INFORMEL
Indo-French Formal Methods Lab

UPPSALA UNIVERSITET

COMMUNICATING RECURSIVE PROGRAMS: CONTROL AND SPLIT-WIDTH

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Joint work with

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LSV, ENS Cachan, France

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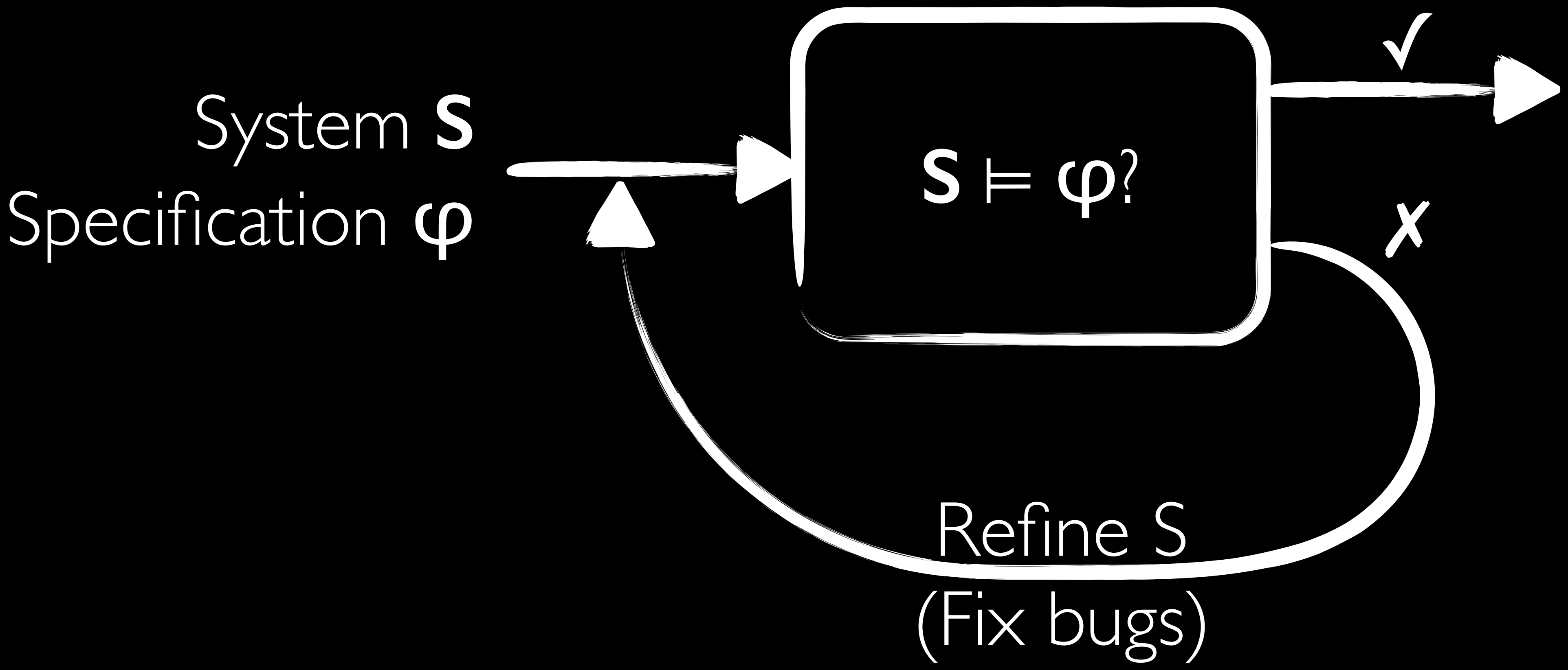
Chennai Mathematical Institute, India

ACTS
09/02/2015, Chennai

VERIFICATION

Model Checking



System **S**

Specification **φ**

S ⊨ φ?

✓

✗

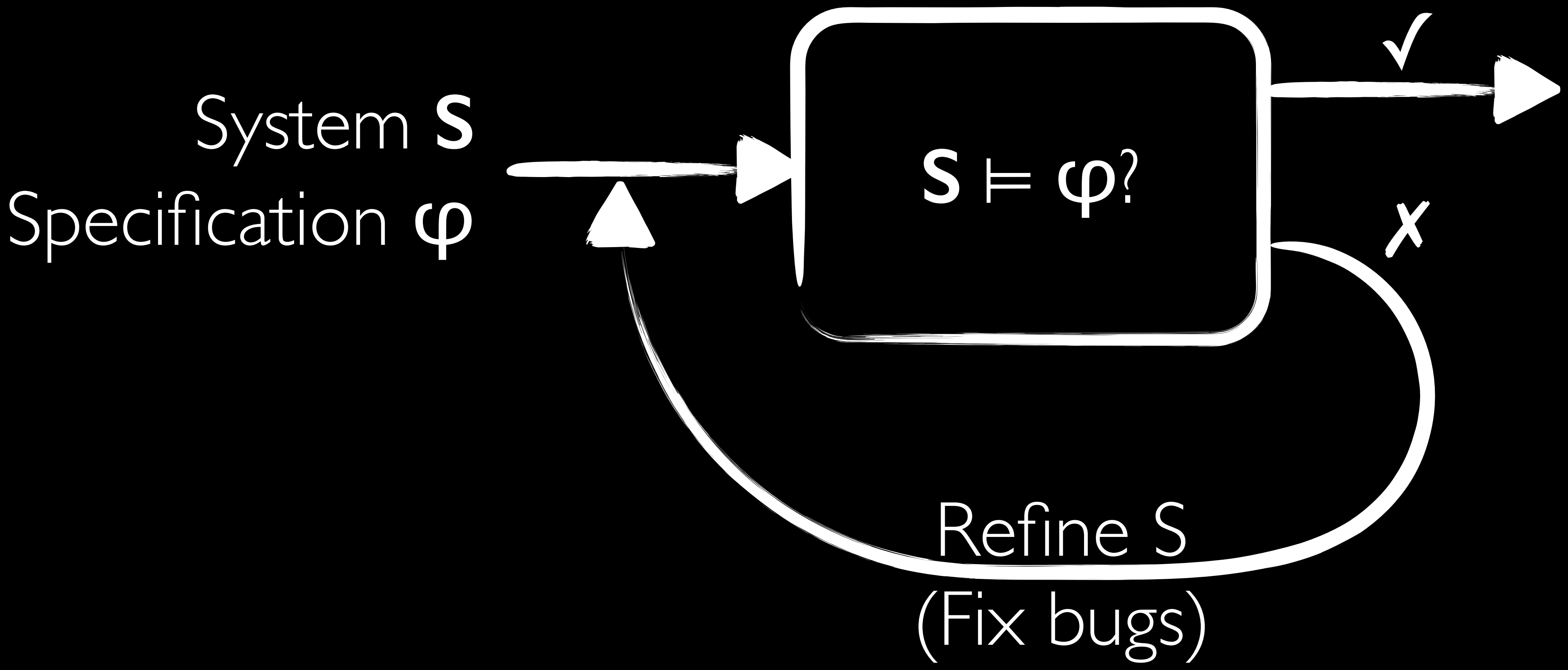
Refine S

(Fix bugs)

VERIFICATION

Model Checking

- **Undecidable in many cases**



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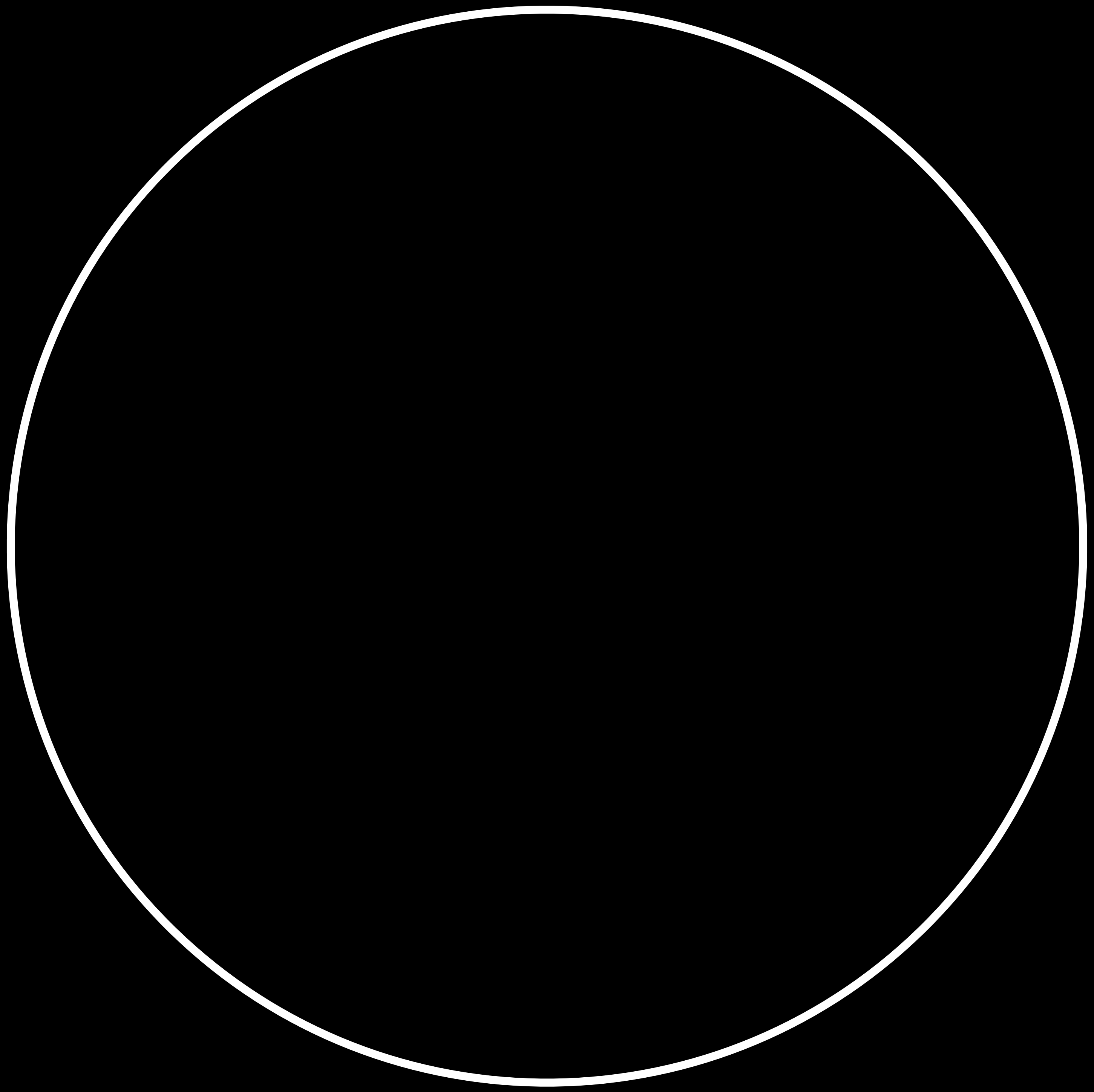
UNDER-APPROXIMATE VERIFICATION



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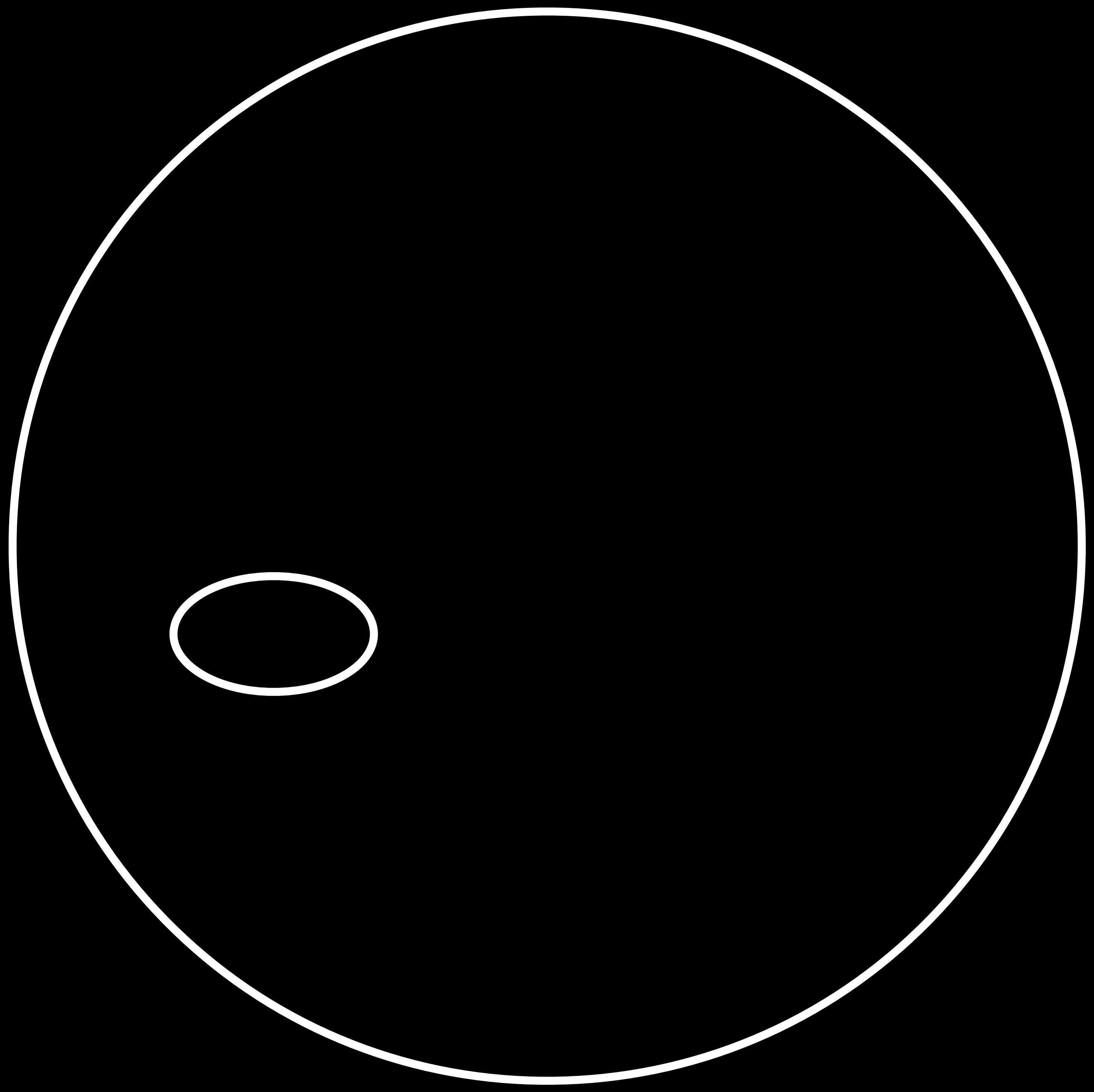
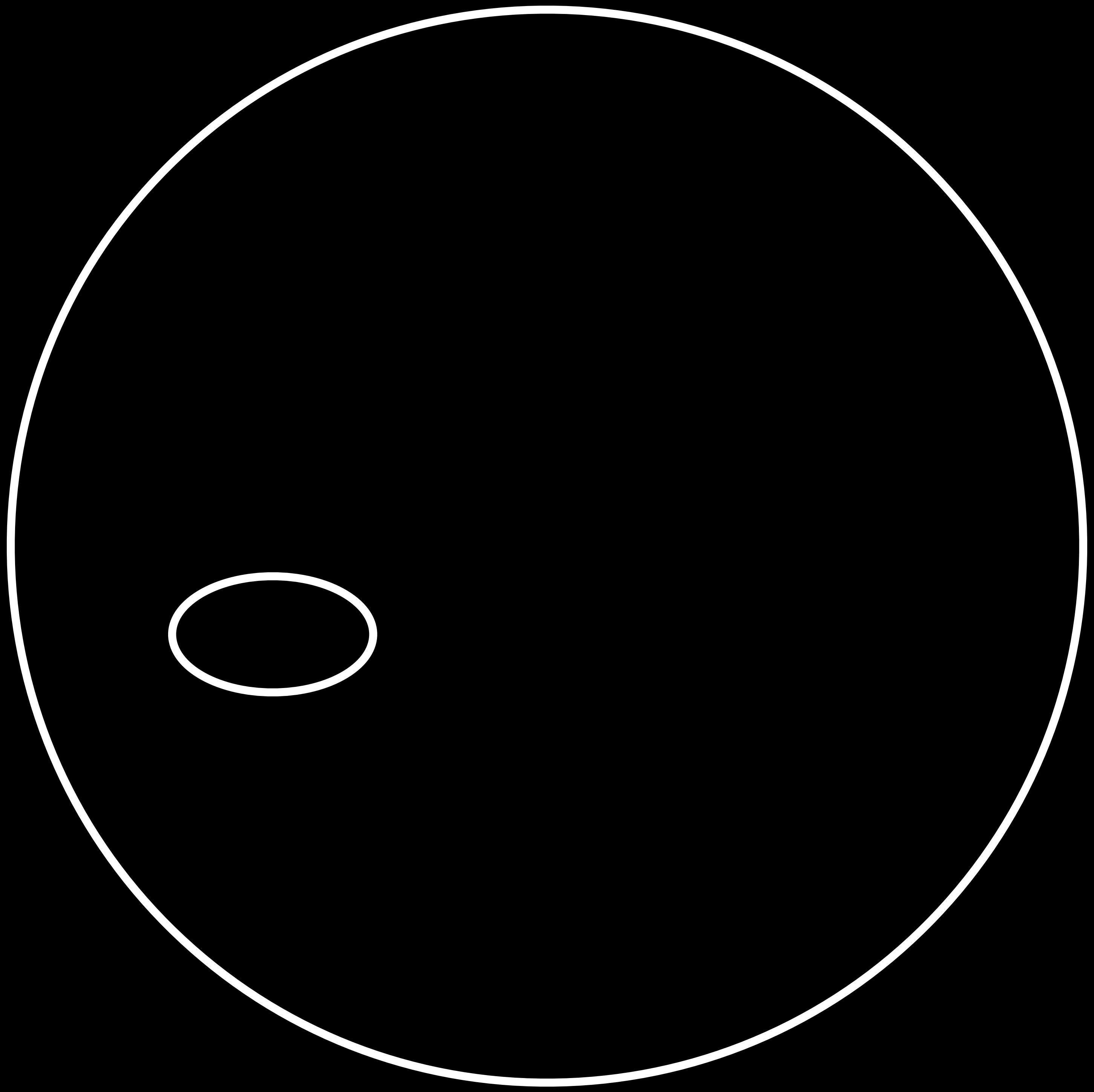


UNDER-APPROXIMATE VERIFICATION

- Parametrised



UNDER-APPROXIMATE VERIFICATION

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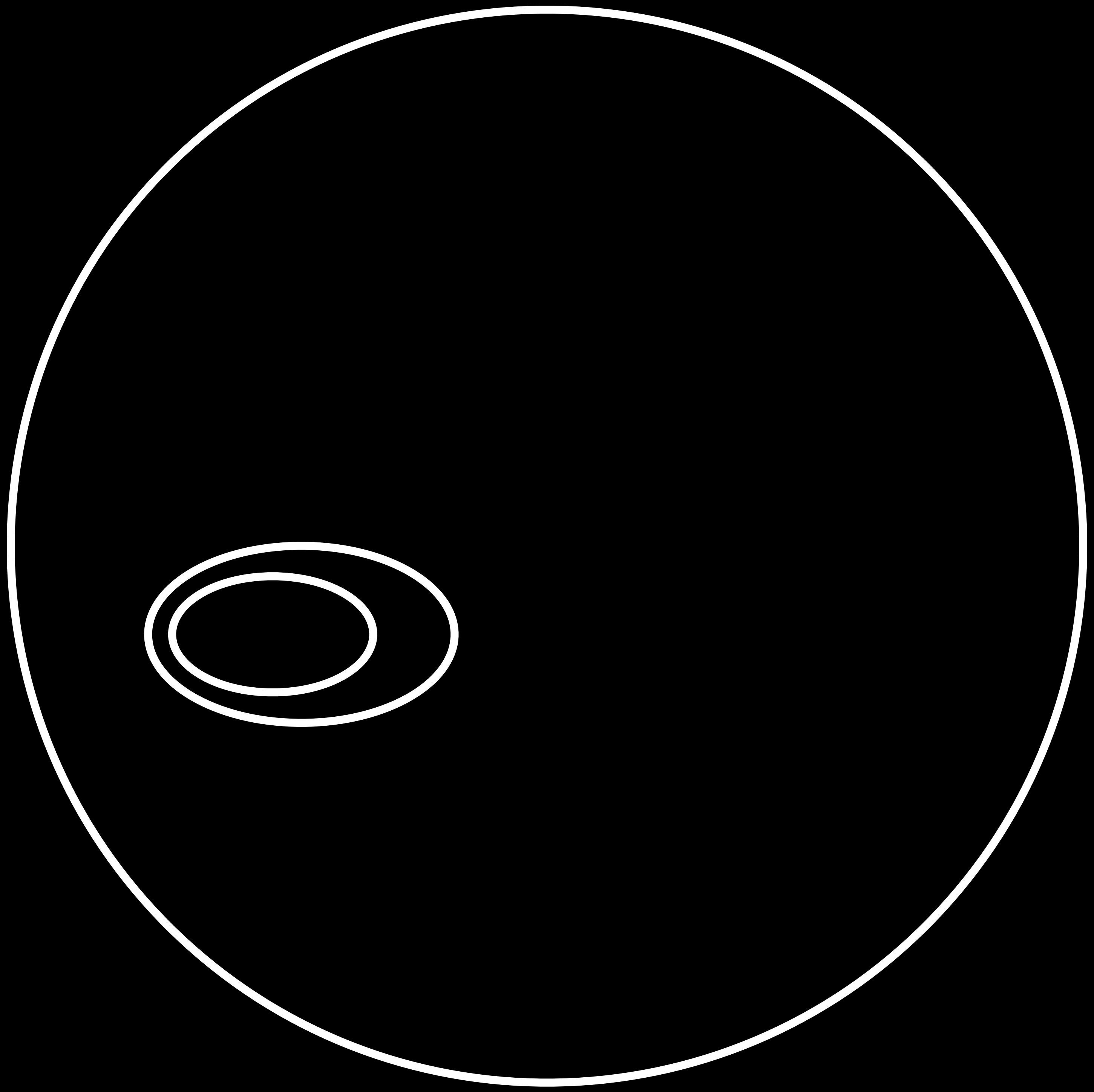
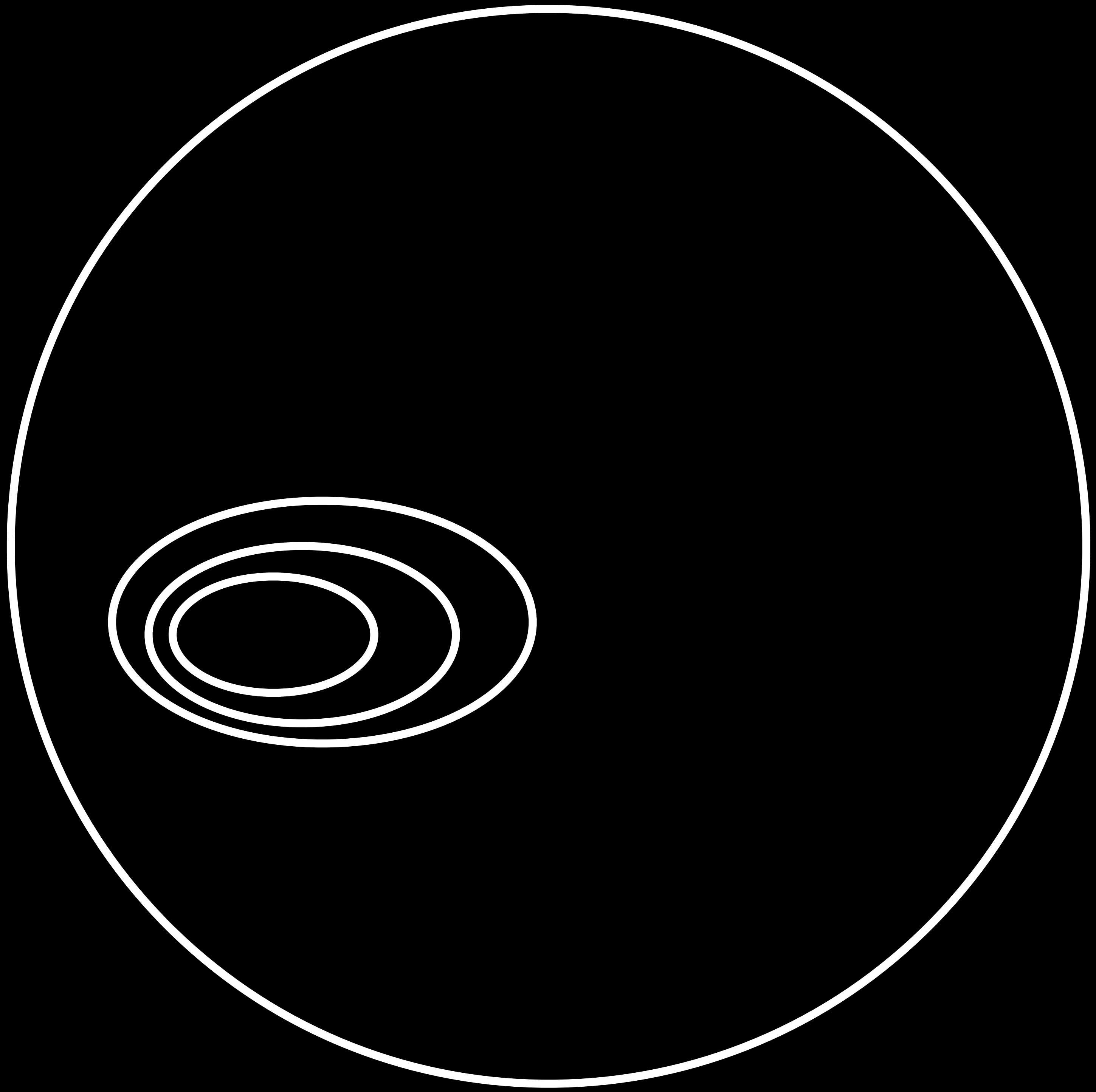


