dependant relations?

Adding $\downarrow$: still decidable?

Complexity: 3ExpSpace (2ExpSpace in normal form)

thank you!

Etc.



$\mathfrak{f}_1$
$(a, \bar{d})$
$\mathfrak{f}_2$
$\mathfrak{f}_5$
$\mathfrak{f}_3$
$\mathfrak{f}_4$
(bounded)
$\mathfrak{f}_1 = \mathfrak{f}'_1 + \cdots + \mathfrak{f}'_n$
$\mathfrak{f}$
$\mathfrak{f}_1 + \cdots + \mathfrak{f}_t$
$(a_1, \bar{d}_1)$
$(a_t, \bar{d}_t)$
$\mathfrak{f}_{3,1}$
$\mathfrak{f}_{3,t}$
$\mathfrak{f}'_{1,1} + \cdots + \mathfrak{f}'_{1,n_1} + \cdots + \mathfrak{f}'_{t,1} + \cdots + \mathfrak{f}'_{t,n_t}$