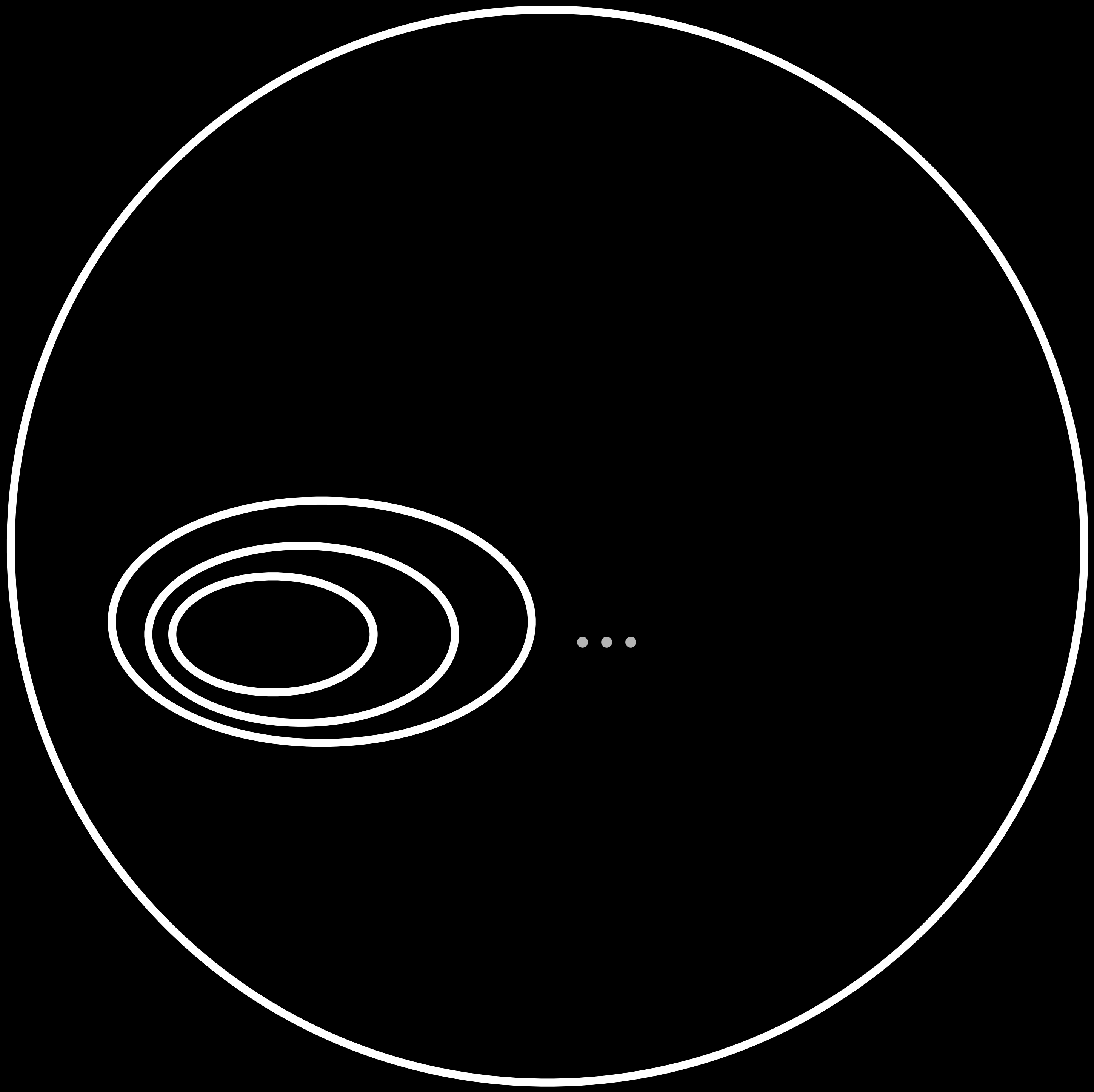
UNDER-APPROXIMATE VERIFICATION

- Parametrised



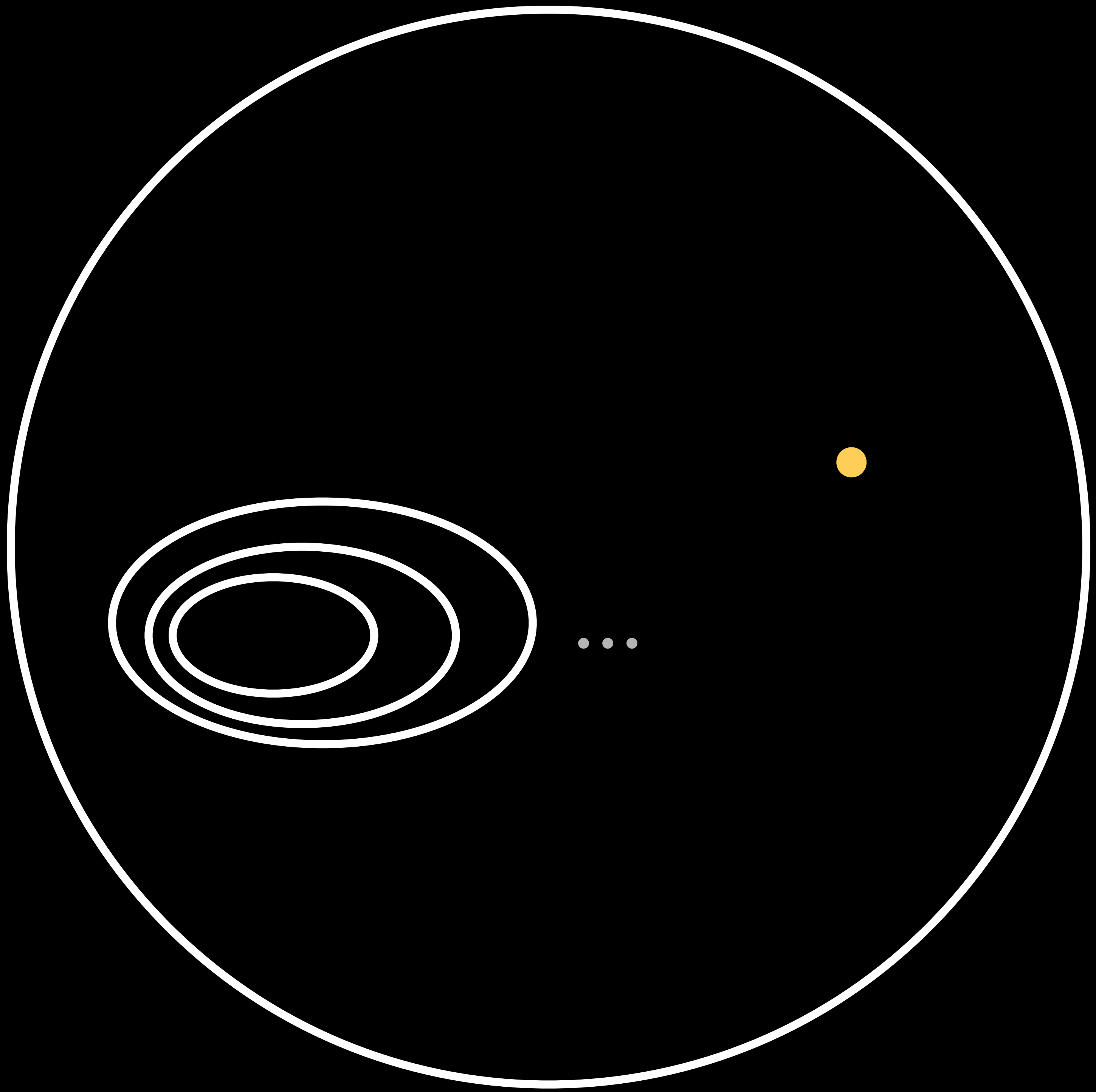
UNDER-APPROXIMATE VERIFICATION

- Parametrised



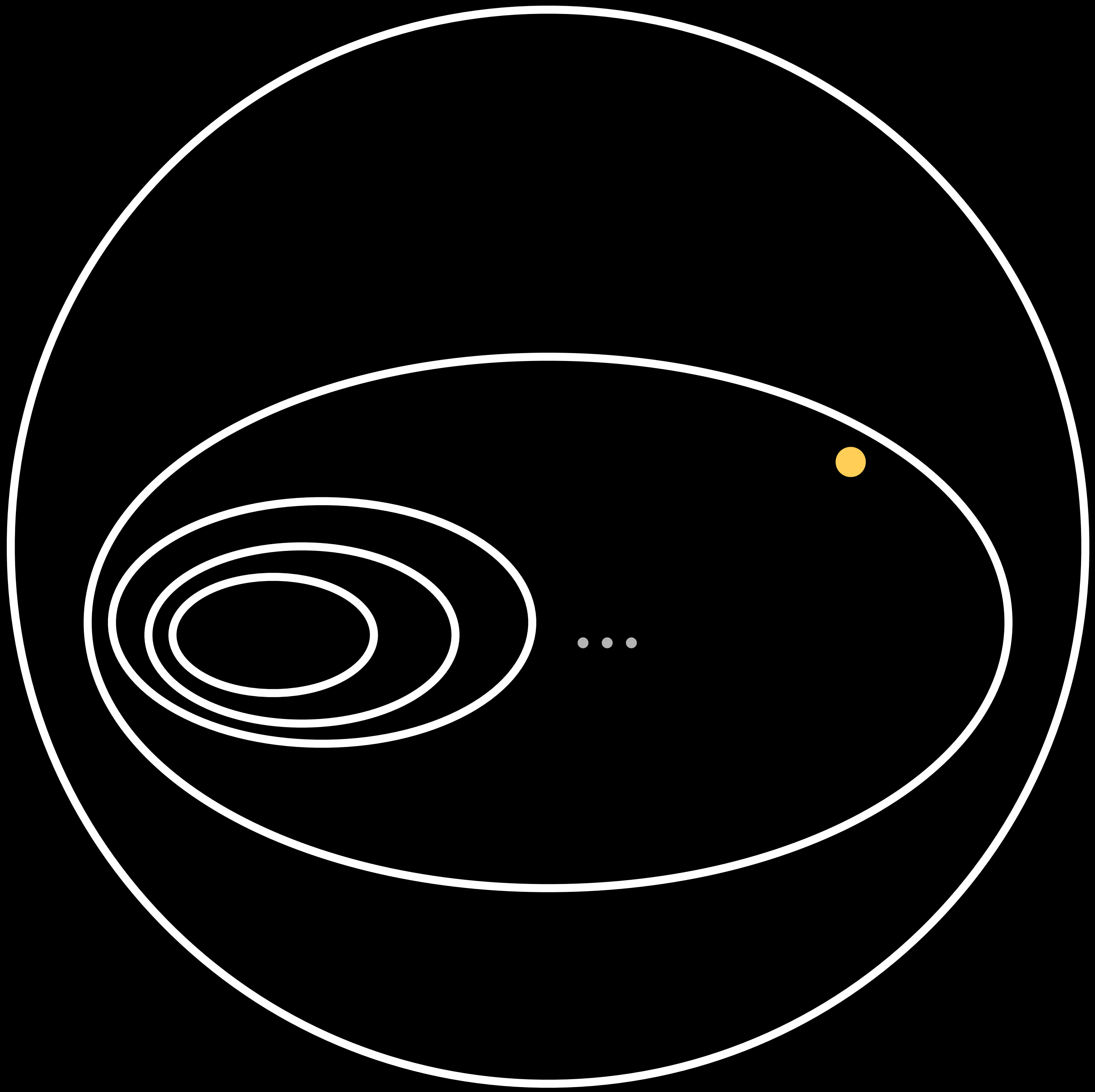
UNDER-APPROXIMATE VERIFICATION

- Parametrised
- Exhaustive



UNDER-APPROXIMATE VERIFICATION

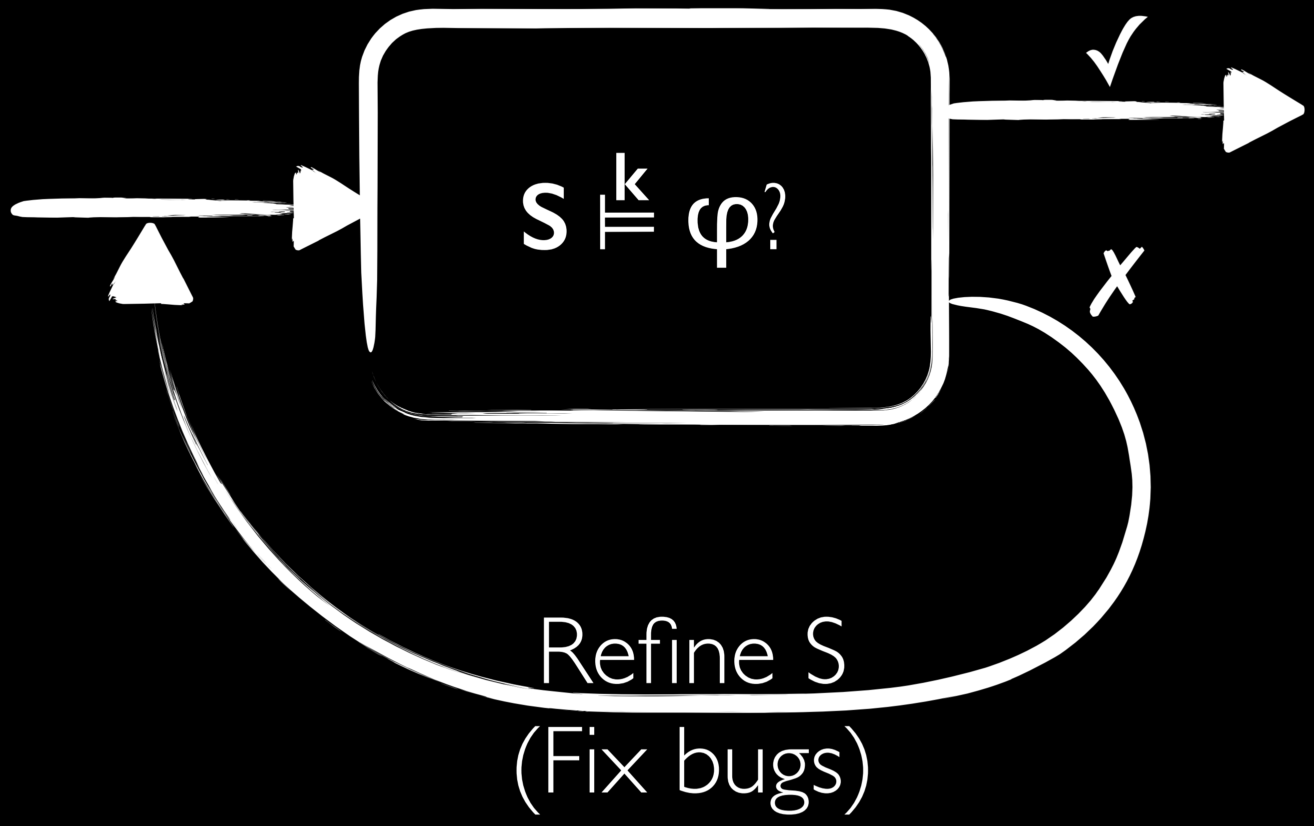
- Parametrised
- Exhaustive



UNDER-APPROXIMATE VERIFICATION

Model Checking

> **Decidable**



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Specification **φ**

$\mathbf{S} \overset{\mathbf{k}}{\models} \varphi$?

✓

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(Fix bugs)

UNDER-APPROXIMATE VERIFICATION

Model Checking

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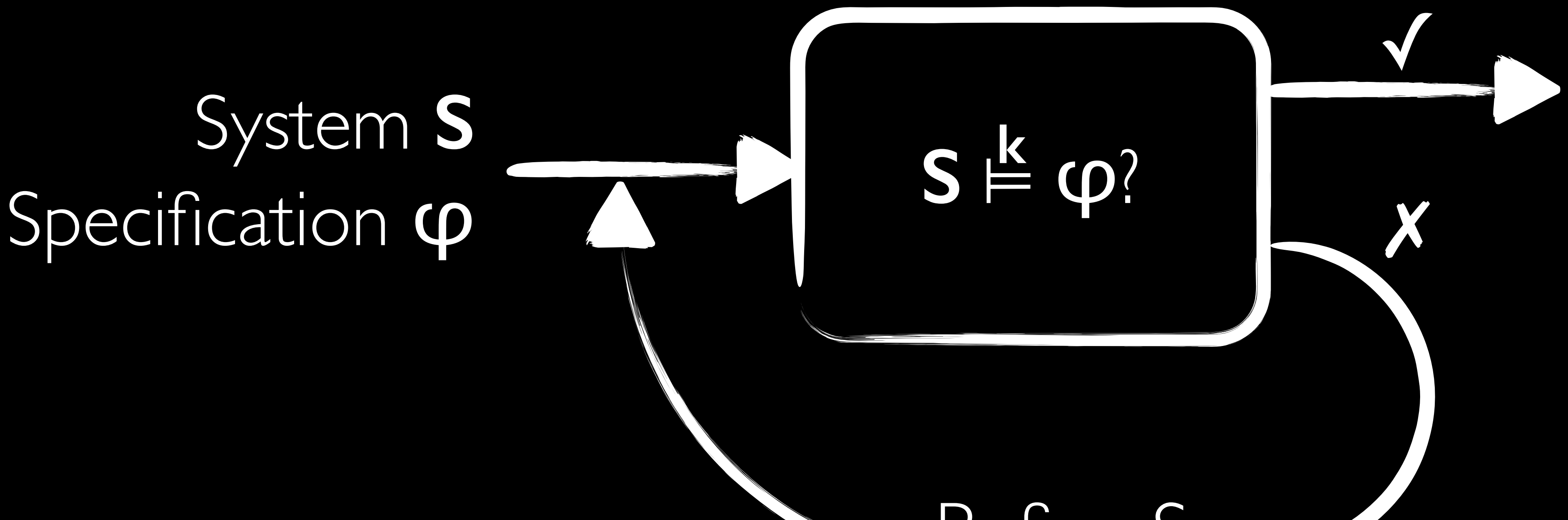

UNDER-APPROXIMATE VERIFICATION

Model Checking

› **Decidable**

System **S**

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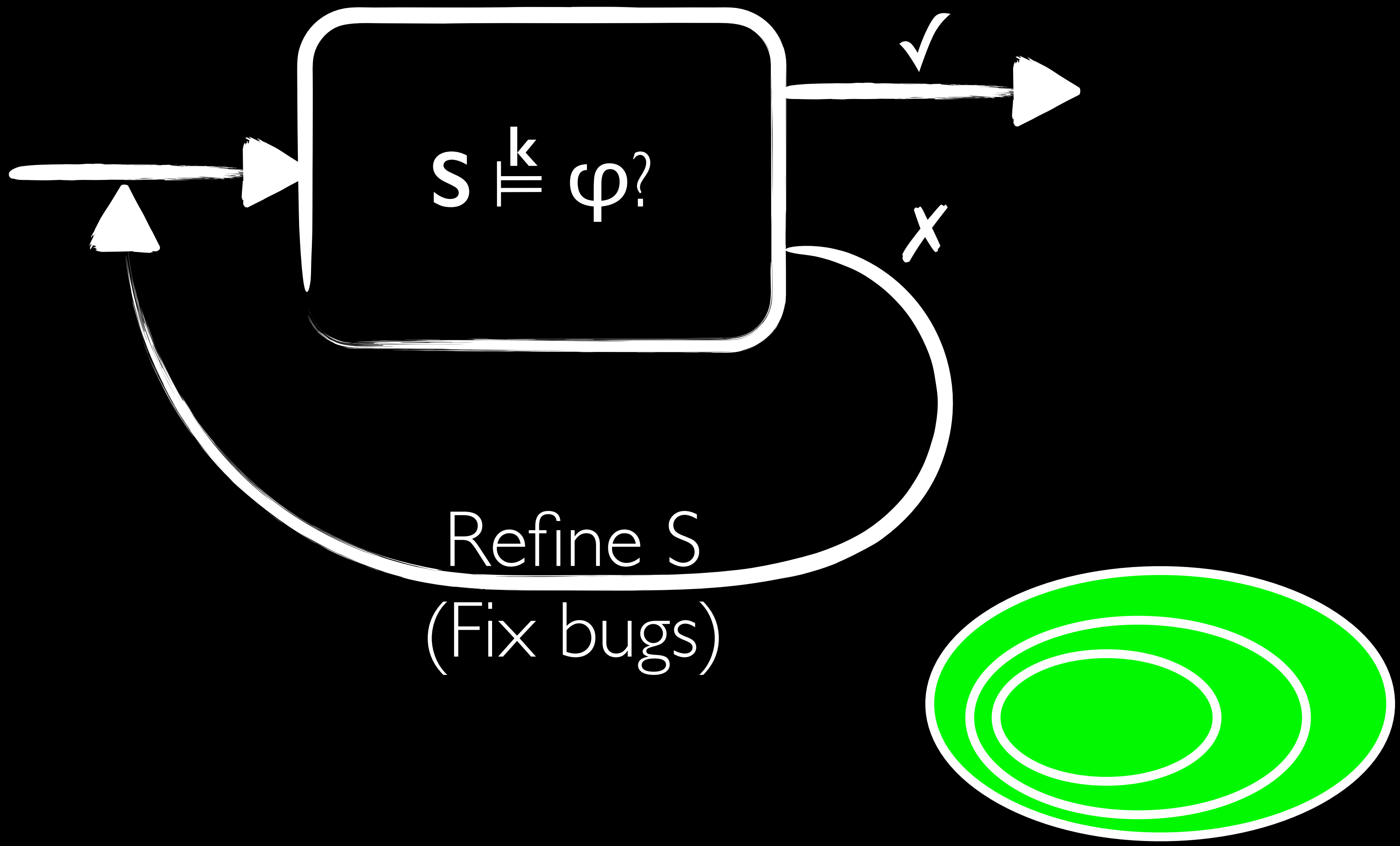

UNDER-APPROXIMATE VERIFICATION

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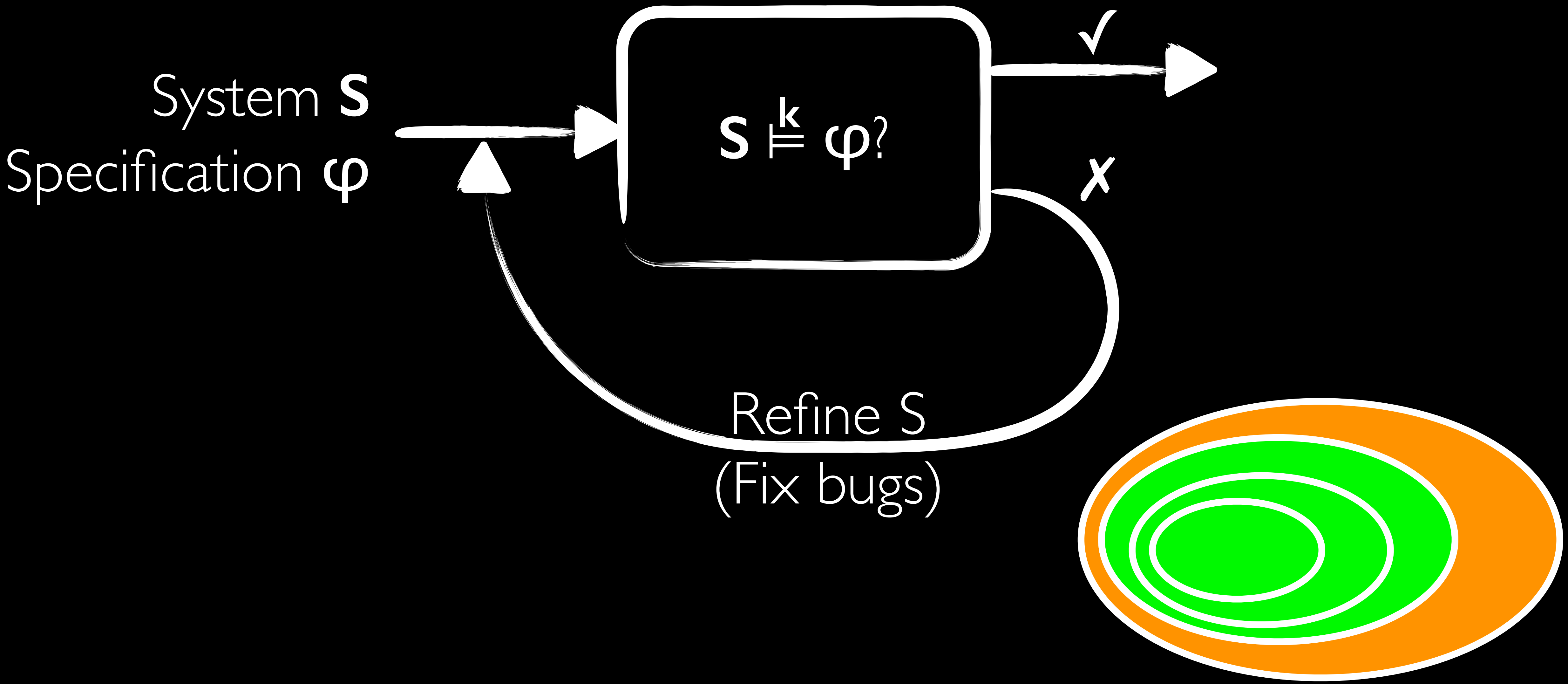

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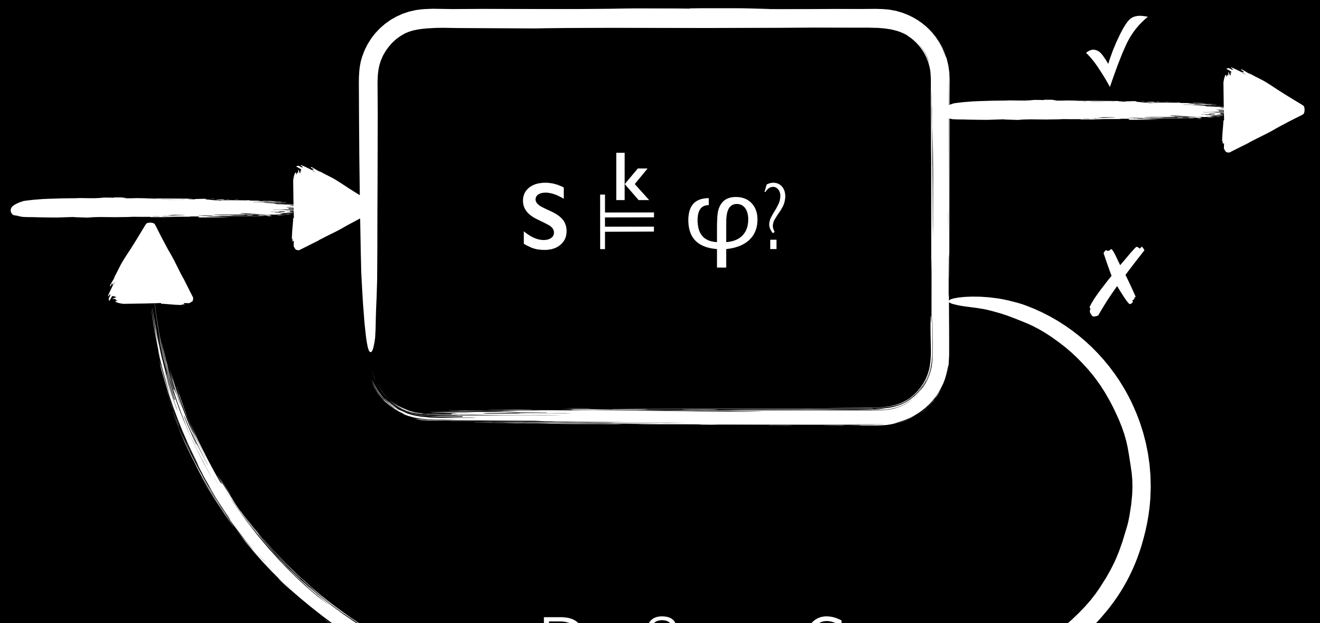

Refine S

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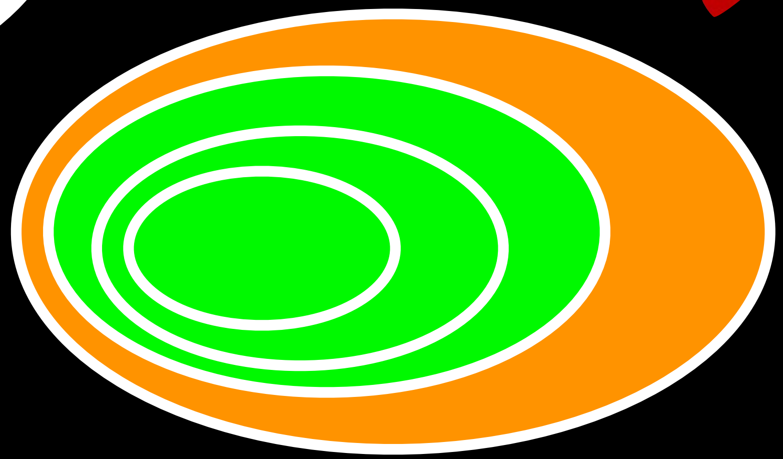
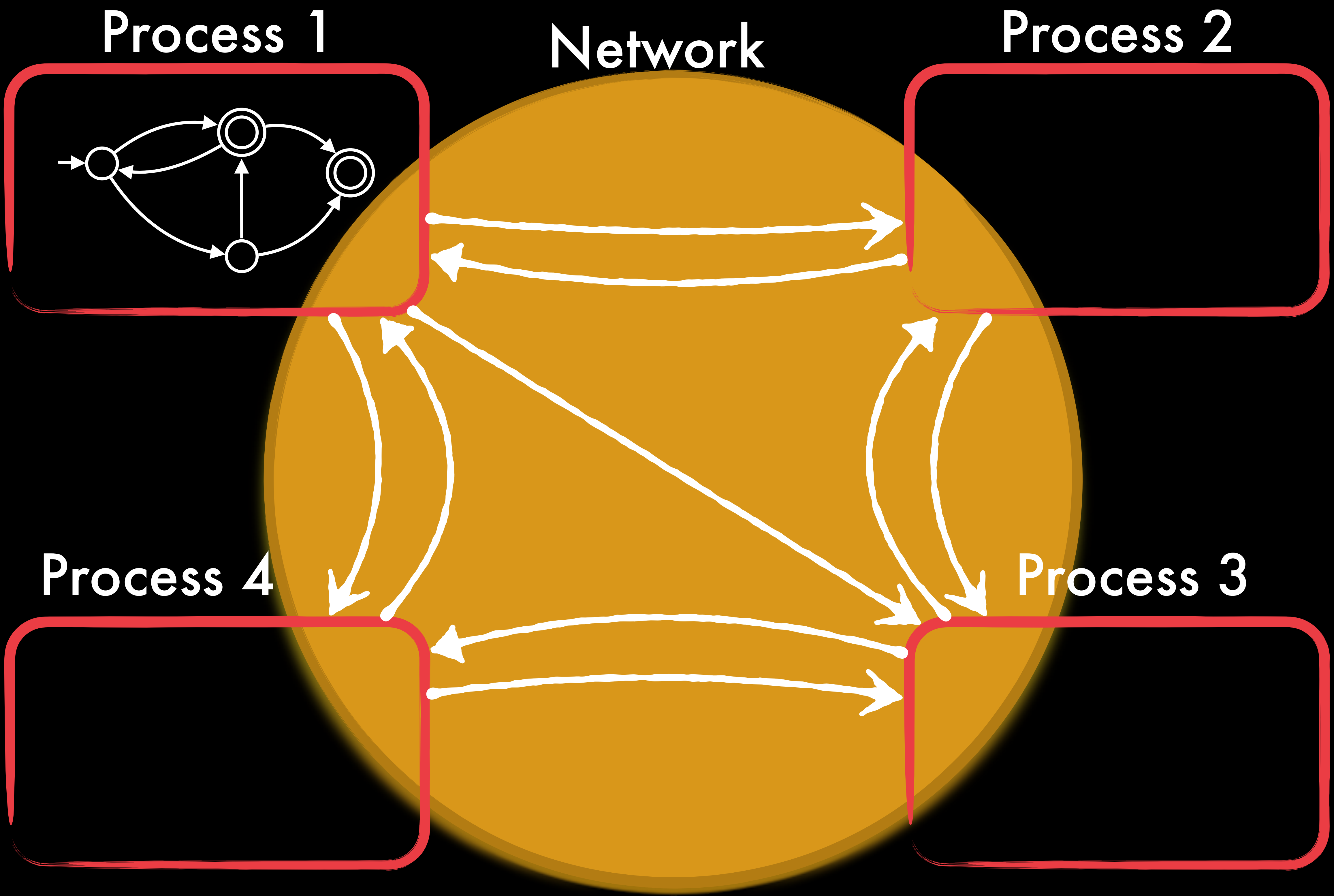
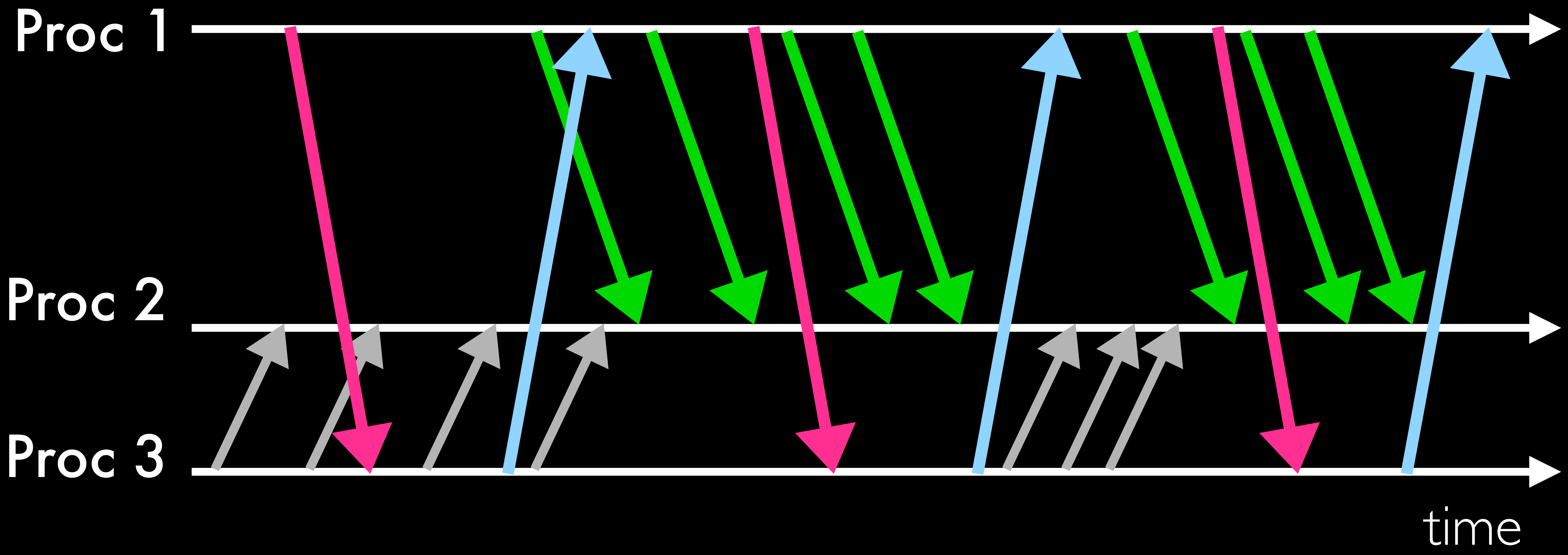


COMMUNICATING RECURSIVE PROGRAMS: CONTROL AND SPLIT-WIDTH

COMMUNICATING DISTRIBUTED SYSTEMS



BEHAVIOURS : MESSAGE SEQUENCE CHARTS



VERIFICATION PROBLEMS

- Emptiness or Reachability
- Inclusion or Universality
- Satisfiability ϕ
- Model Checking: $S \models \phi$
  - Temporal logics
  - Propositional dynamic logics
  - Monadic second order logic

COMMUNICATING RECURSIVE PROGRAMS:

- Turing powerful: verification undecidable
- Under-upproximations
    - Decidable
    - Controllable

COMMUNICATING RECURSIVE PROGRAMS: CONTROL AND SPLIT-WIDTH

- Turing powerful: verification undecidable
- Under-upproximations
    - Decidable
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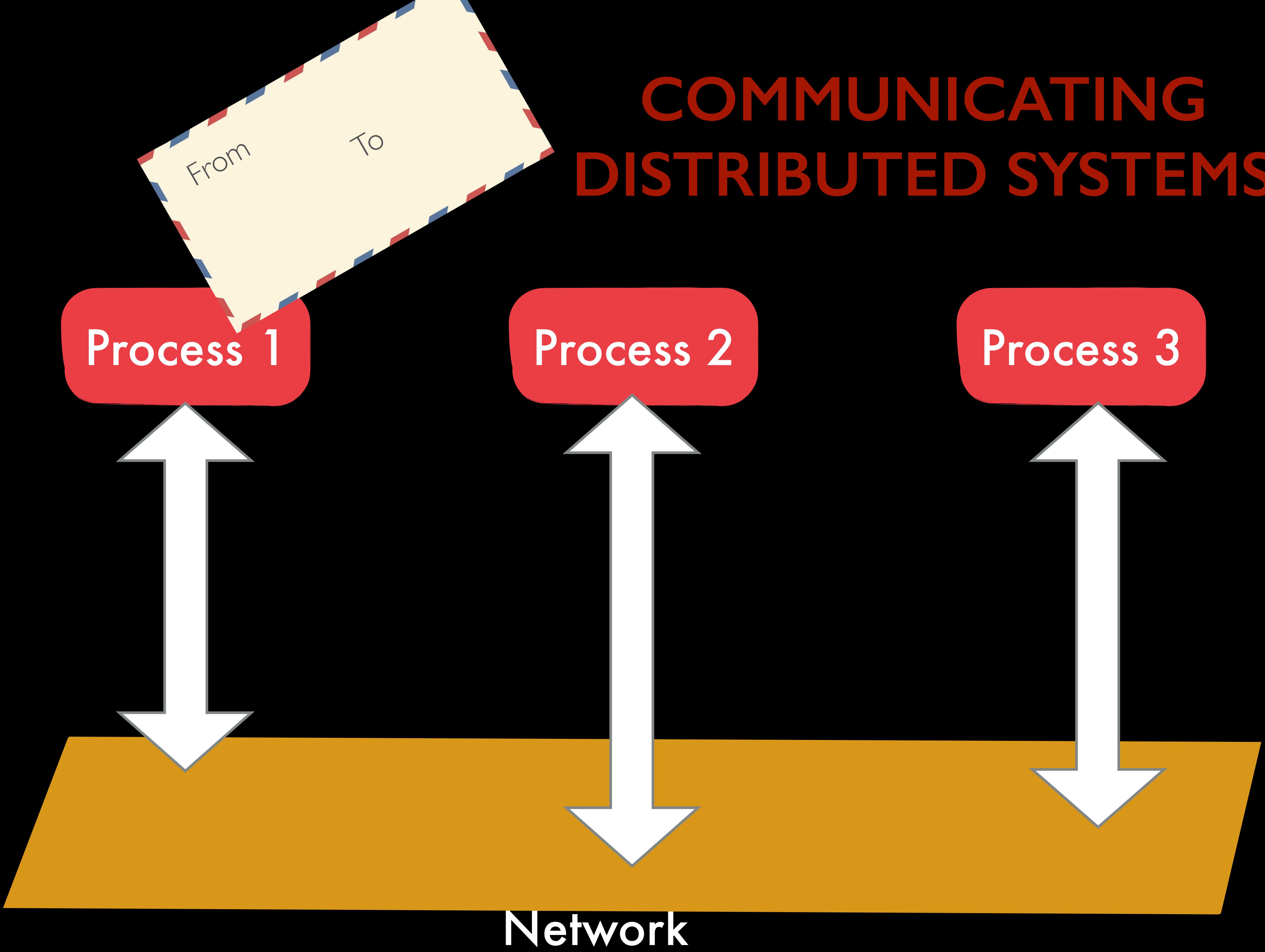
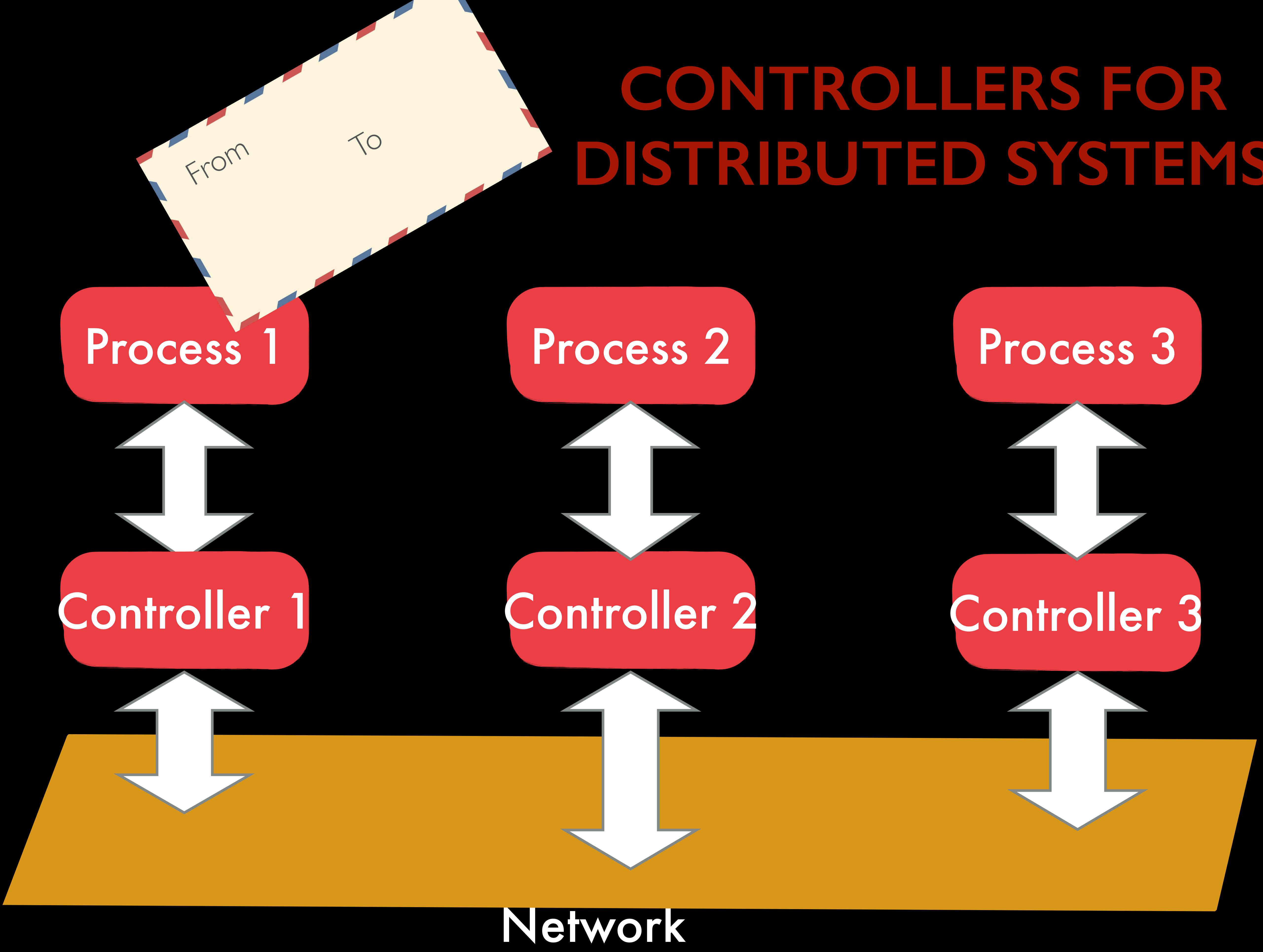
CONTROLLERS FOR VERIFICATION OF COMMUNICATING SYSTEMS

COMMUNICATING DISTRIBUTED SYSTEMS



CONTROLLERS FOR DISTRIBUTED SYSTEMS



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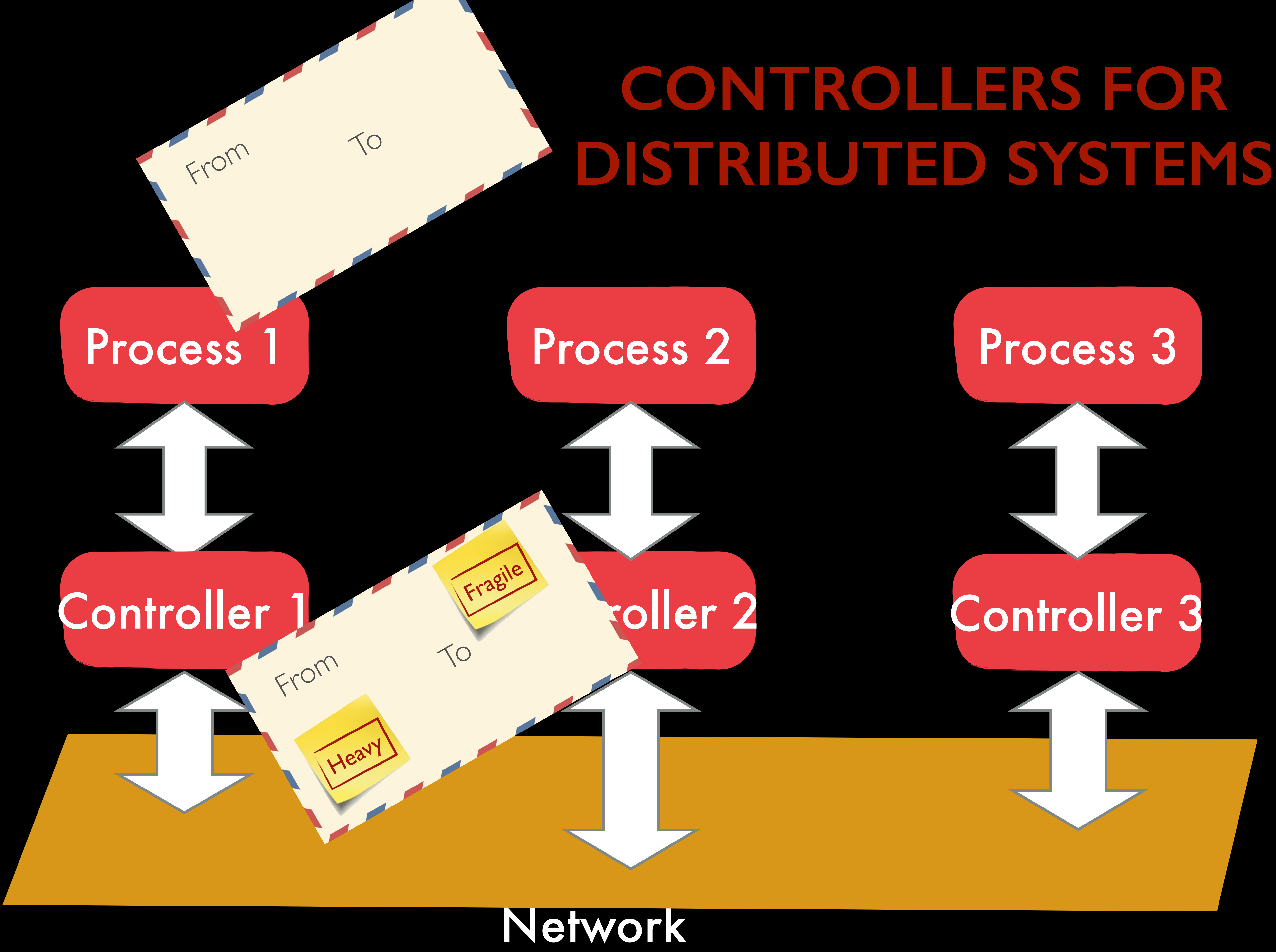
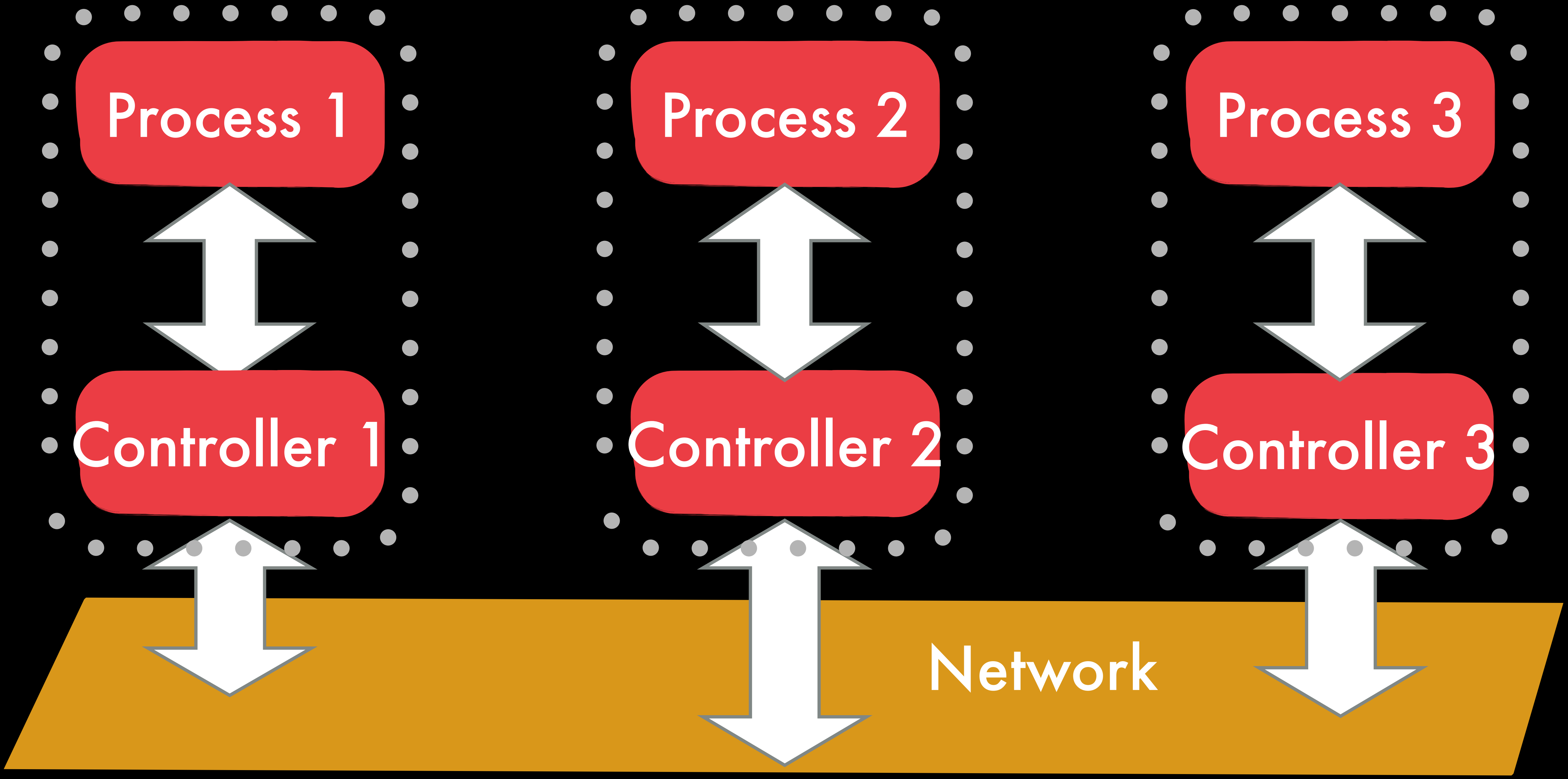
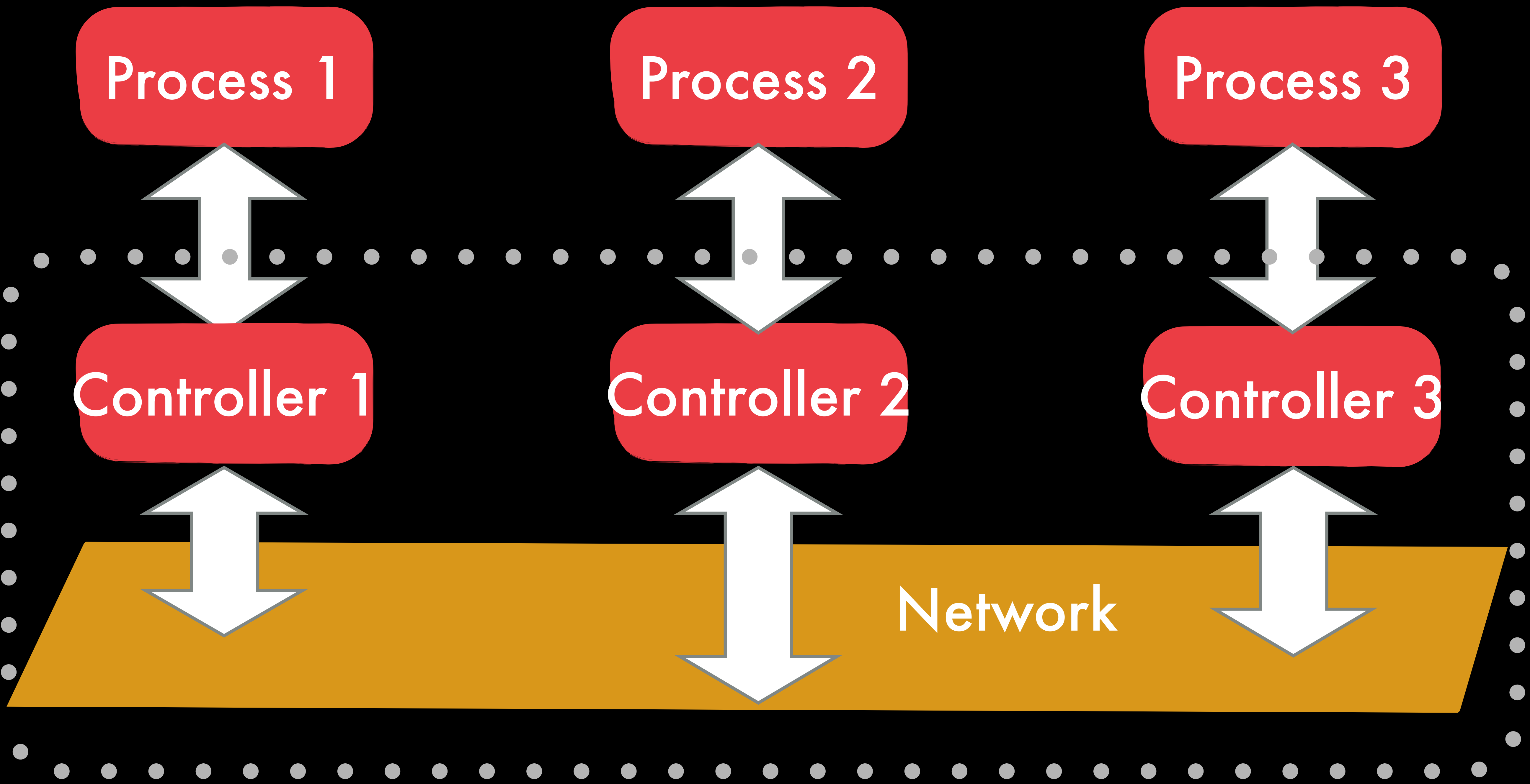



Process 1
Process 2
Process 3
Controller 1
Controller 2
Controller 3
Network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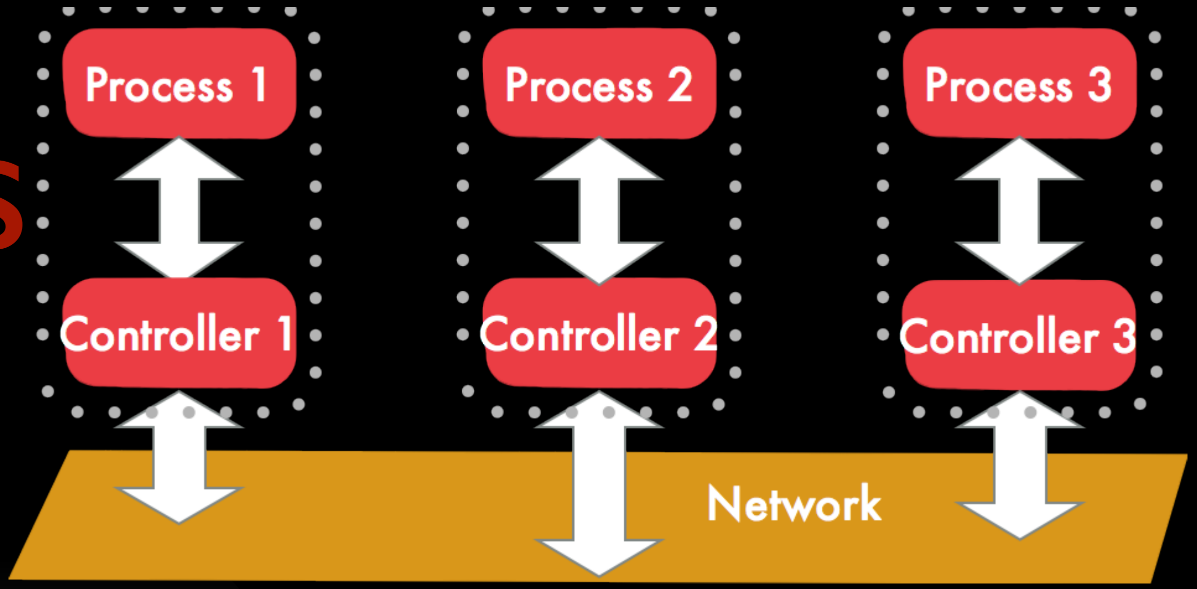
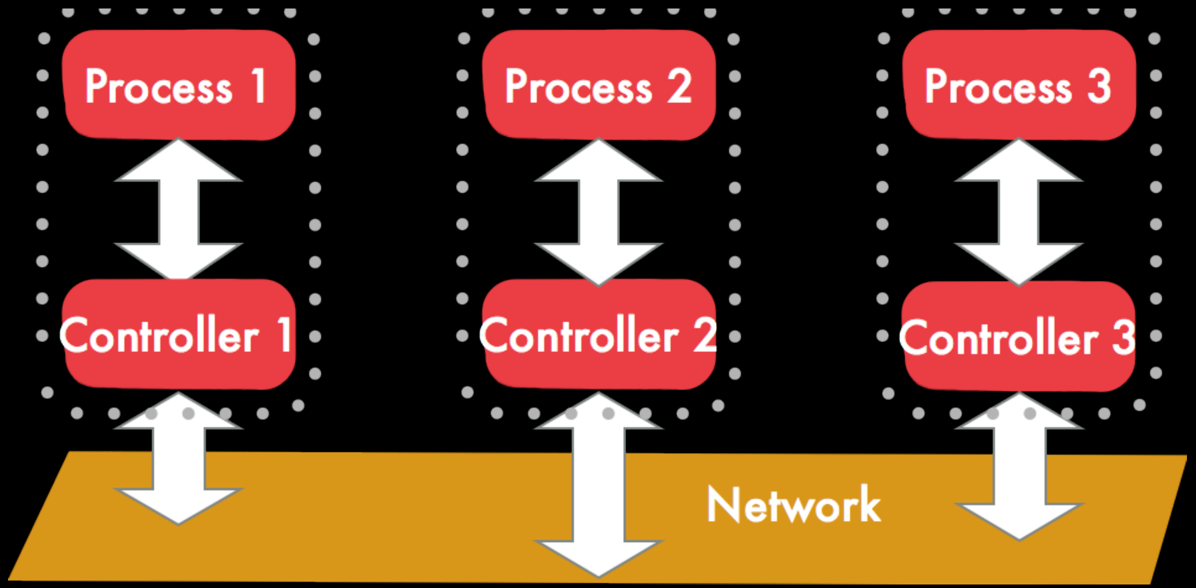
CONTROLLERS FOR DISTRIBUTED SYSTEMS



- Collection of local controllers
- Communication via piggy-backing
- Privacy: Do NOT read states/messages

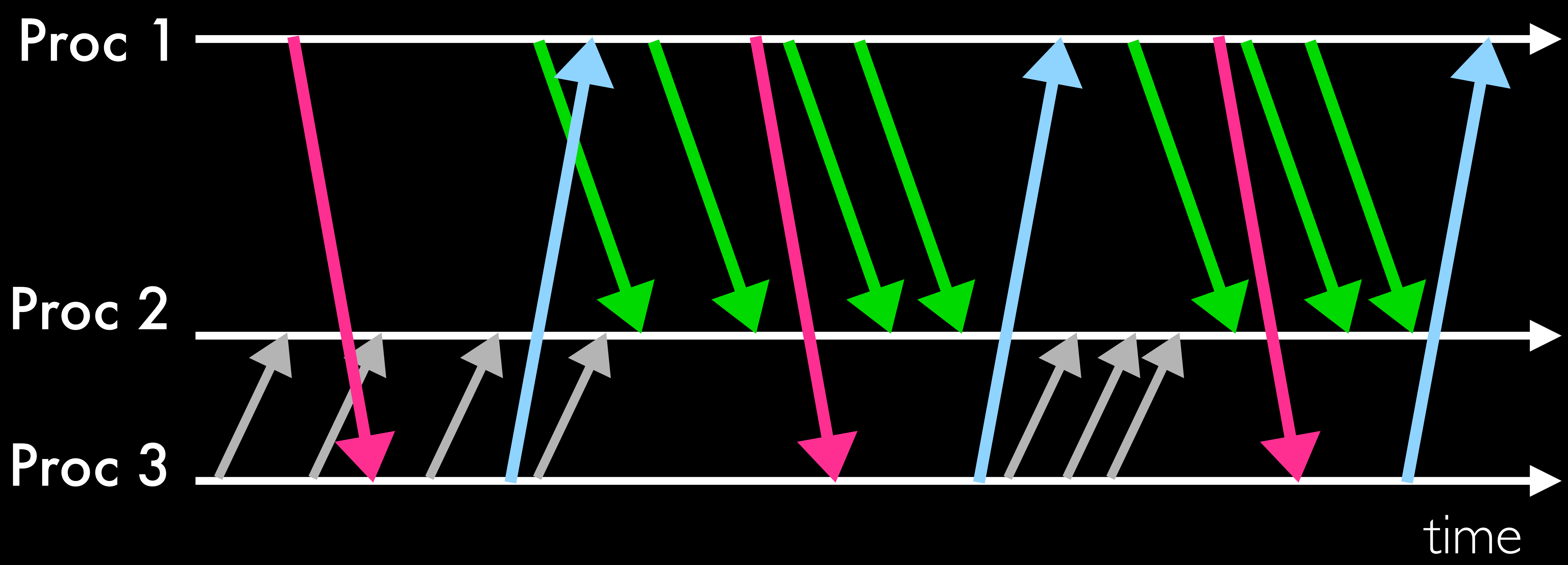
LET'S DESIGN A CONTROLLER



UNDER-APPROXIMATION: BOUNDED (*K*) PHASE

PHASE

Receive from one process, send to all processes



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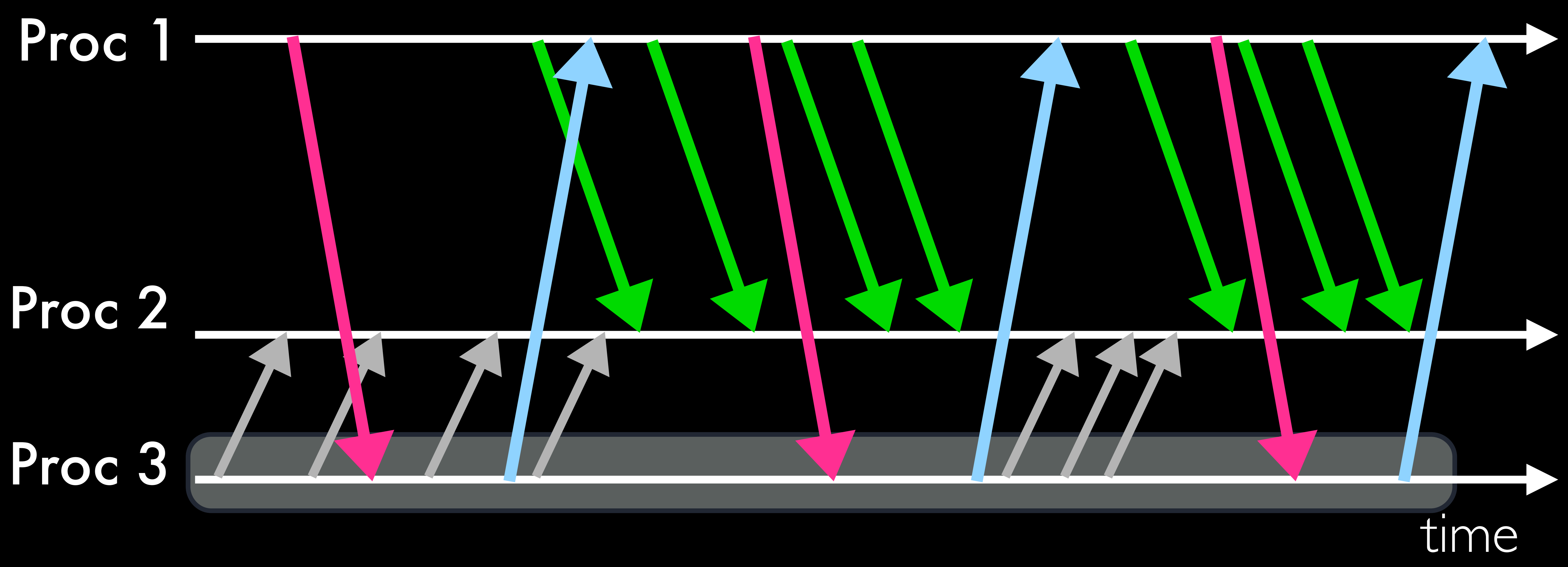

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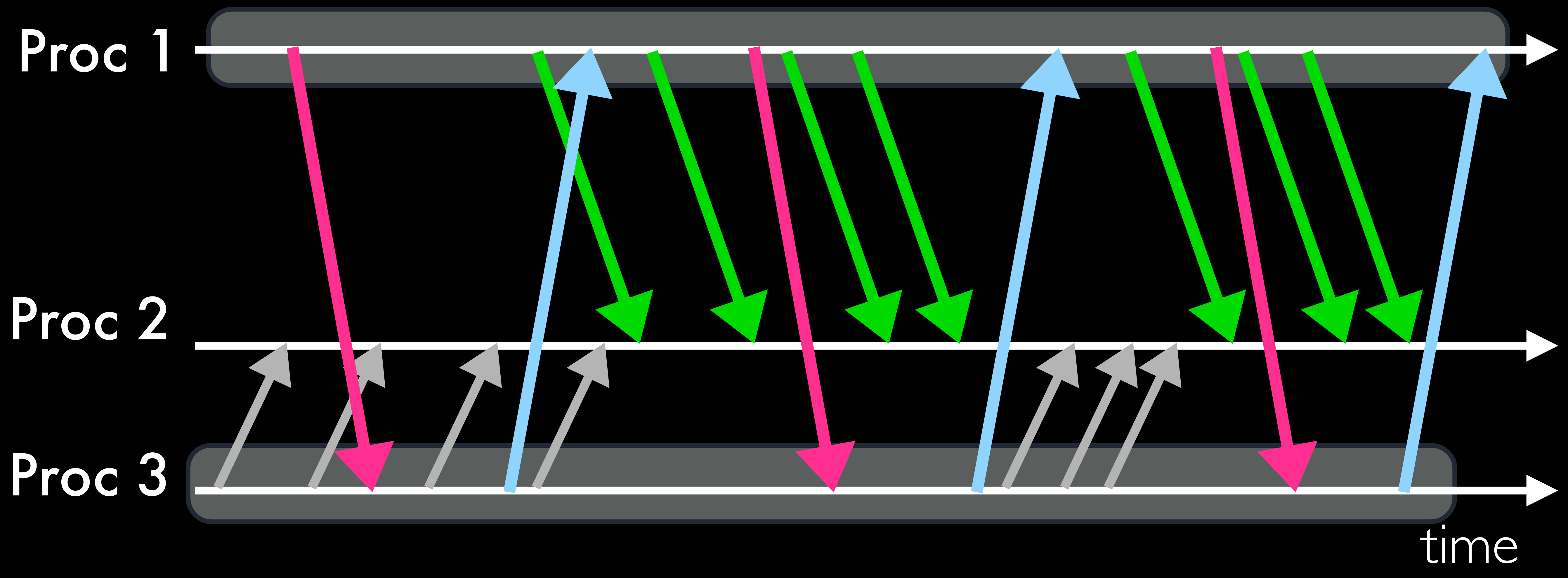

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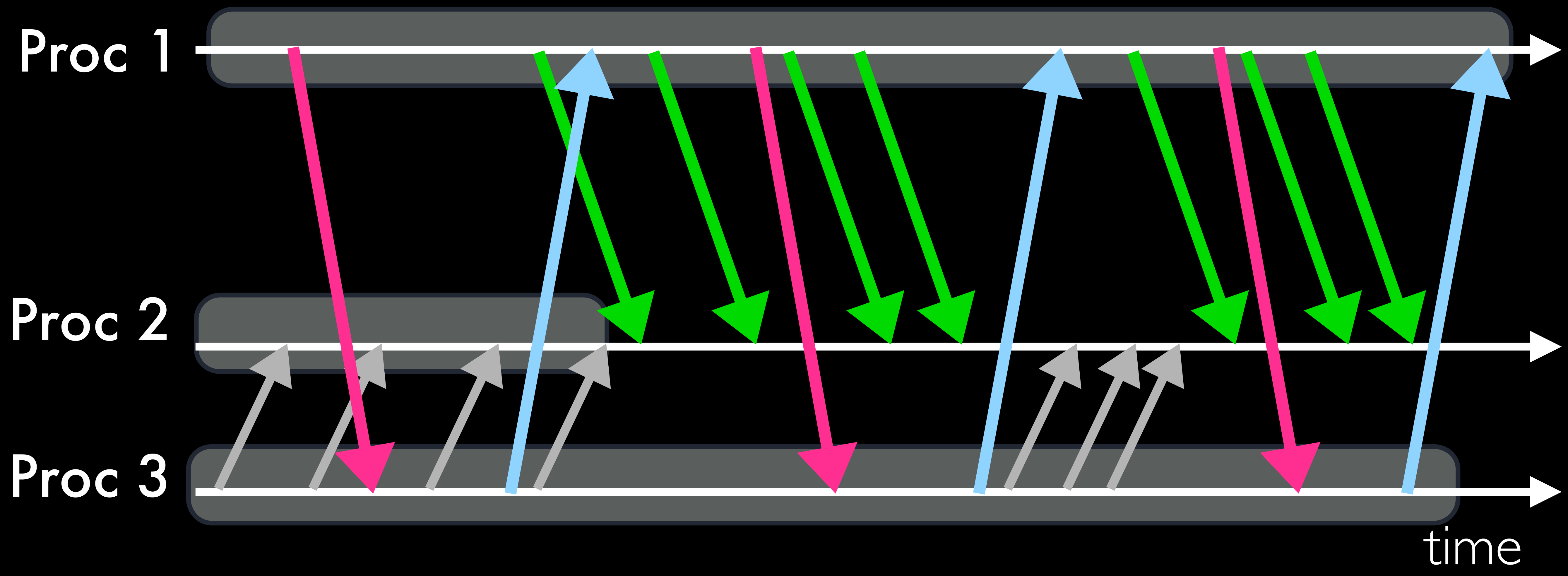

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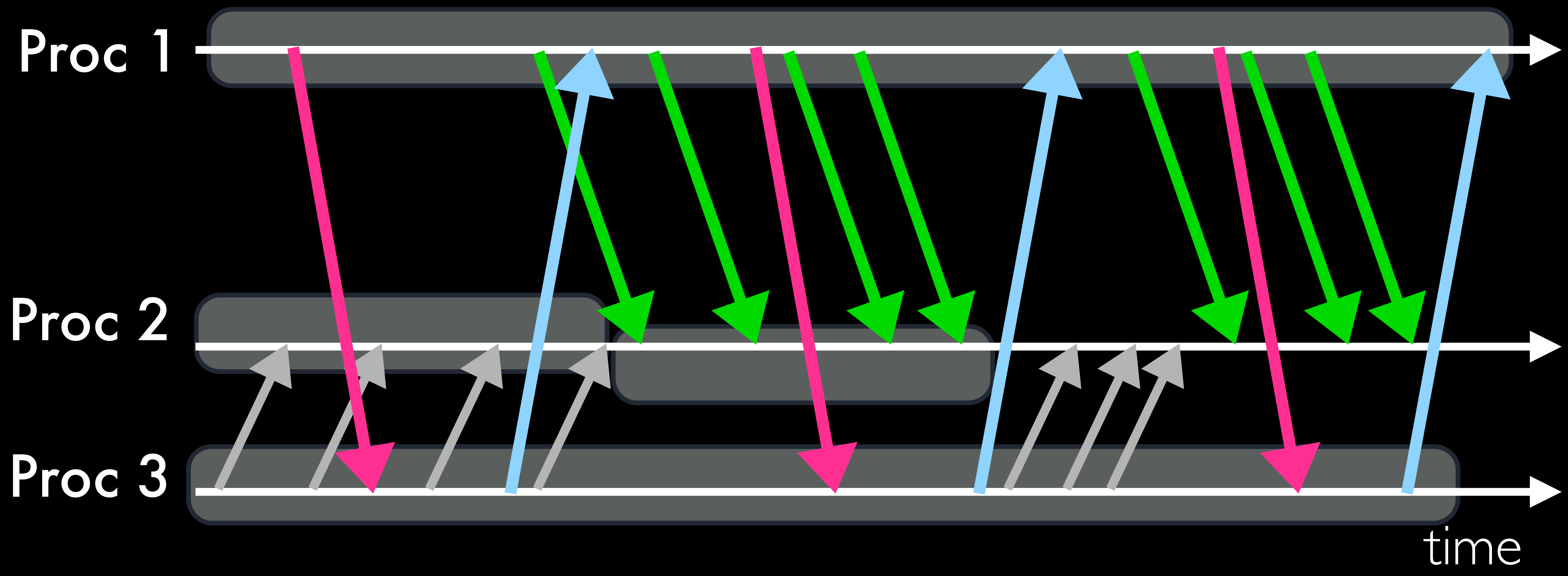

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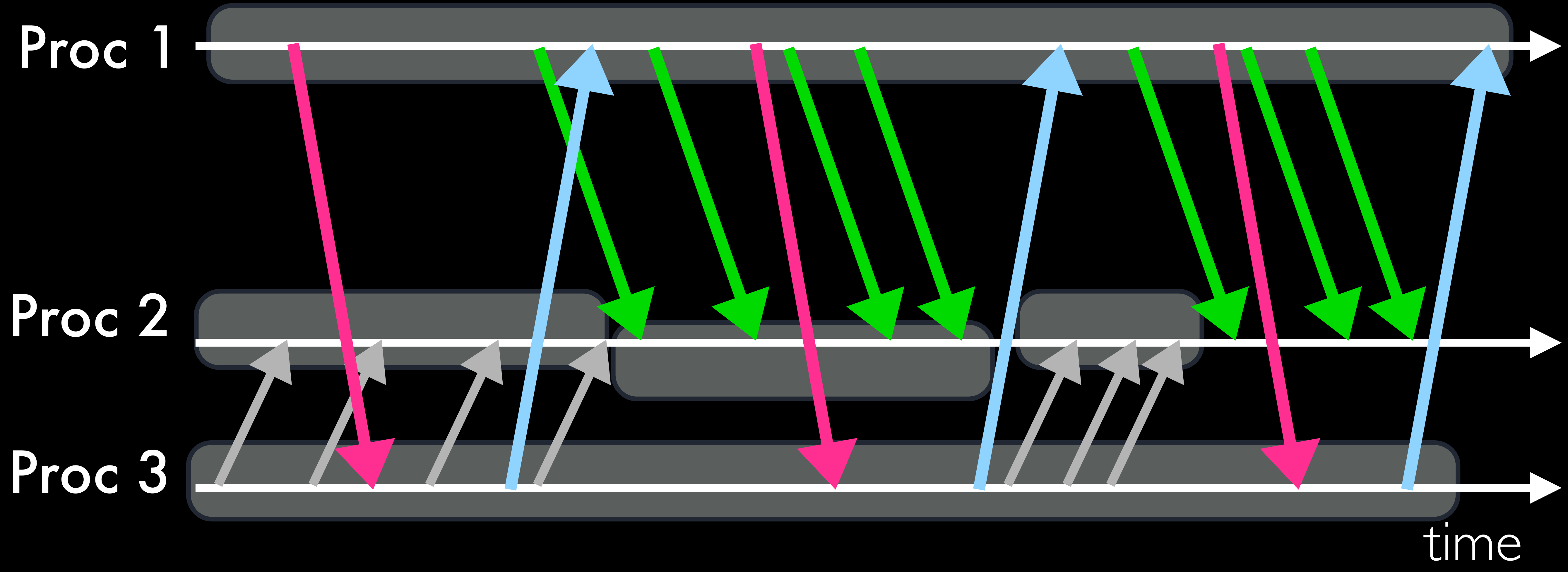

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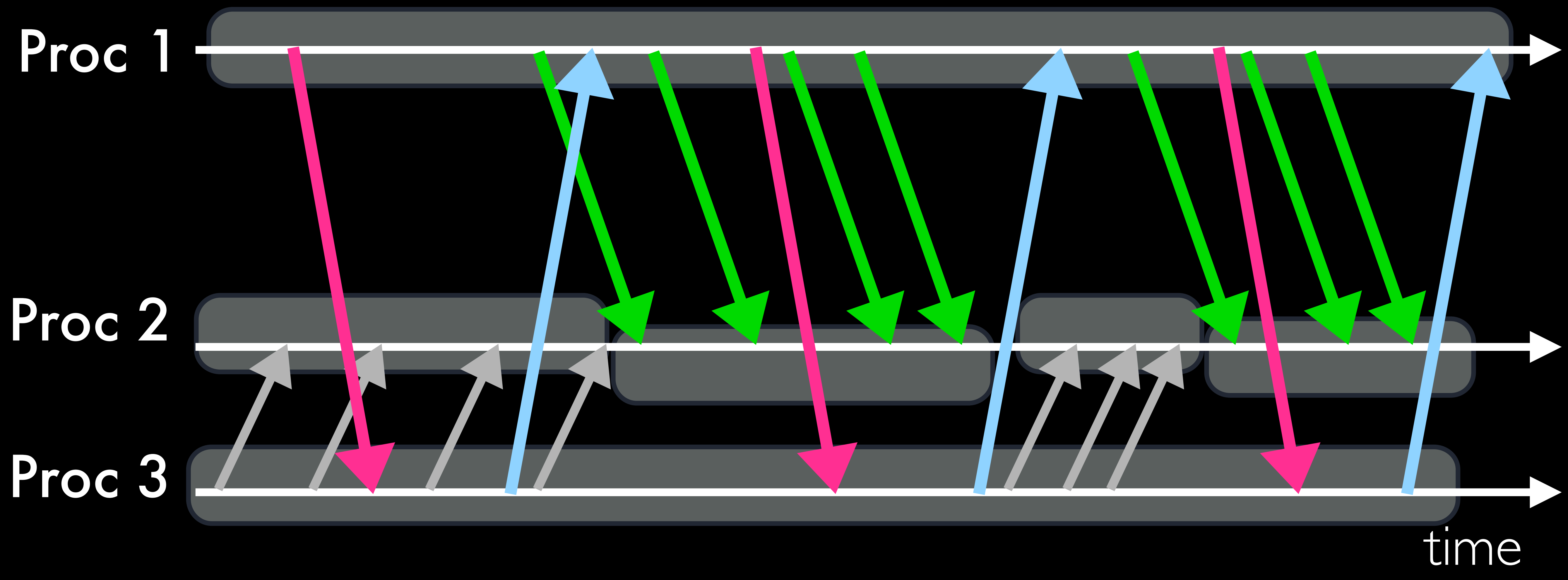
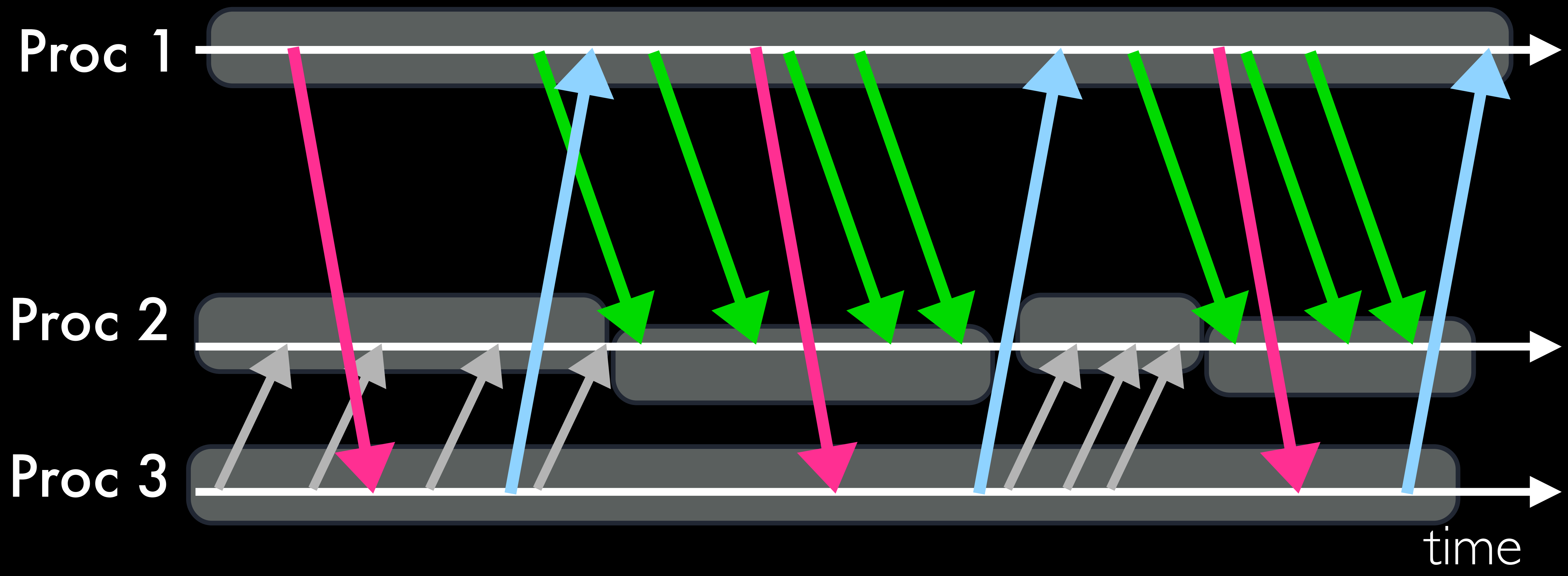


PHASE

Receive from one process, send to all processes

k-BOUNDED PHASE

1. At most k phases on each process
2. No cycles



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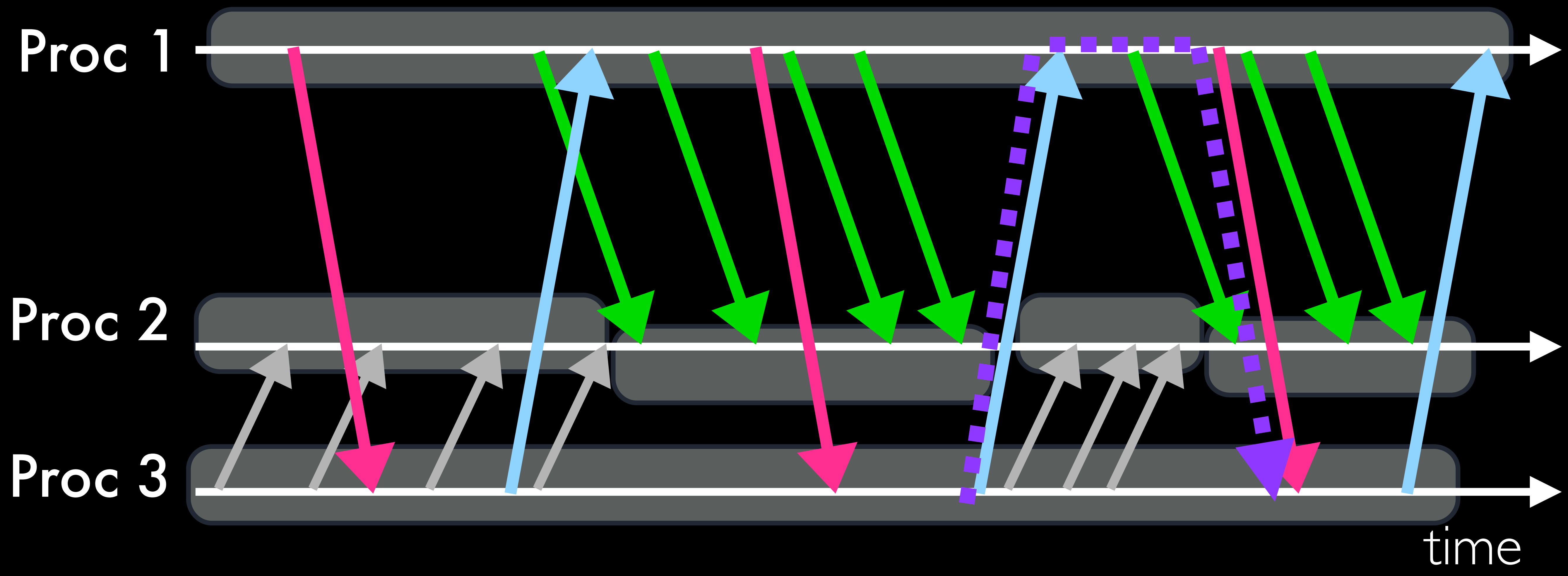

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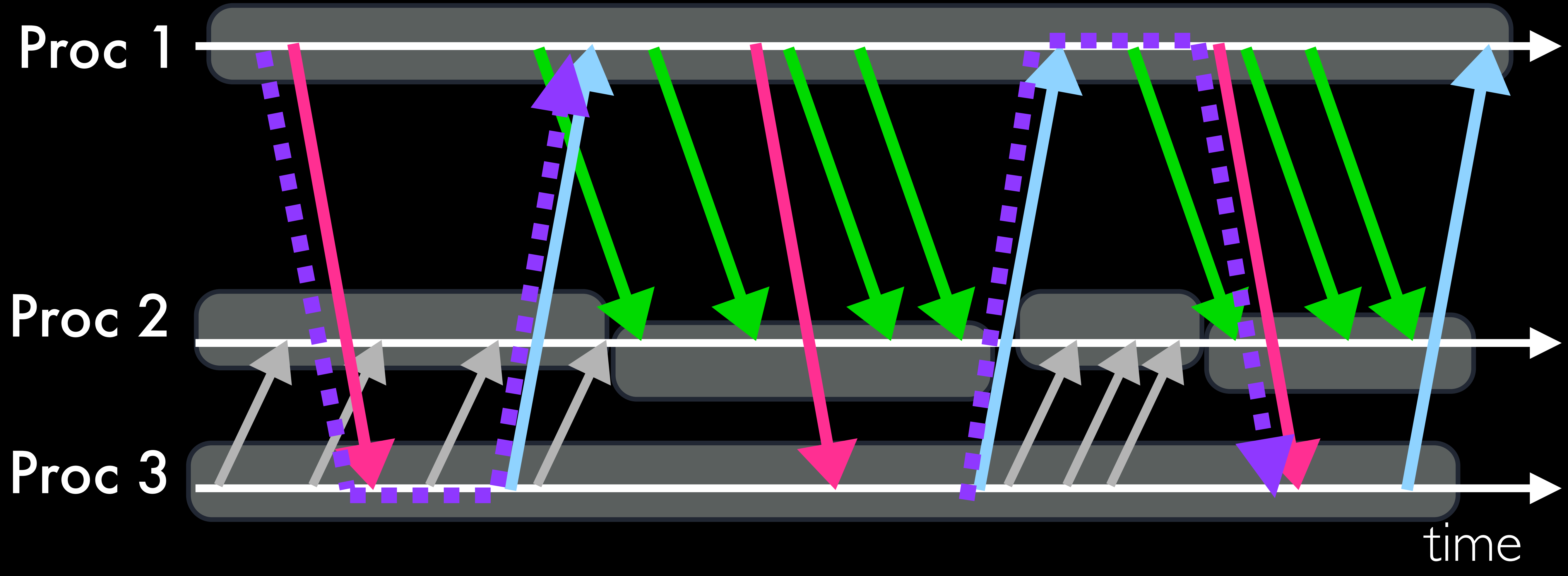

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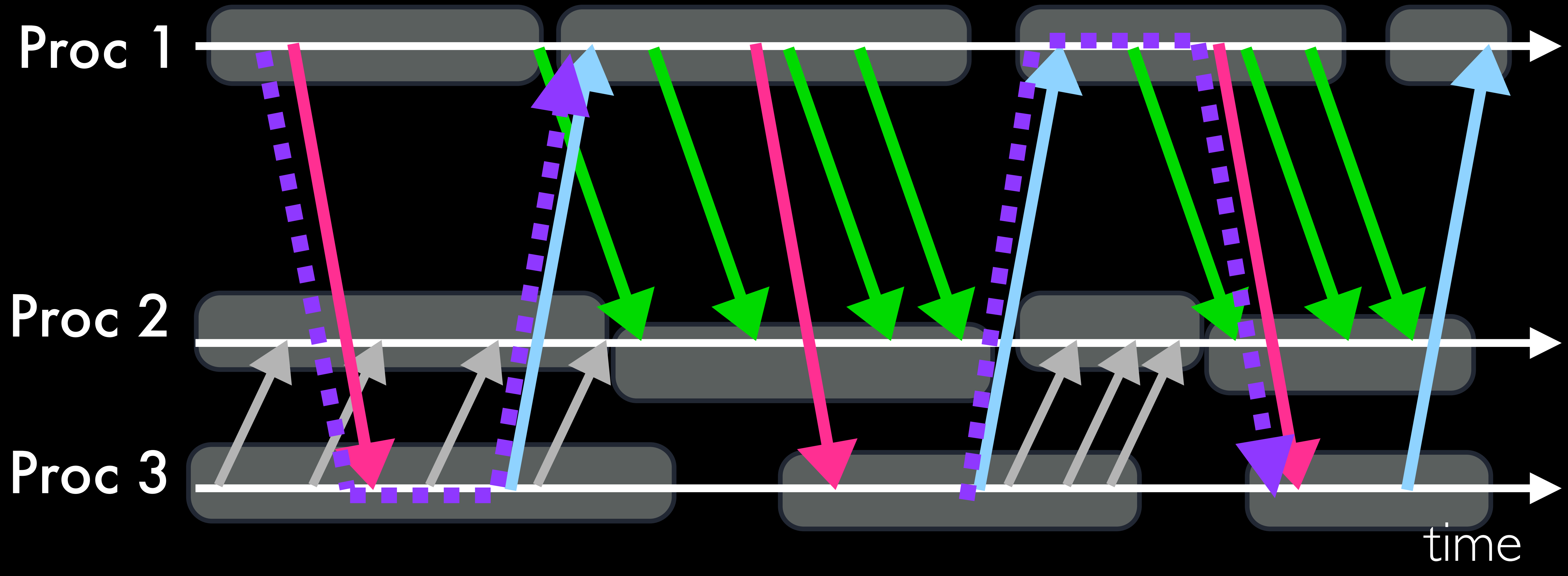
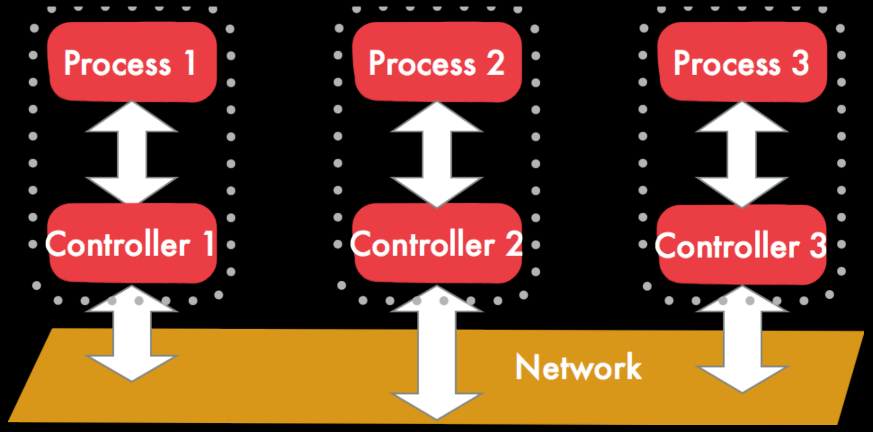
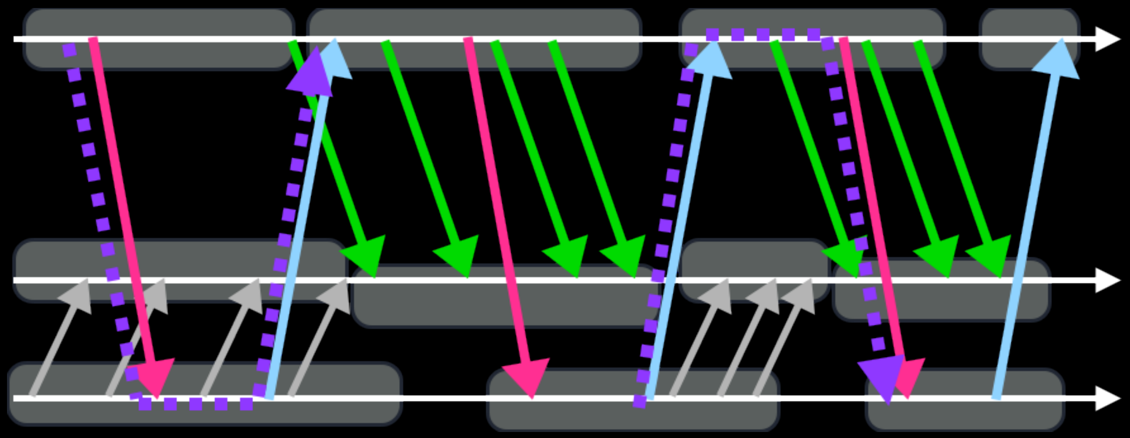


DISTRIBUTED CONTROLLER FOR K-BOUNDED PHASE U-A





A local controller for each process

State

Has a Phase Counter

Remembers current sender

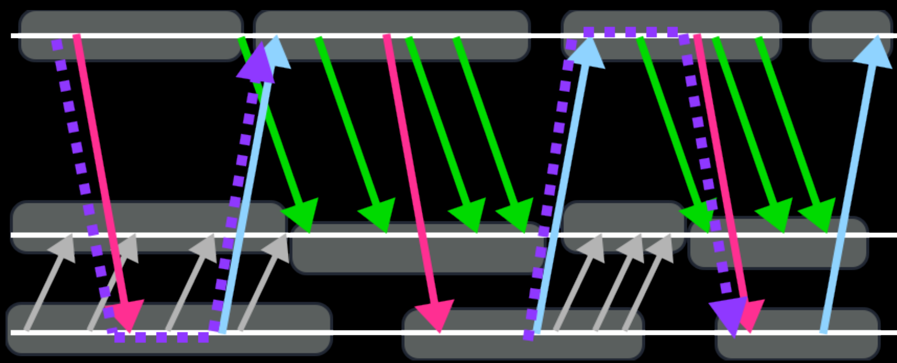
Transitions

Different sender?

Detect Cycle?

Increment counter,
Update sender

DISTRIBUTED CONTROLLER FOR K-BOUNDED PHASE U-A



Detect Cycle?

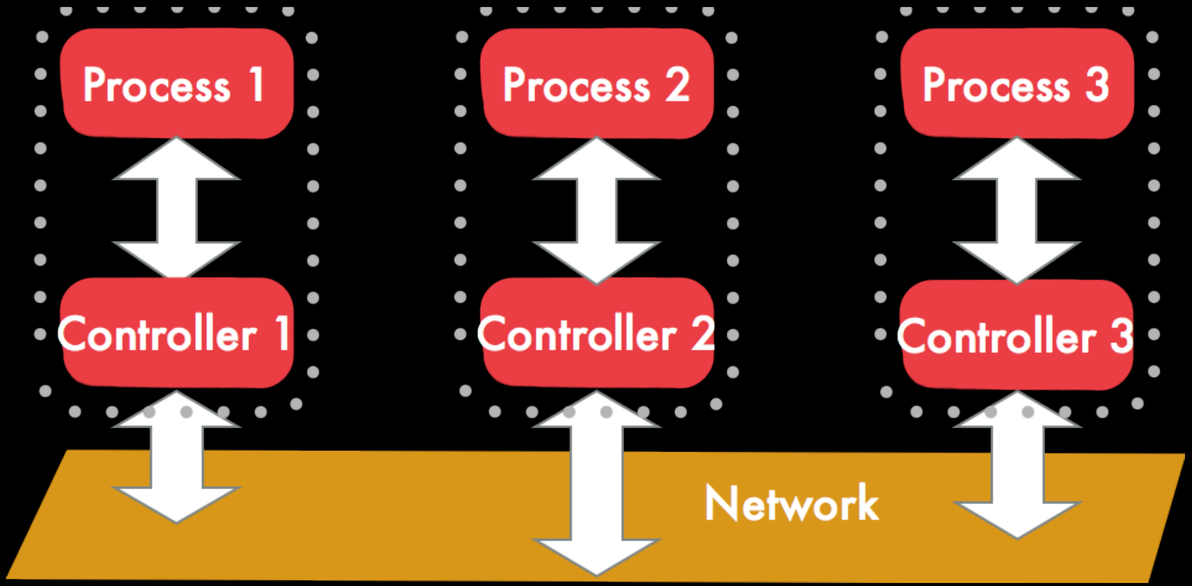
Phase Vectors

best info about phase number of other processes

Sends: tag with phase vector

Receives: update phase vector by taking MAX

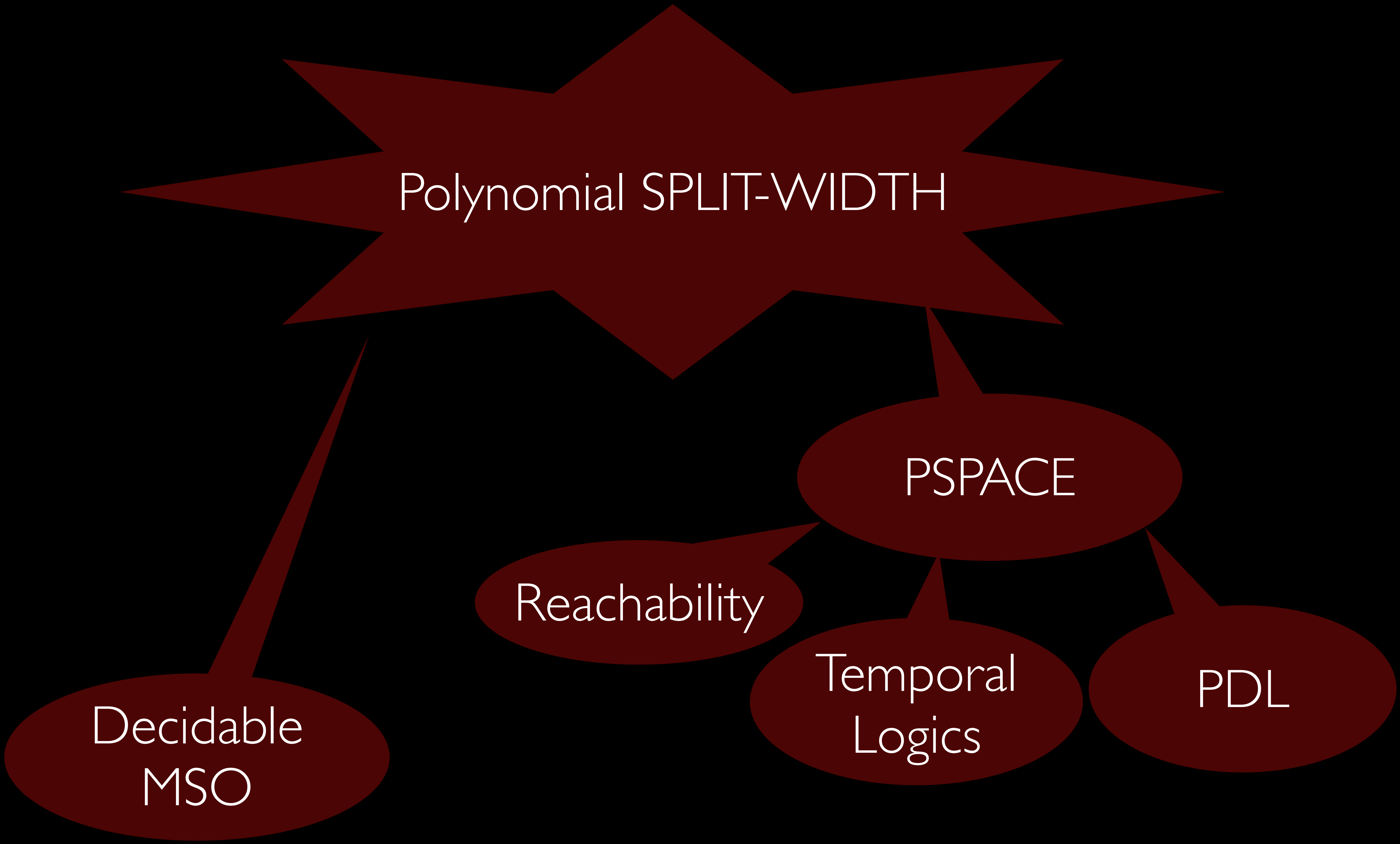
CONTROLLERS FOR BOUNDED PHASE DISTRIBUTED SYSTEMS



- Collection of local controllers
- Communication via piggy-backing
- Privacy: Do NOT read states/messages

- System independent
- Generic
- Deterministic
- Finite state

DECIDABILITY OF K BOUNDED PHASE



Polynomial SPLIT-WIDTH

PSPACE

Reachability

Temporal Logics

PDL

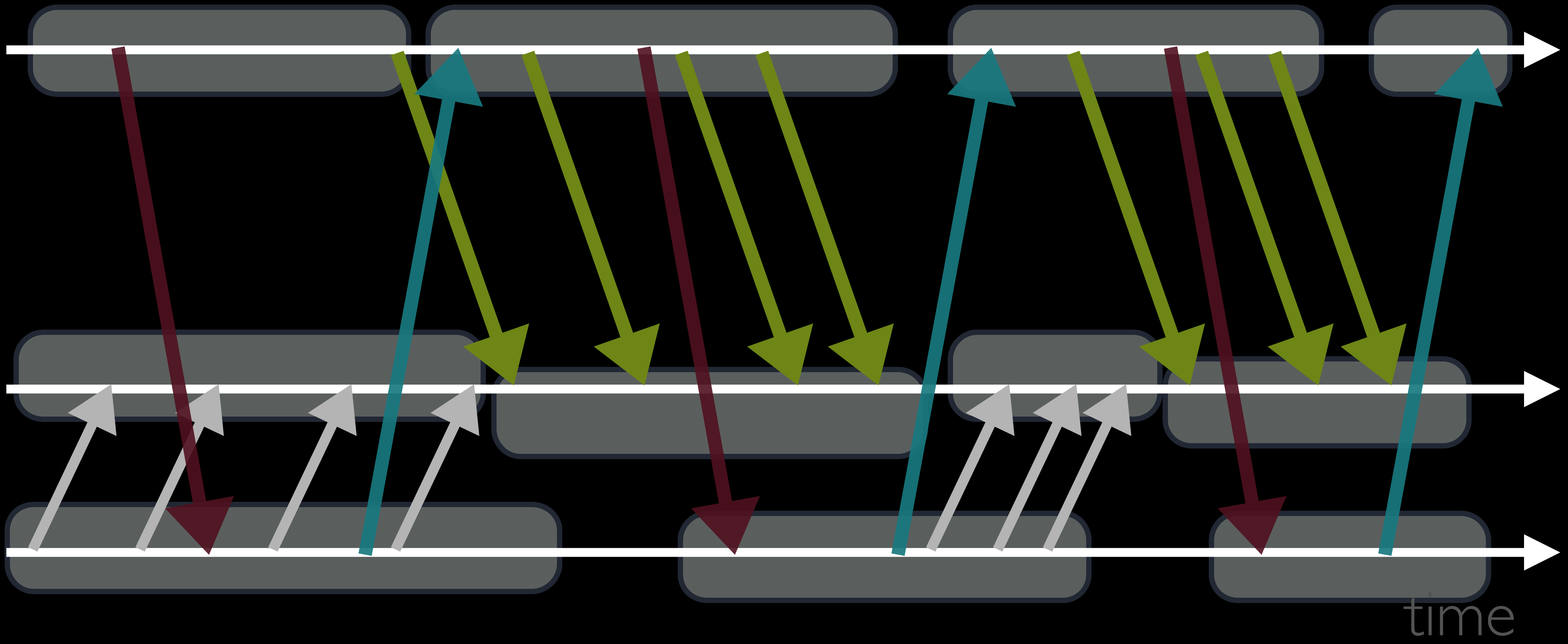
Decidable MSO

Polynomial SPLIT-WIDTH

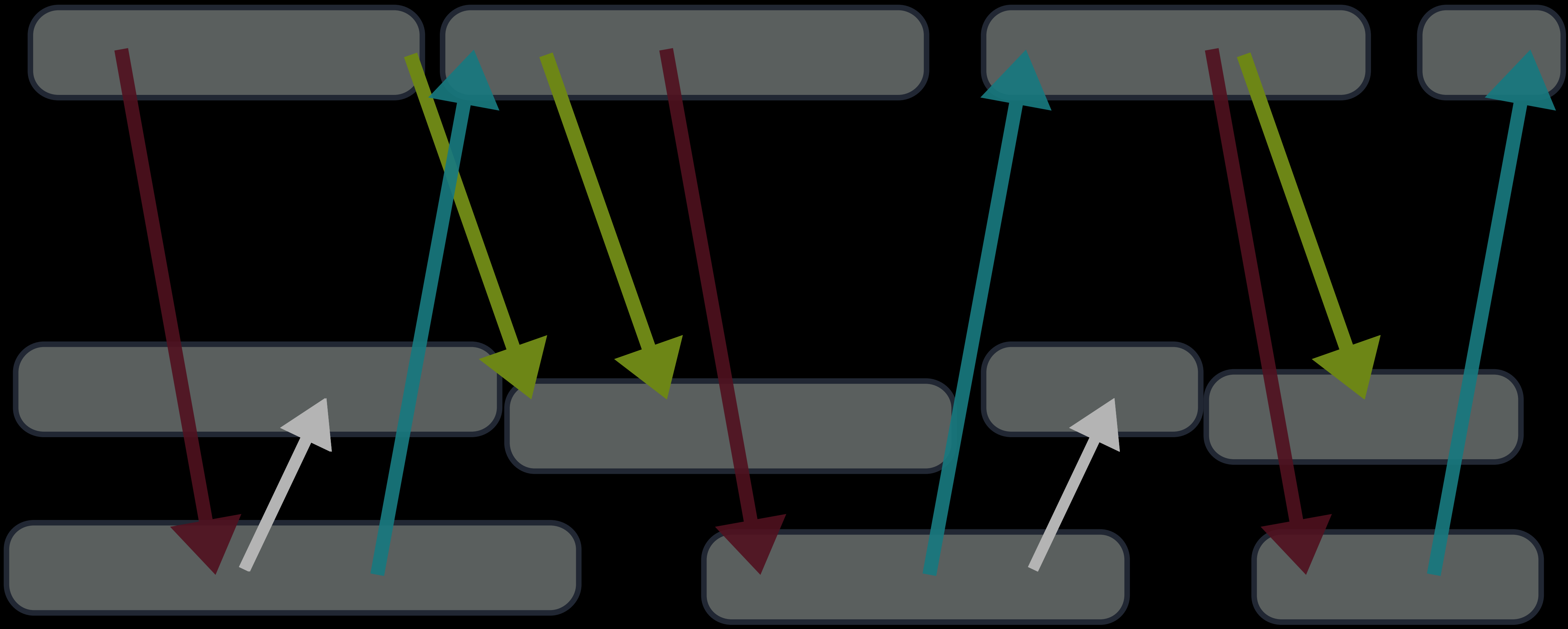
Refine phases to tree-like

bound split-width

ACYCLIC PHASE DECOMPOSITION



INDUCED GRAPH ON PHASE



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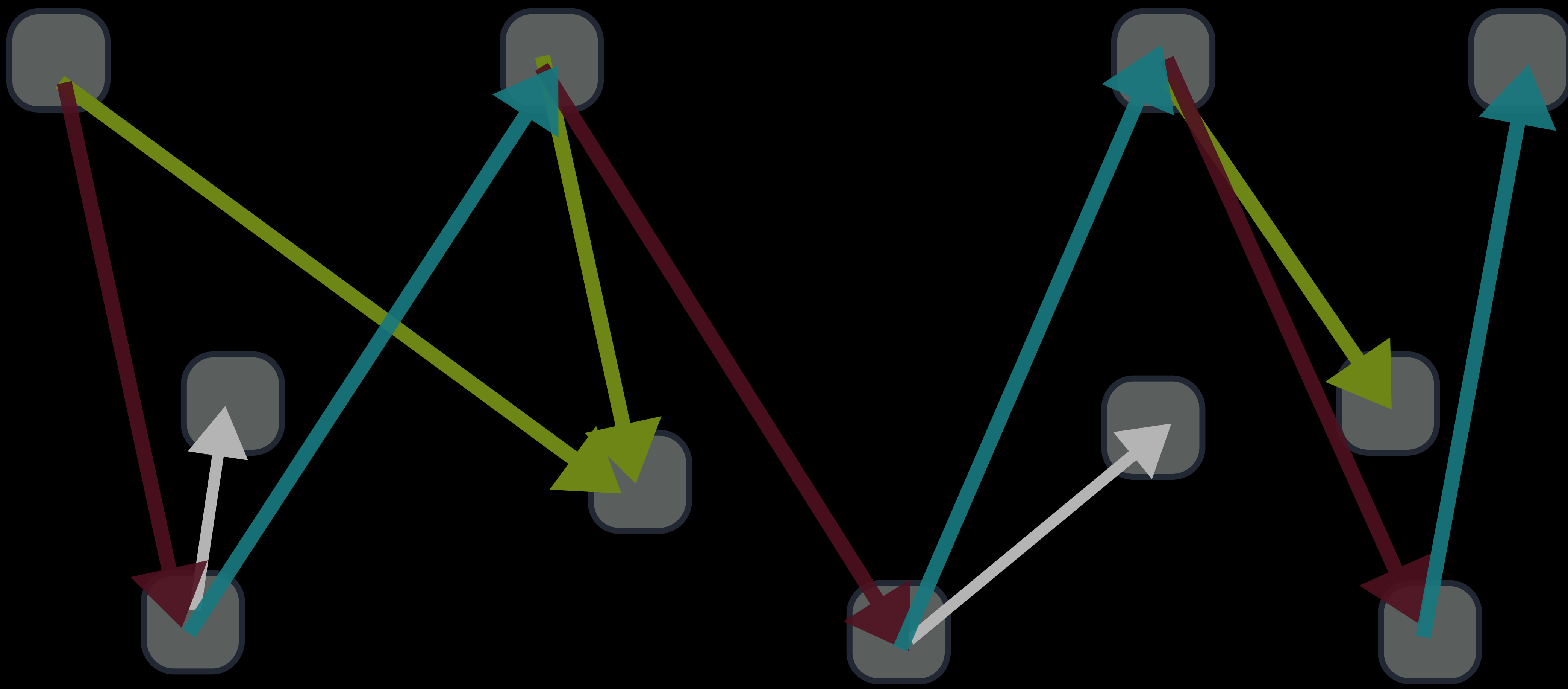

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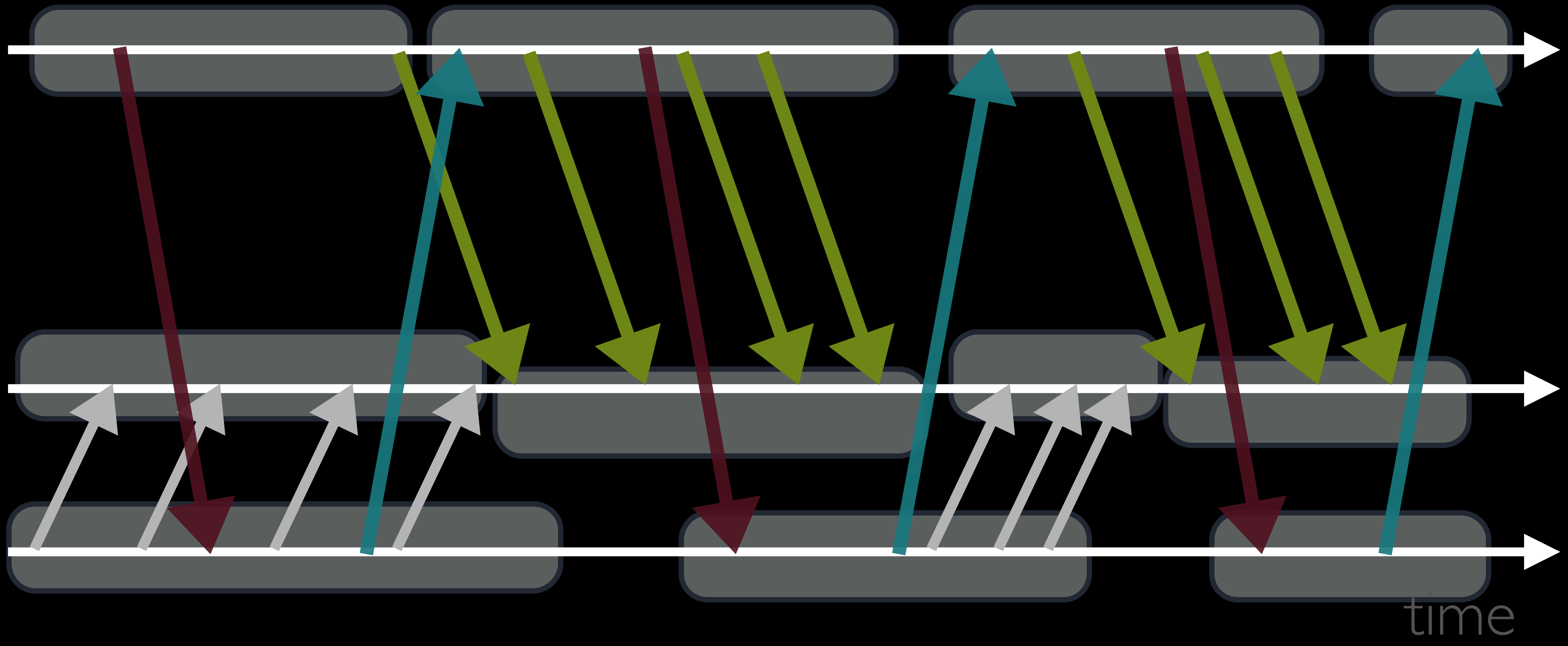

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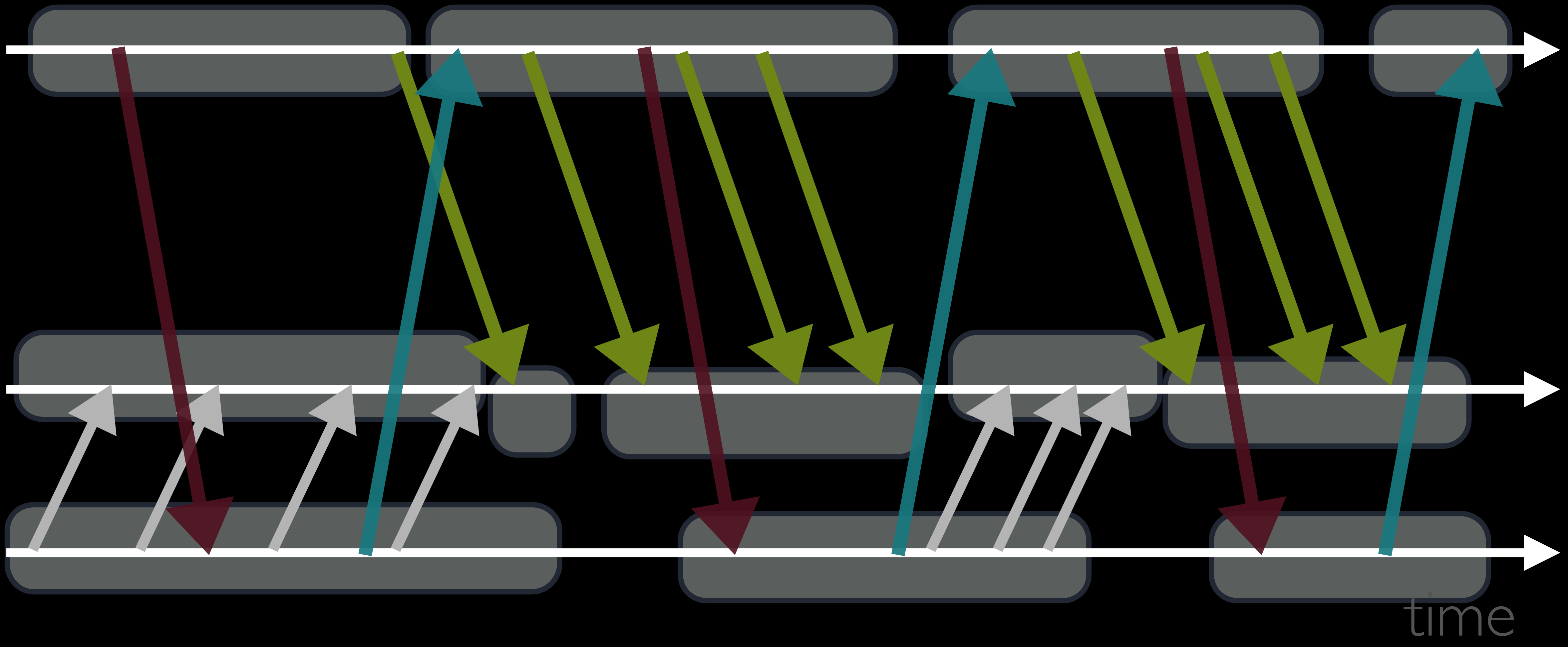
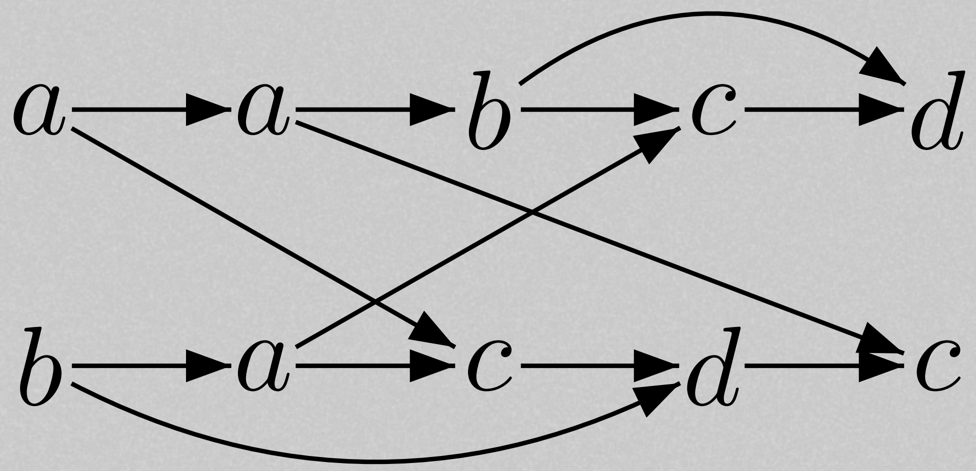


Tree-like

Polynomial SPLIT-WIDTH

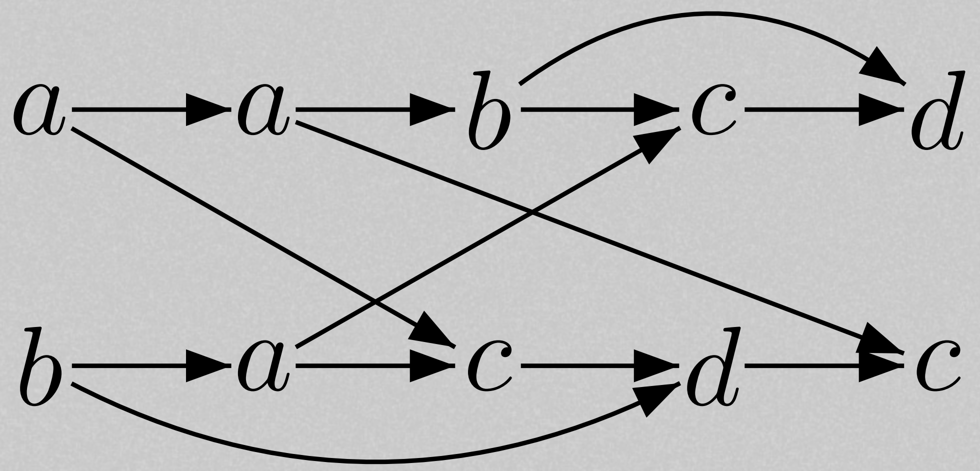
Split-width

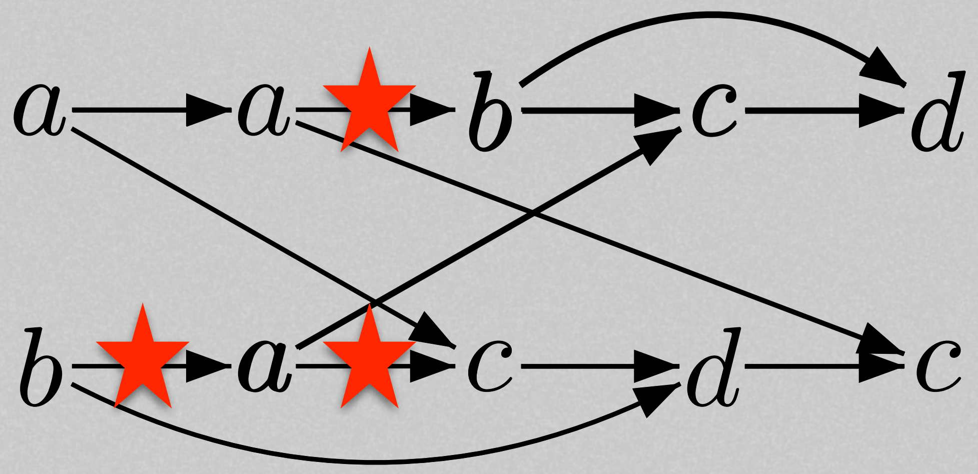


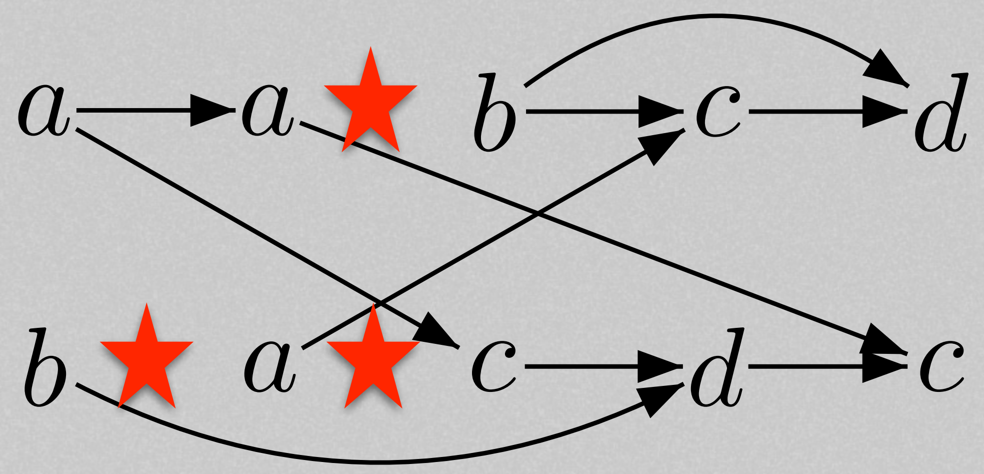
Split-width

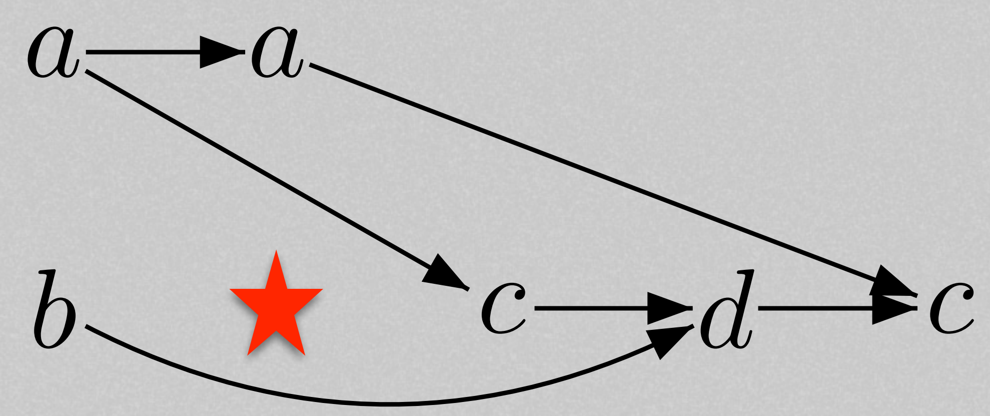
★ ★ ★

BUDGET




a
a
b
c
d
b
a
c
d
c


a a b c d
b a c d c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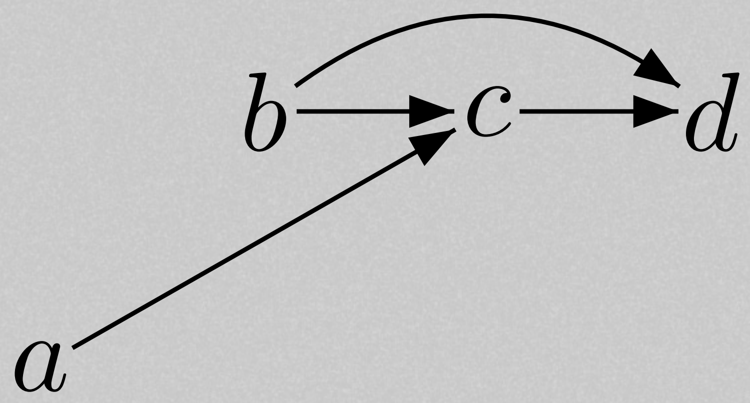
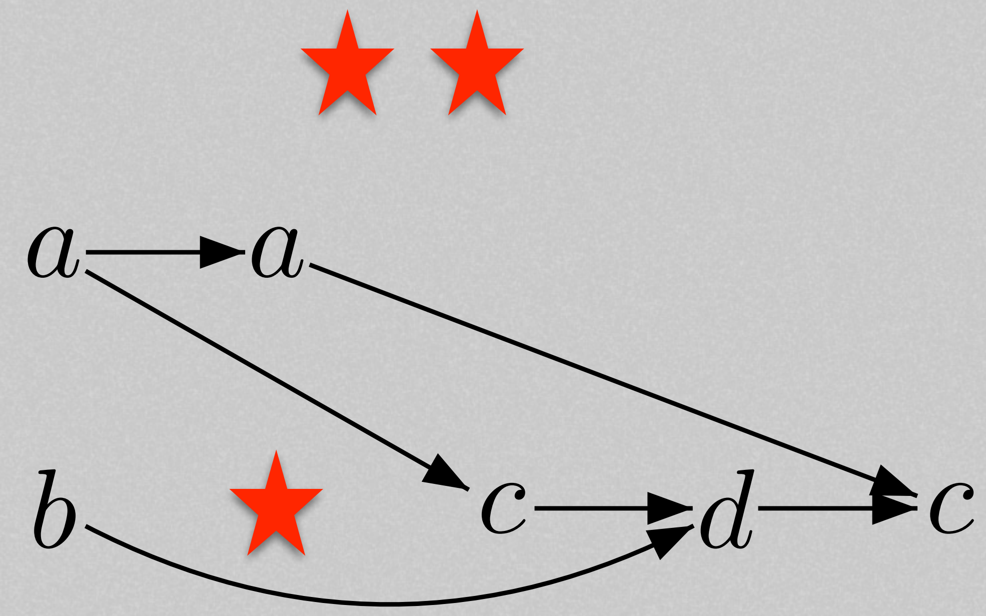
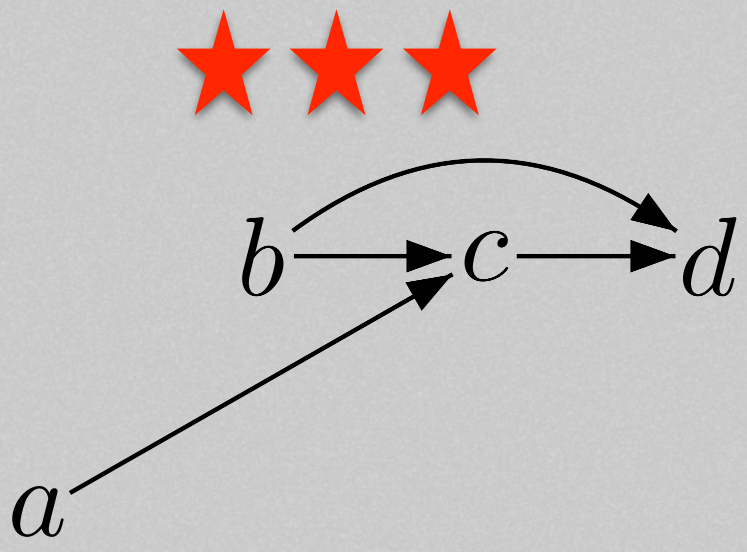
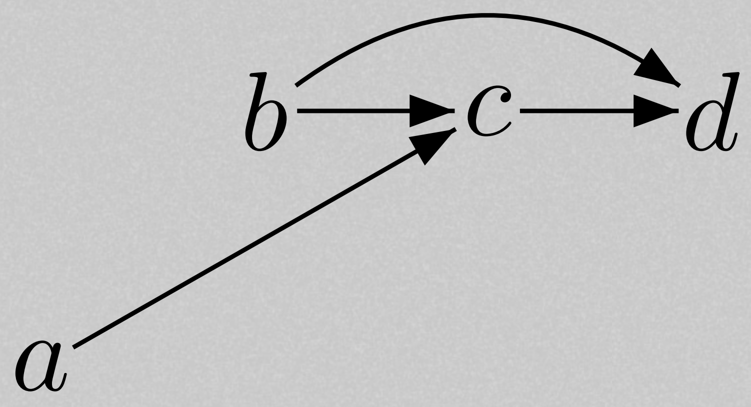
a
b
c
d
c

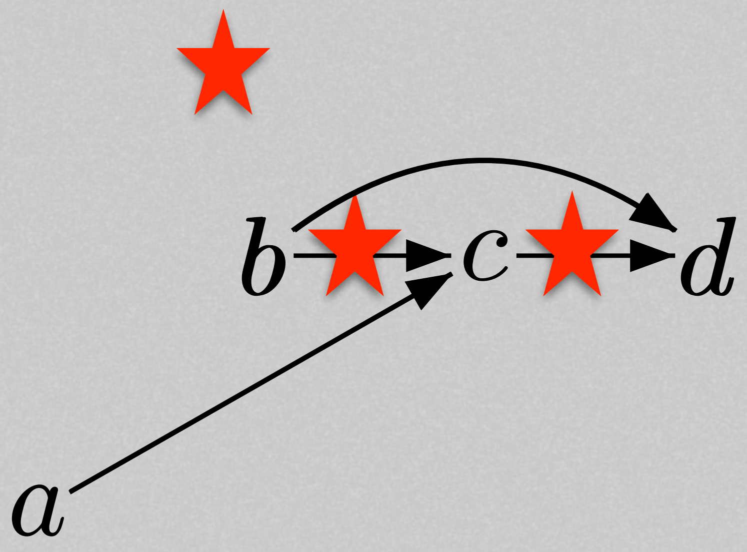

b
c
d
a

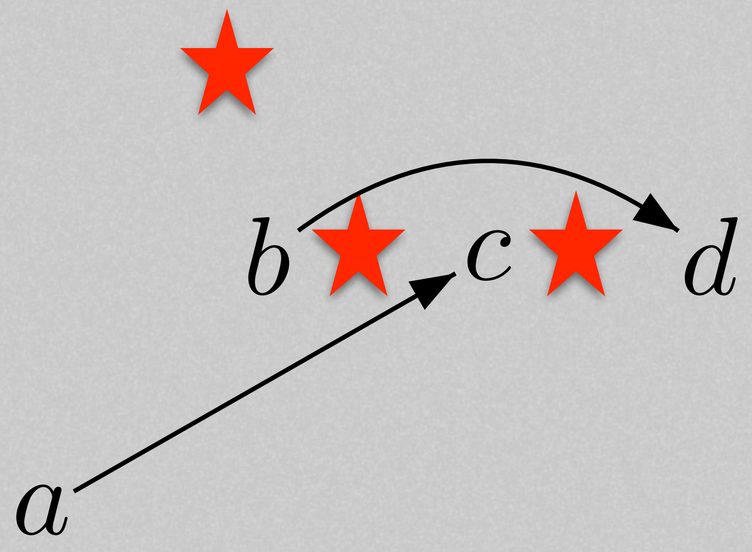

★ ★
a
a
b
★
c
d
c

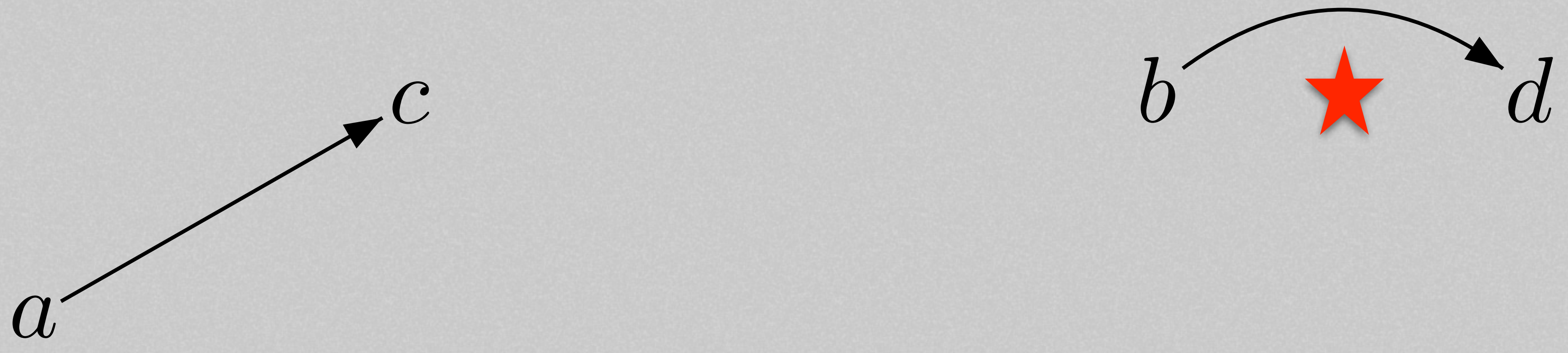

★ ★ ★
b
c
d
a

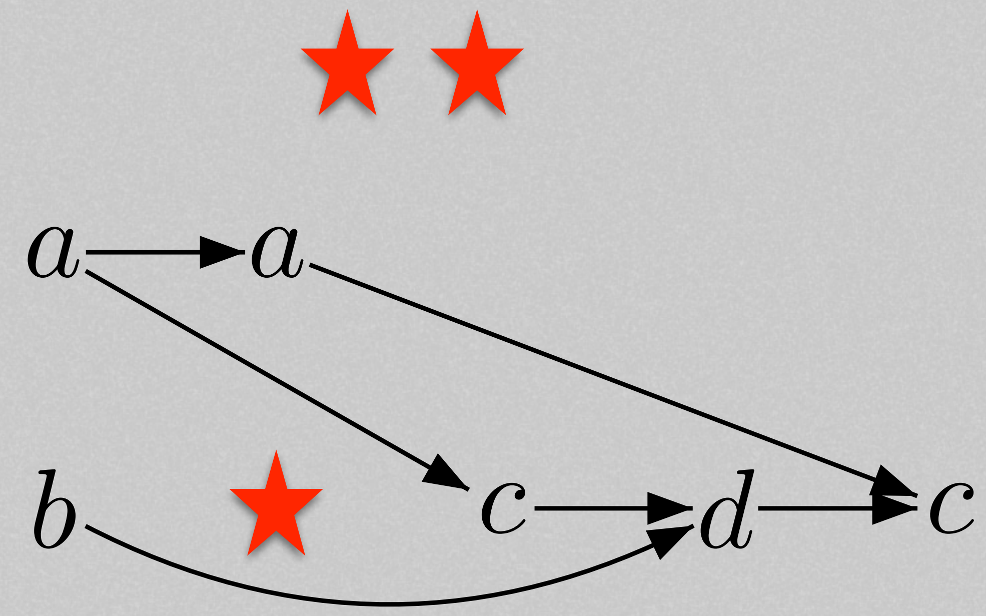
★★★

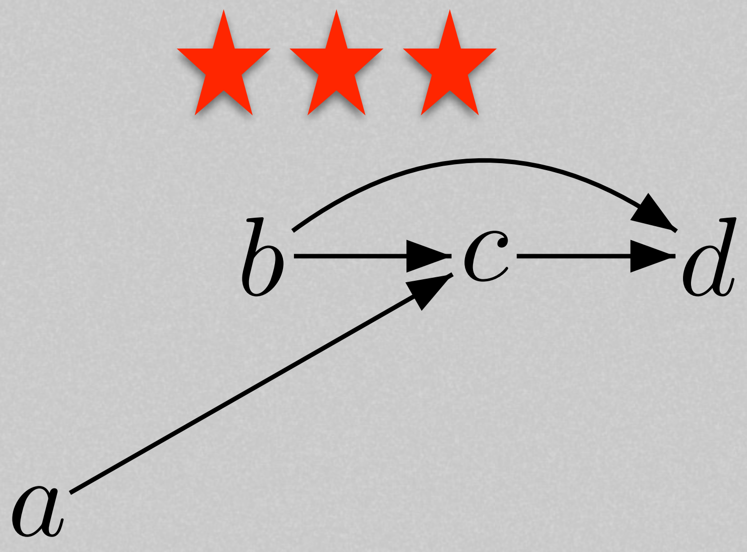

b
c
d
a

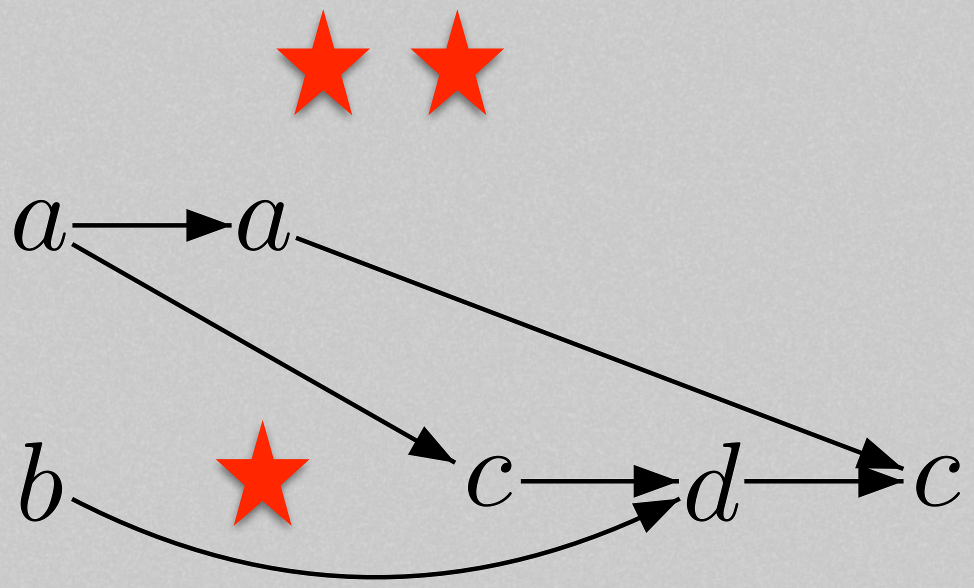

b
c
d
a

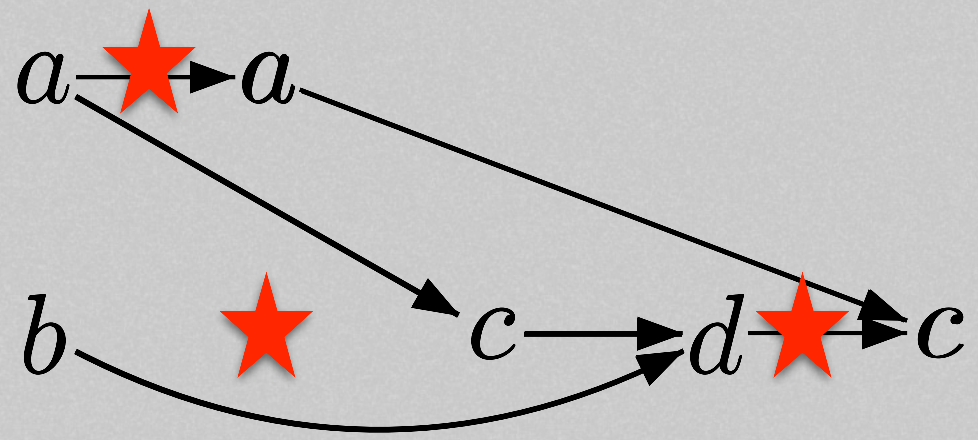

b
c
d
a


a
c
b
d


★★
a
a
b
★
c
d
c


★★★
b
c
d
a


a
a
b
c
d
c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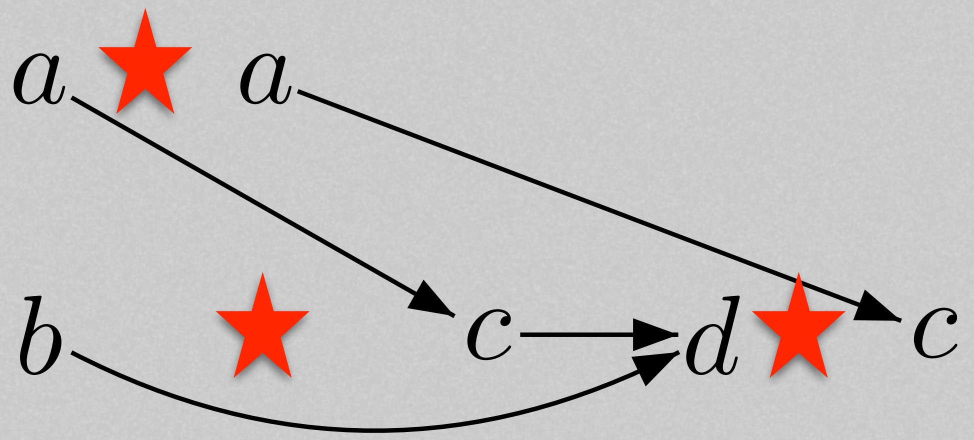
a
b
c
d
c



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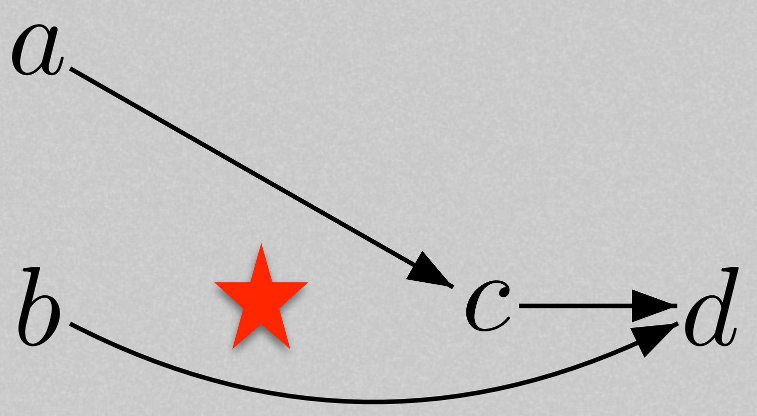
a
b
c
d
c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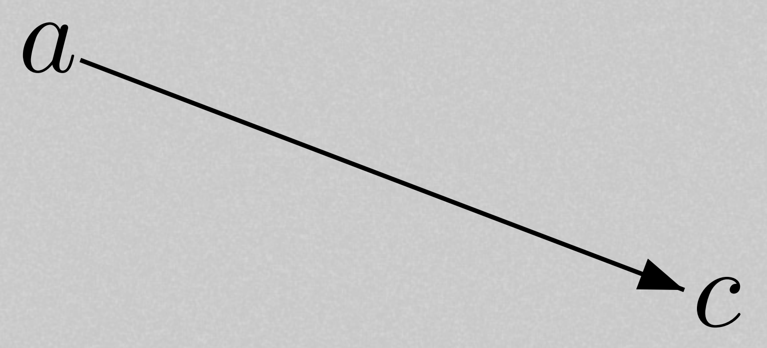
d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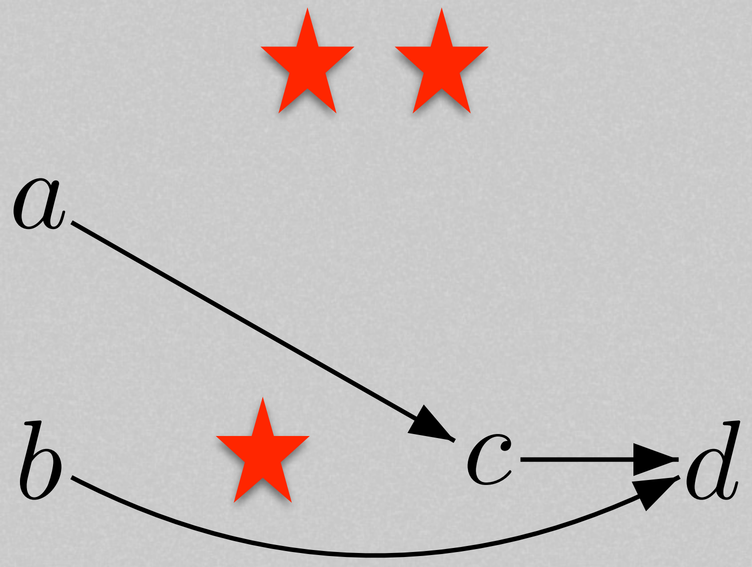
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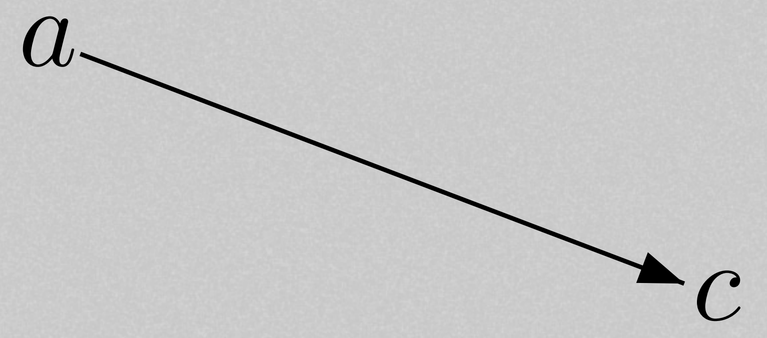
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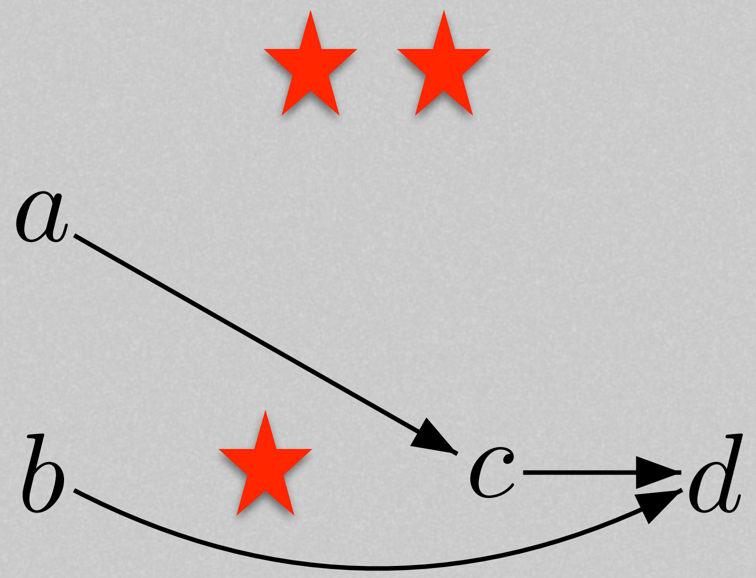
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c
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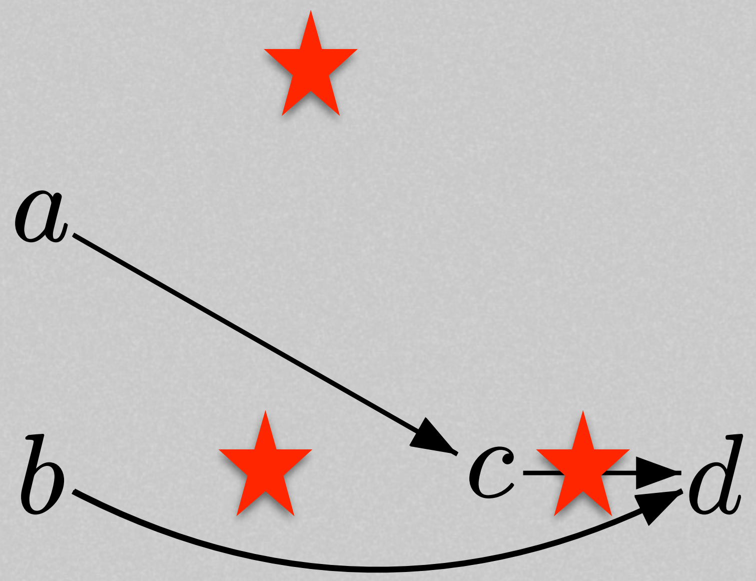
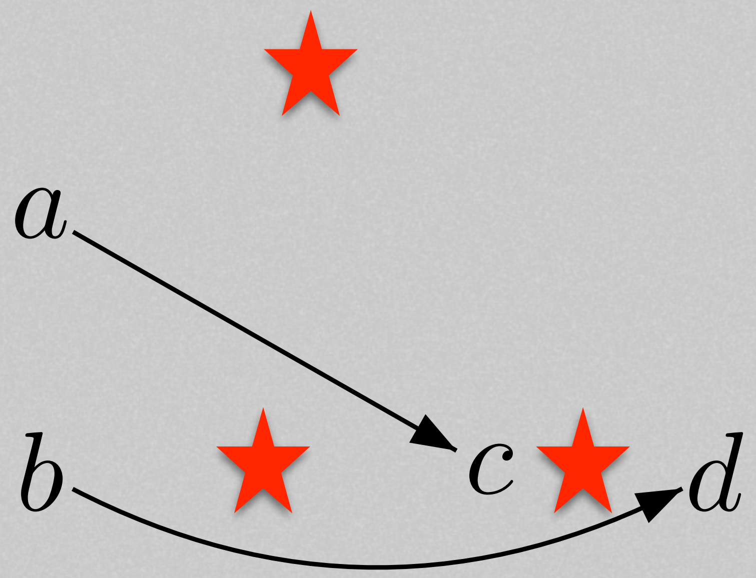
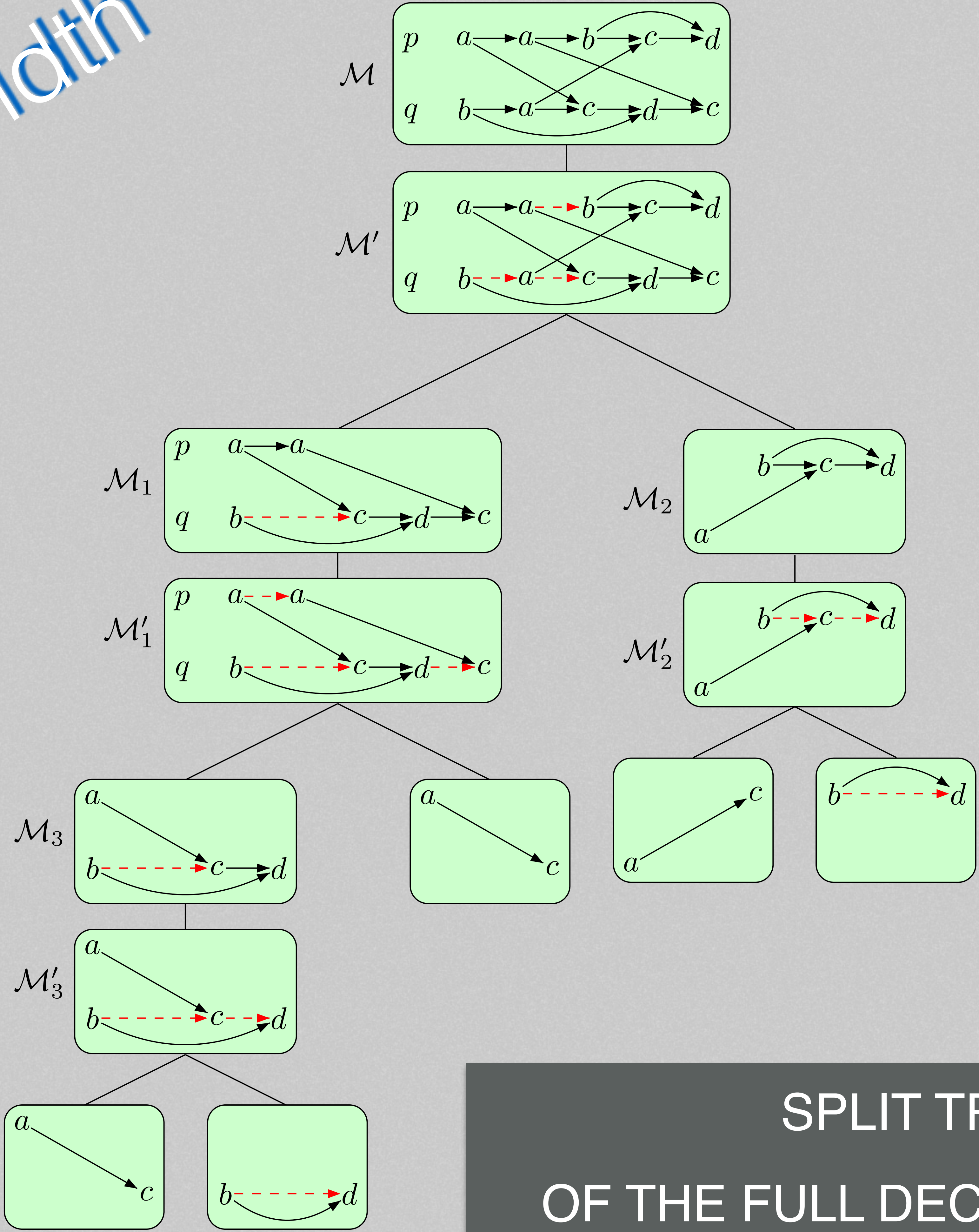


a
b
c
d


a
b
c
d

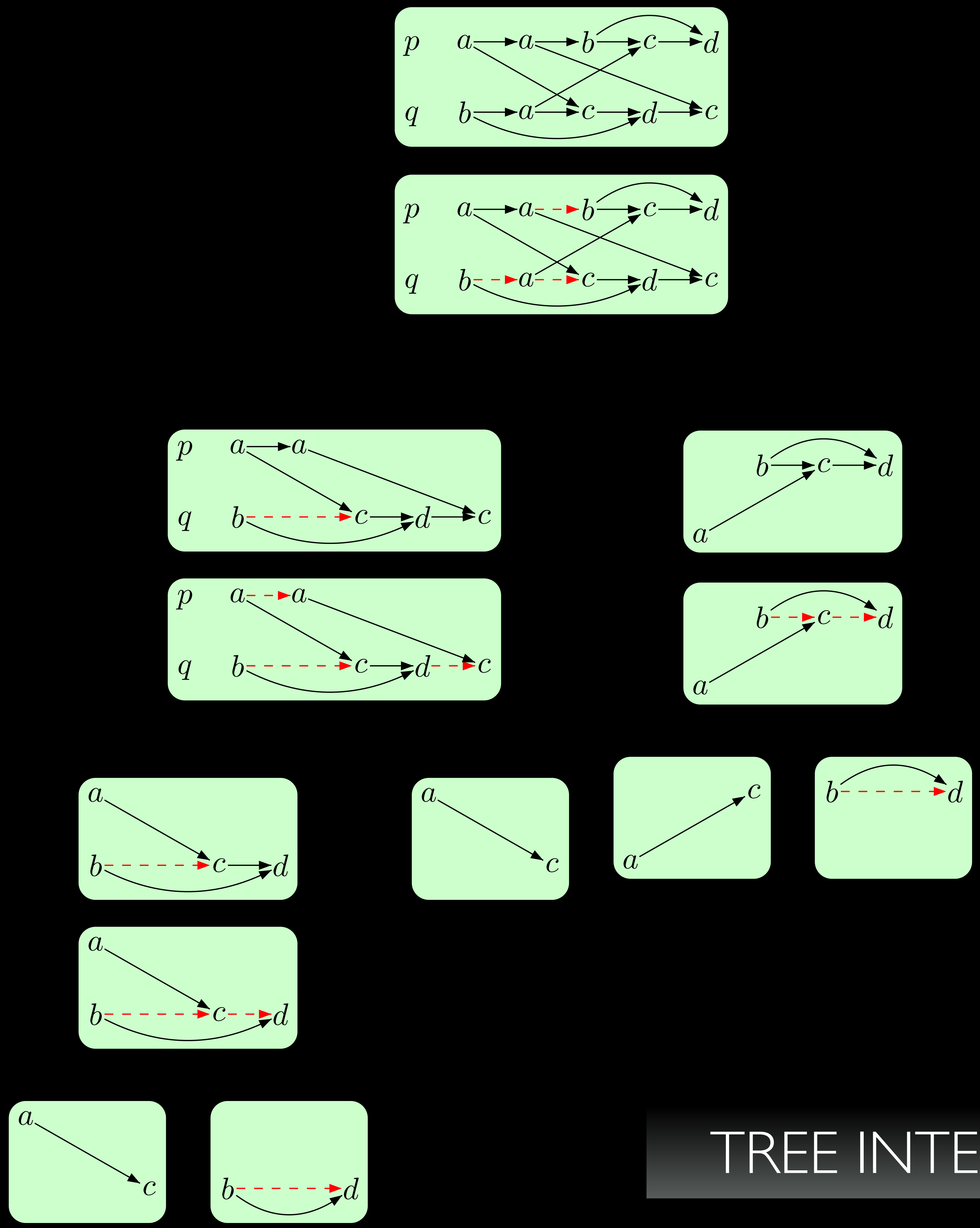
a
c
b
d

Split-width

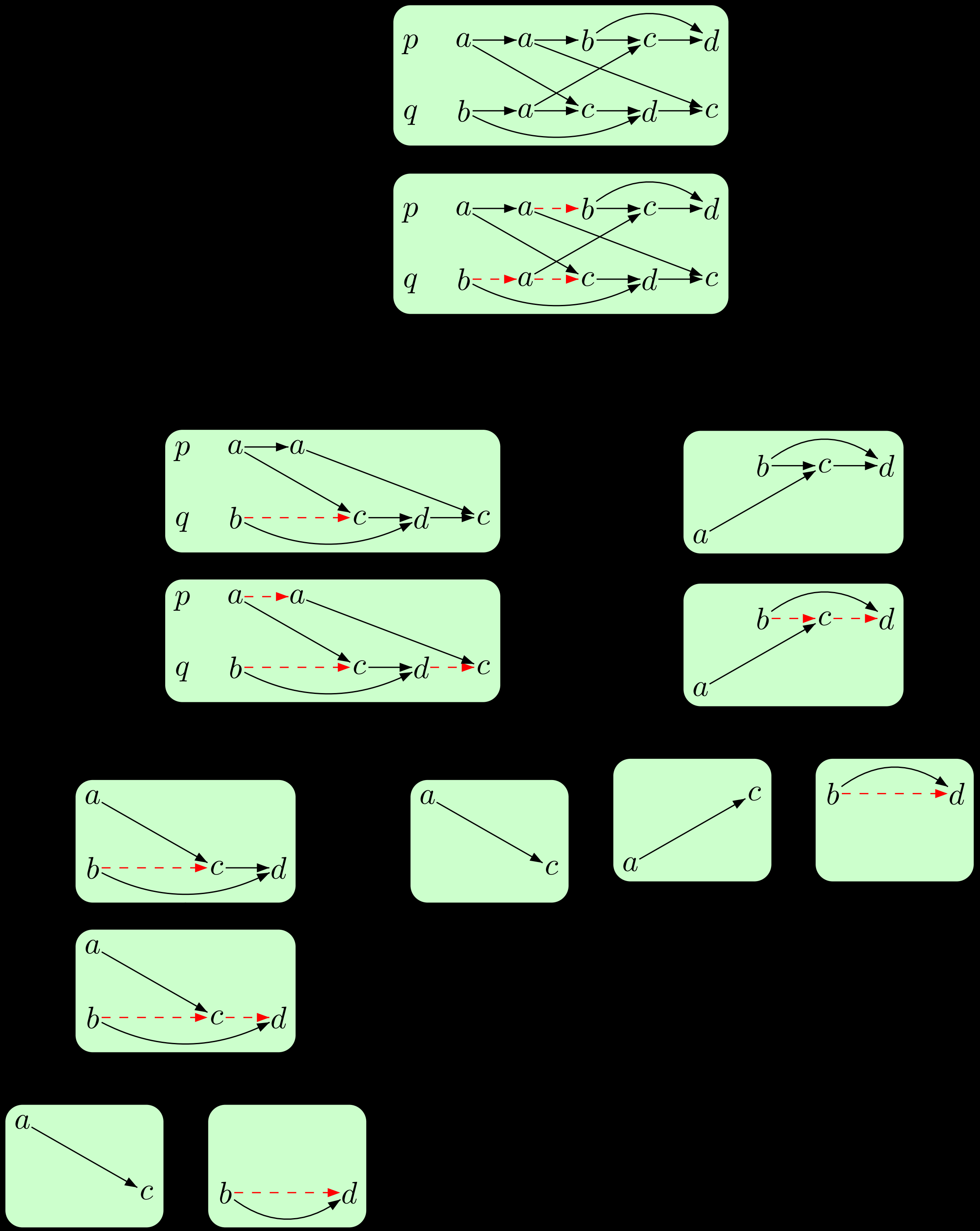


SPLIT TREE

OF THE FULL DECOMPOSITION



TREE INTERPRETATION



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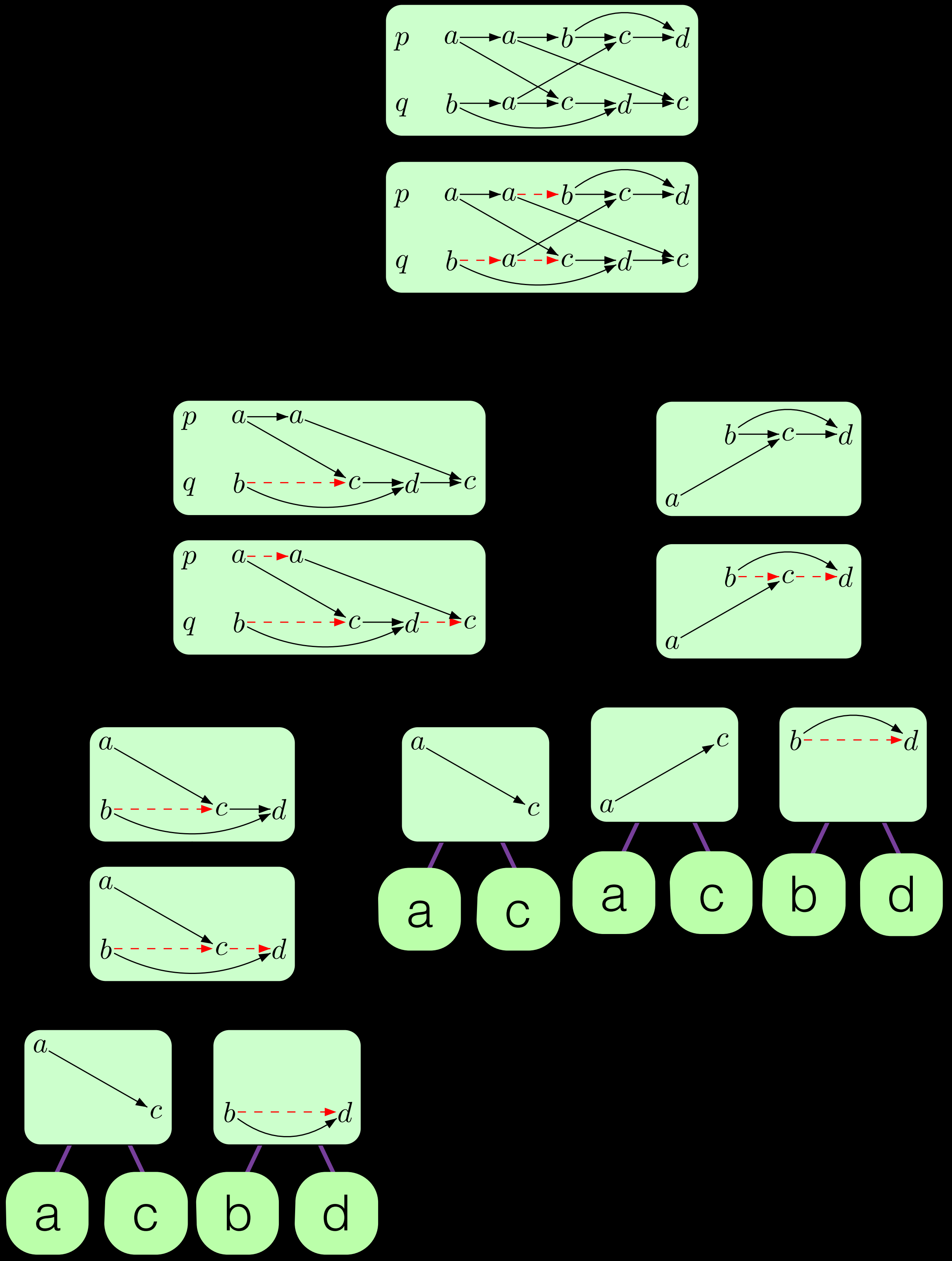

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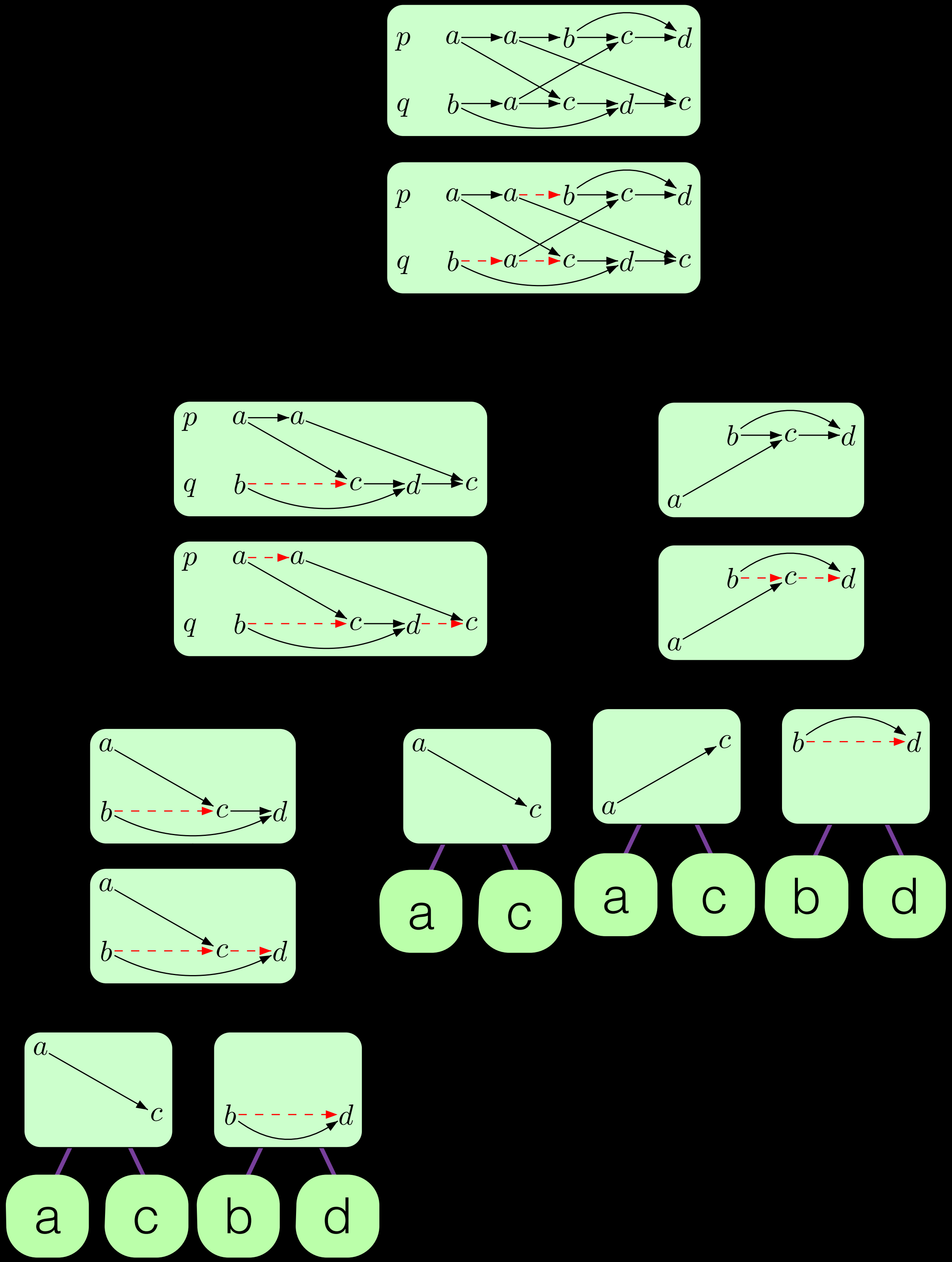

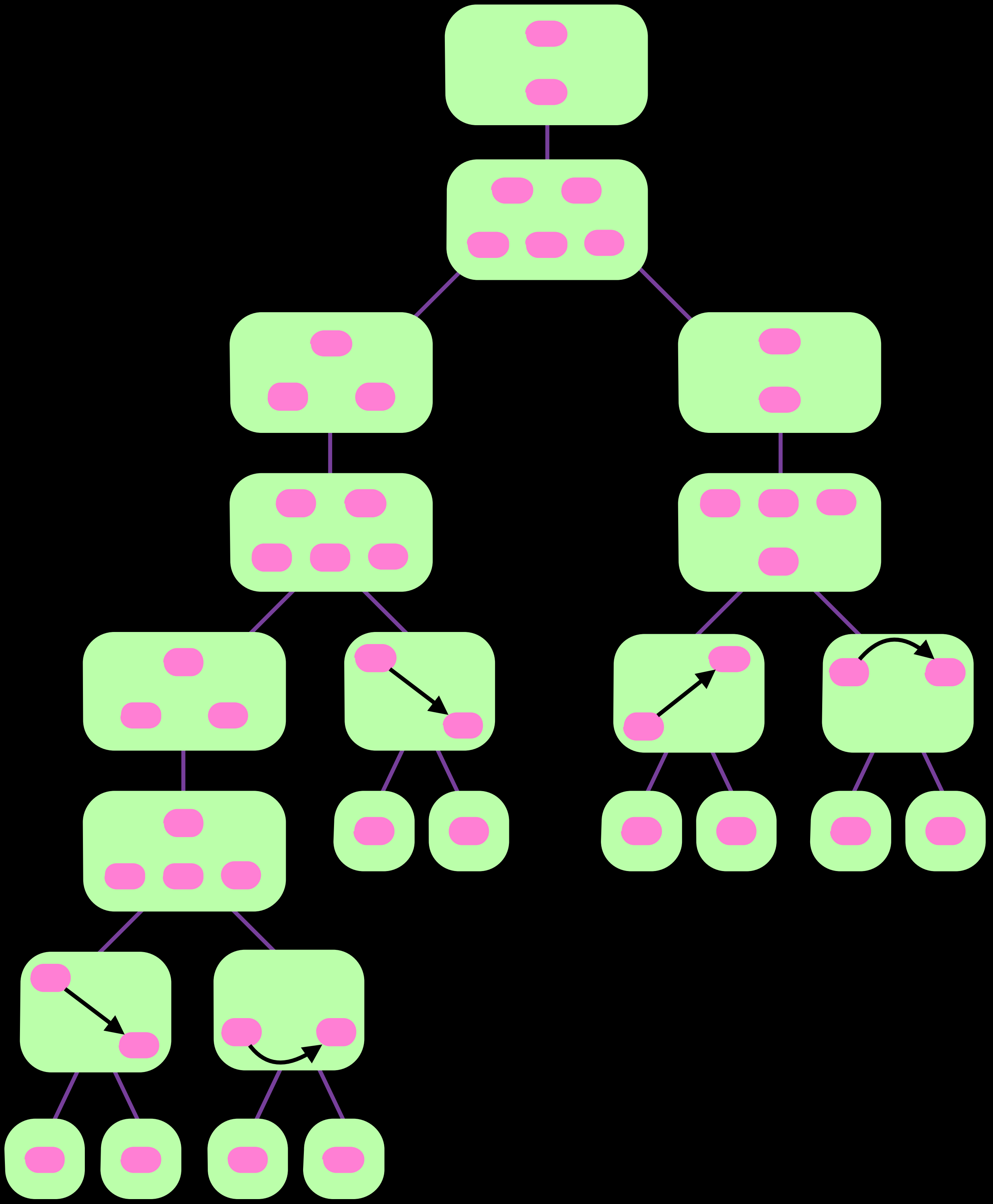

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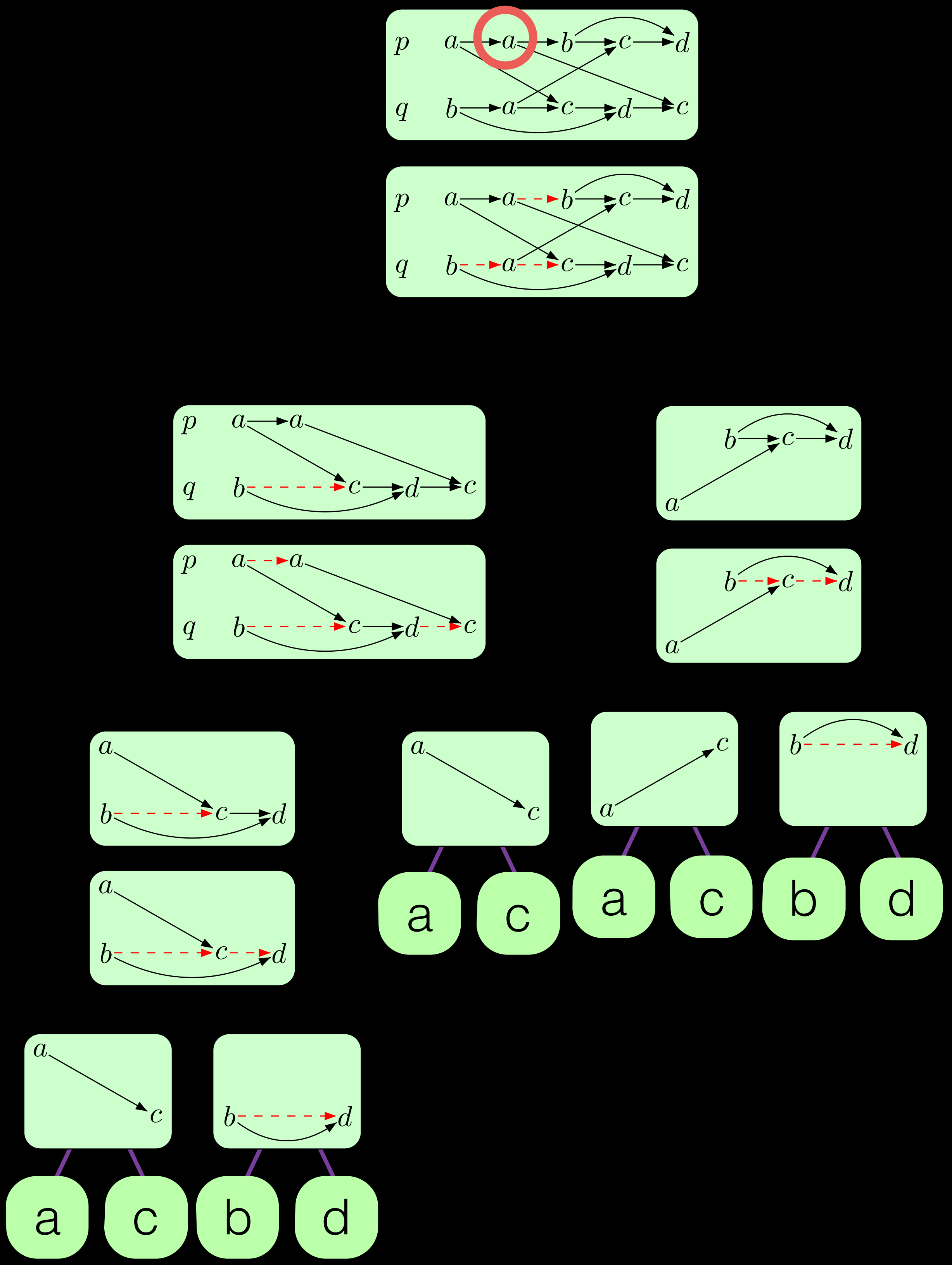

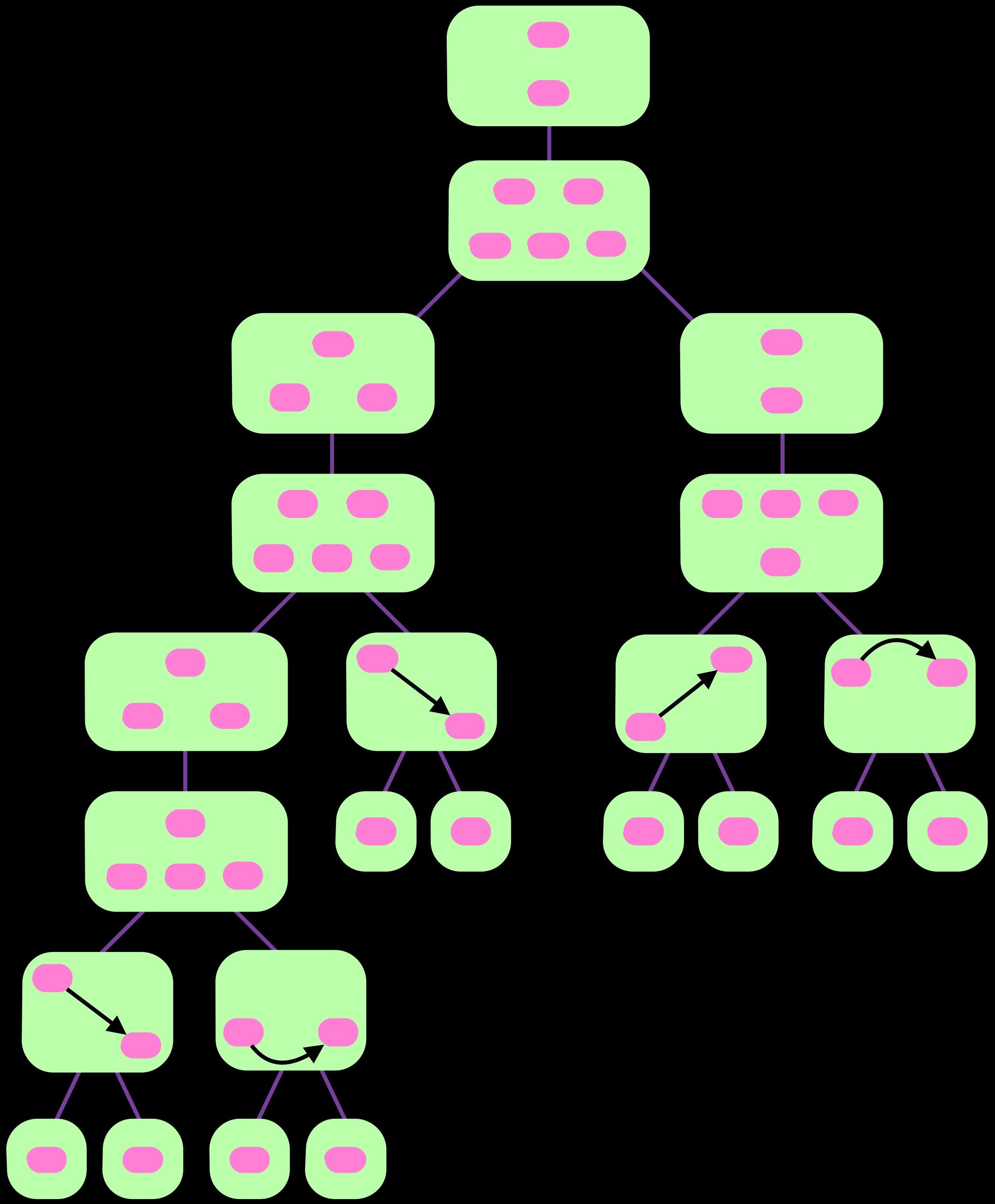

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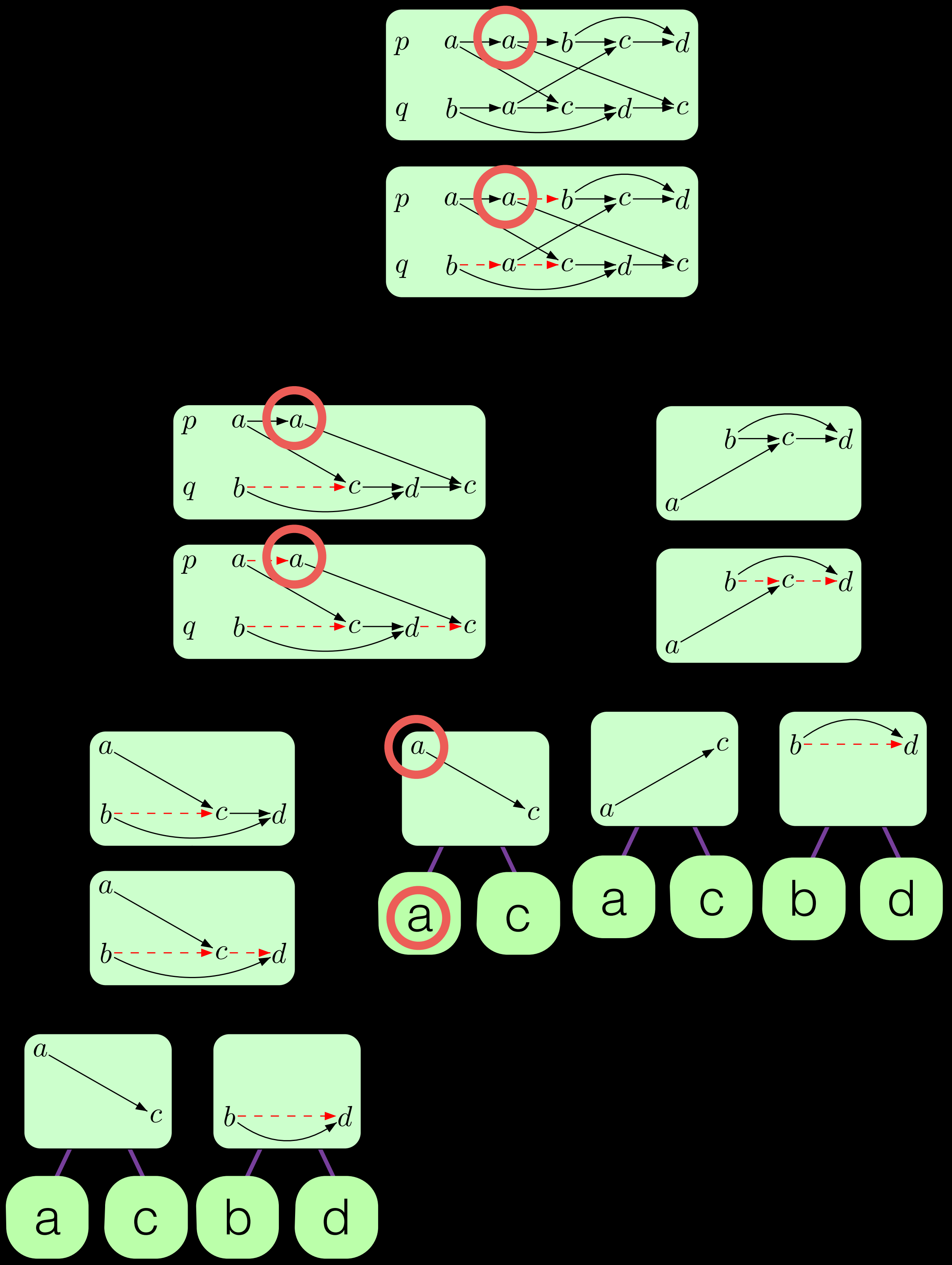

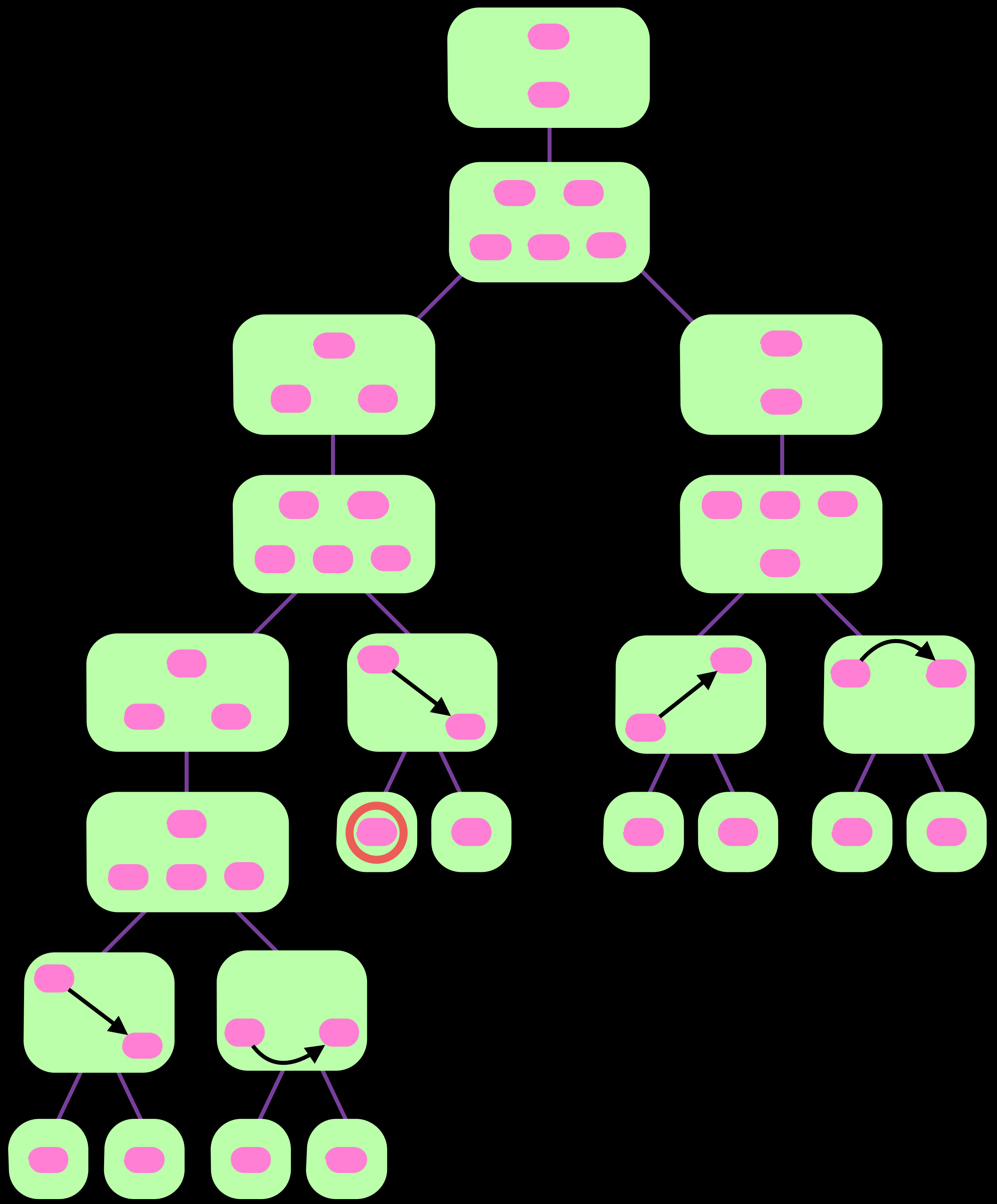

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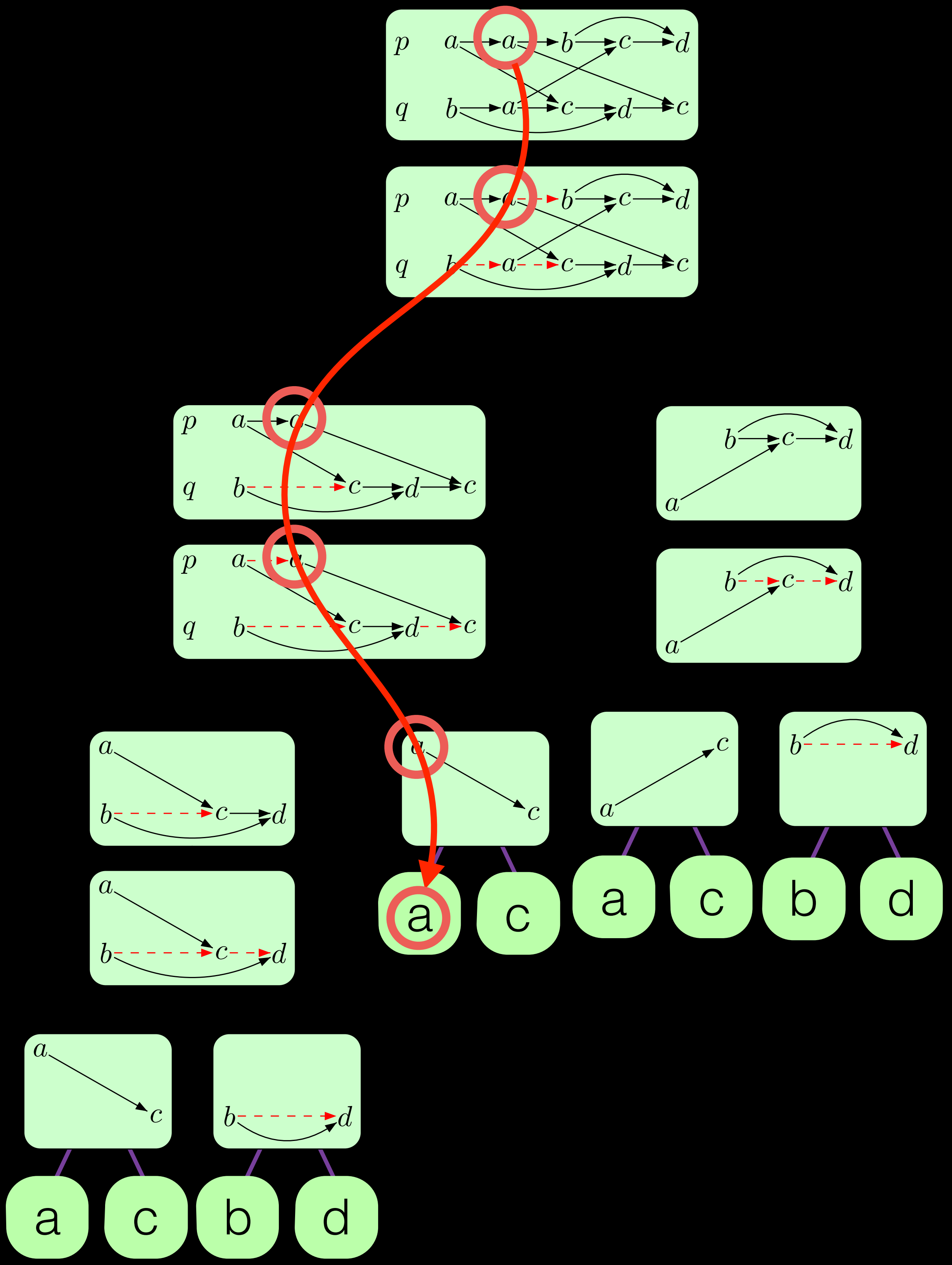

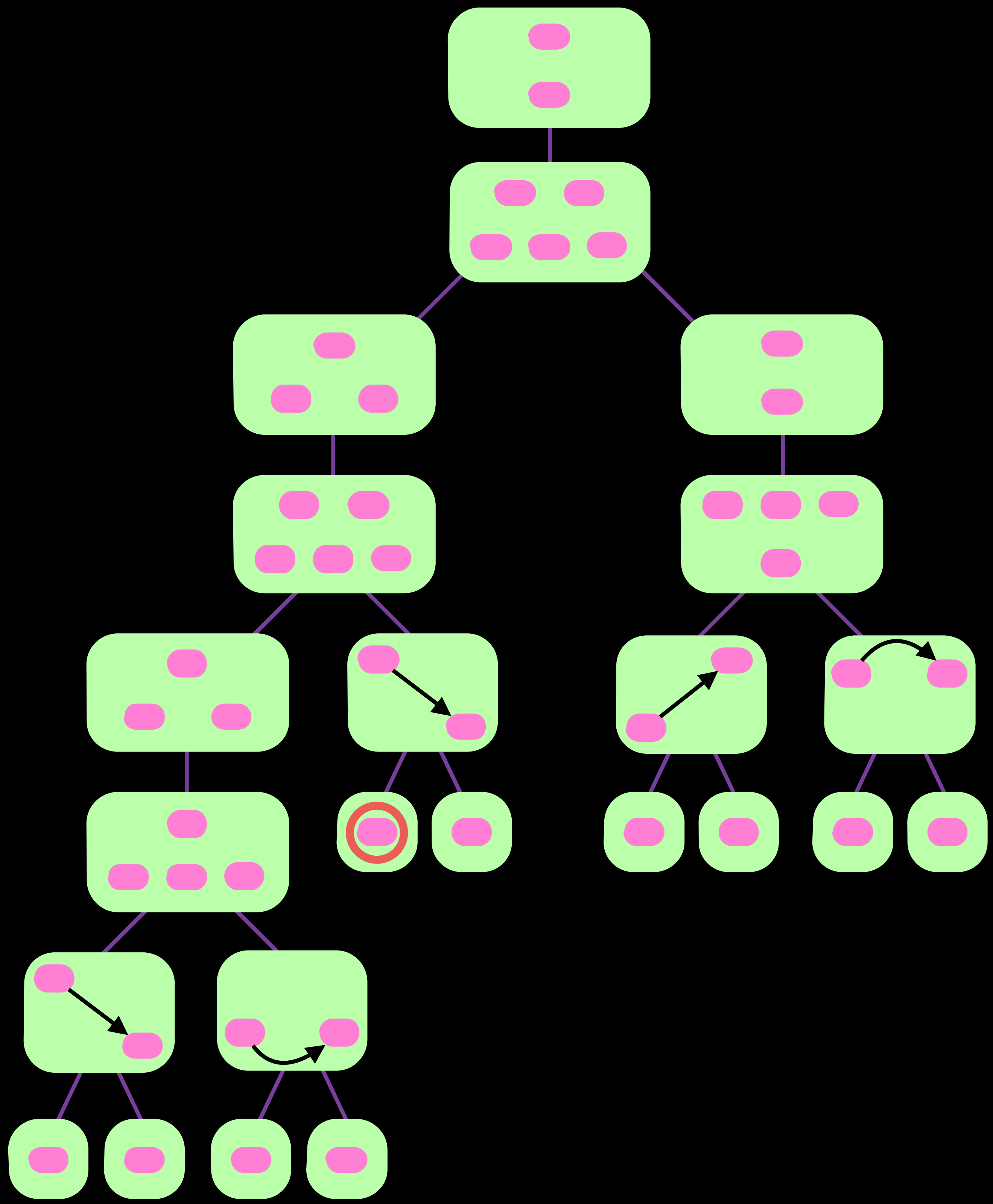

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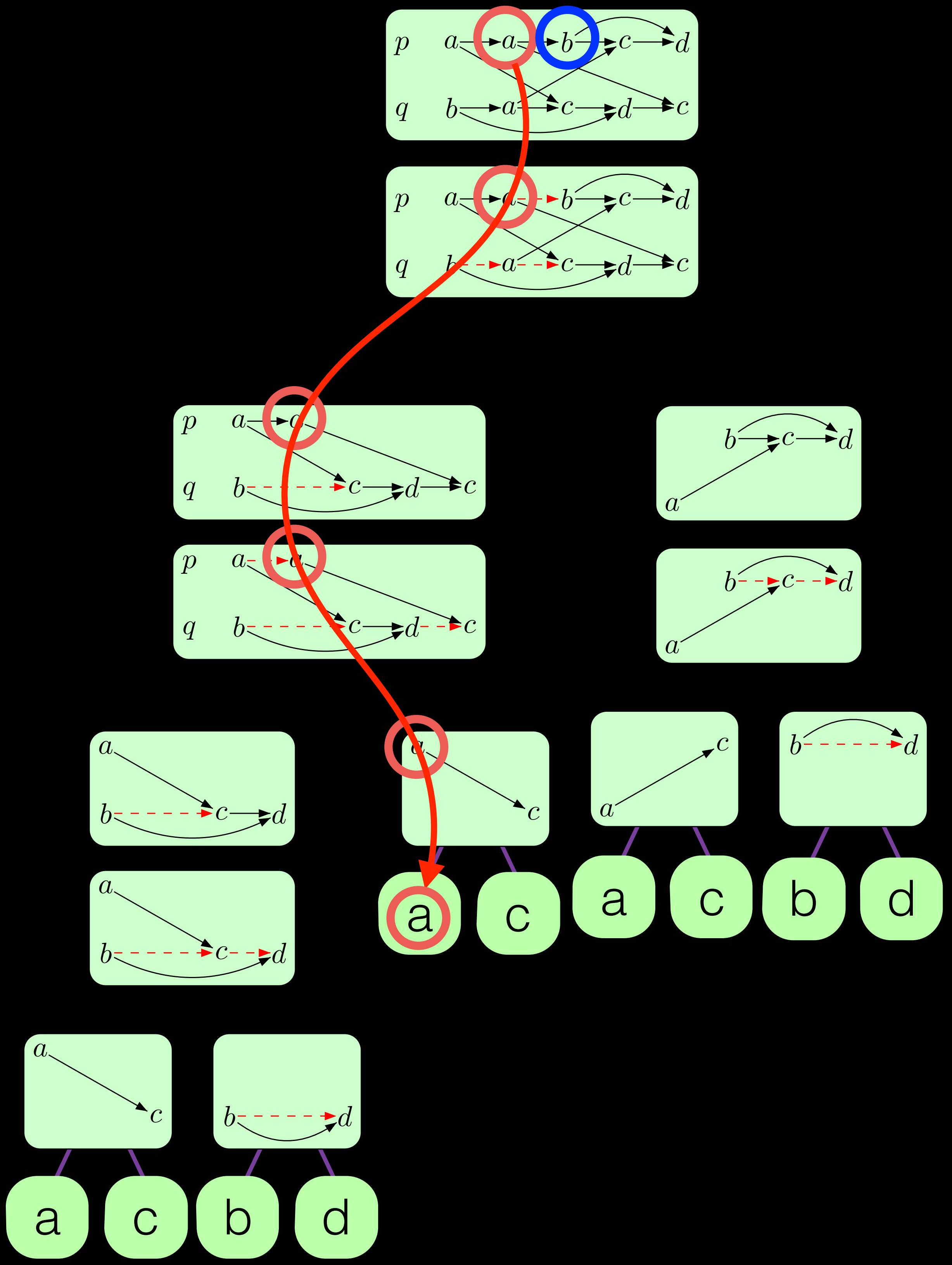

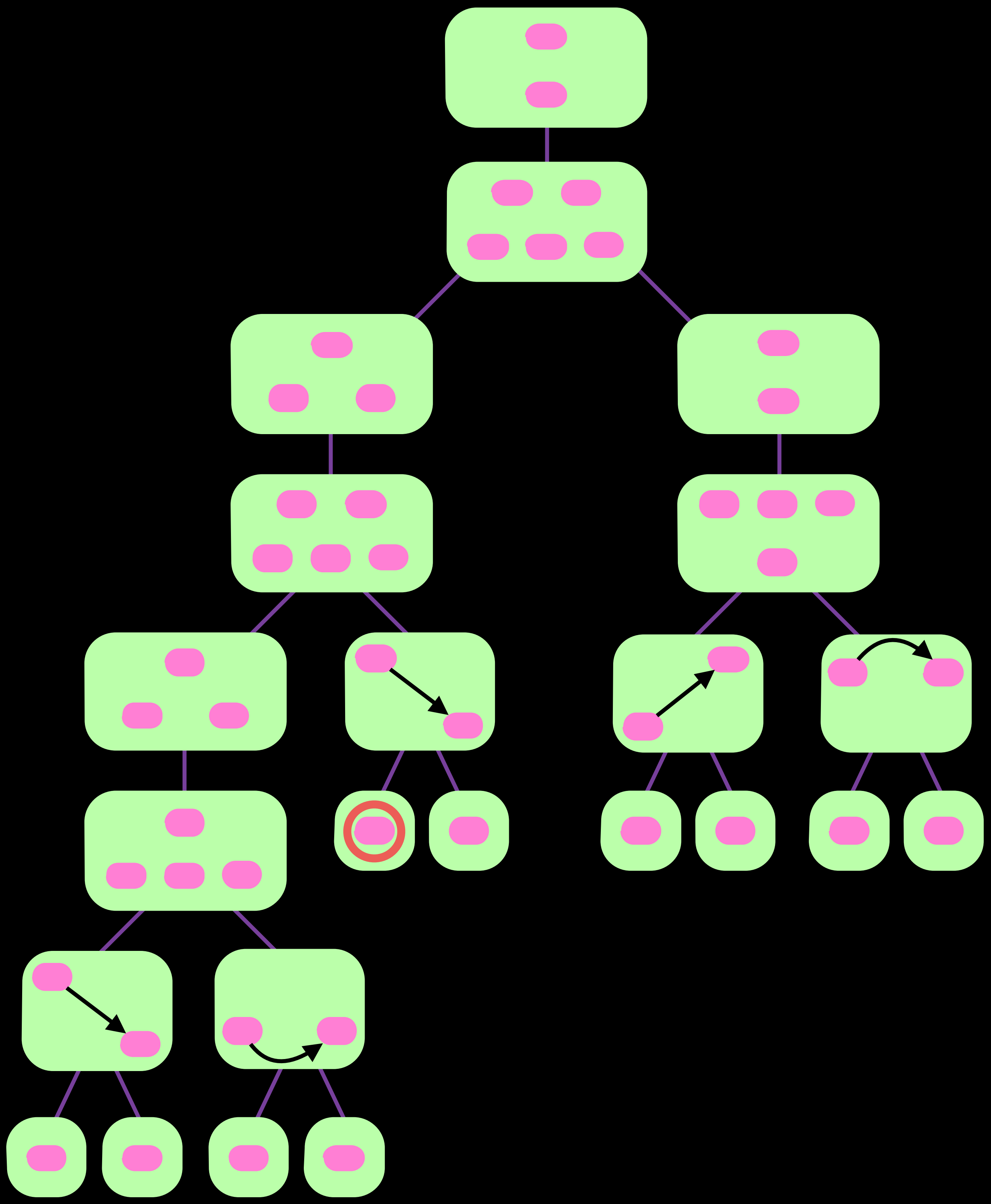

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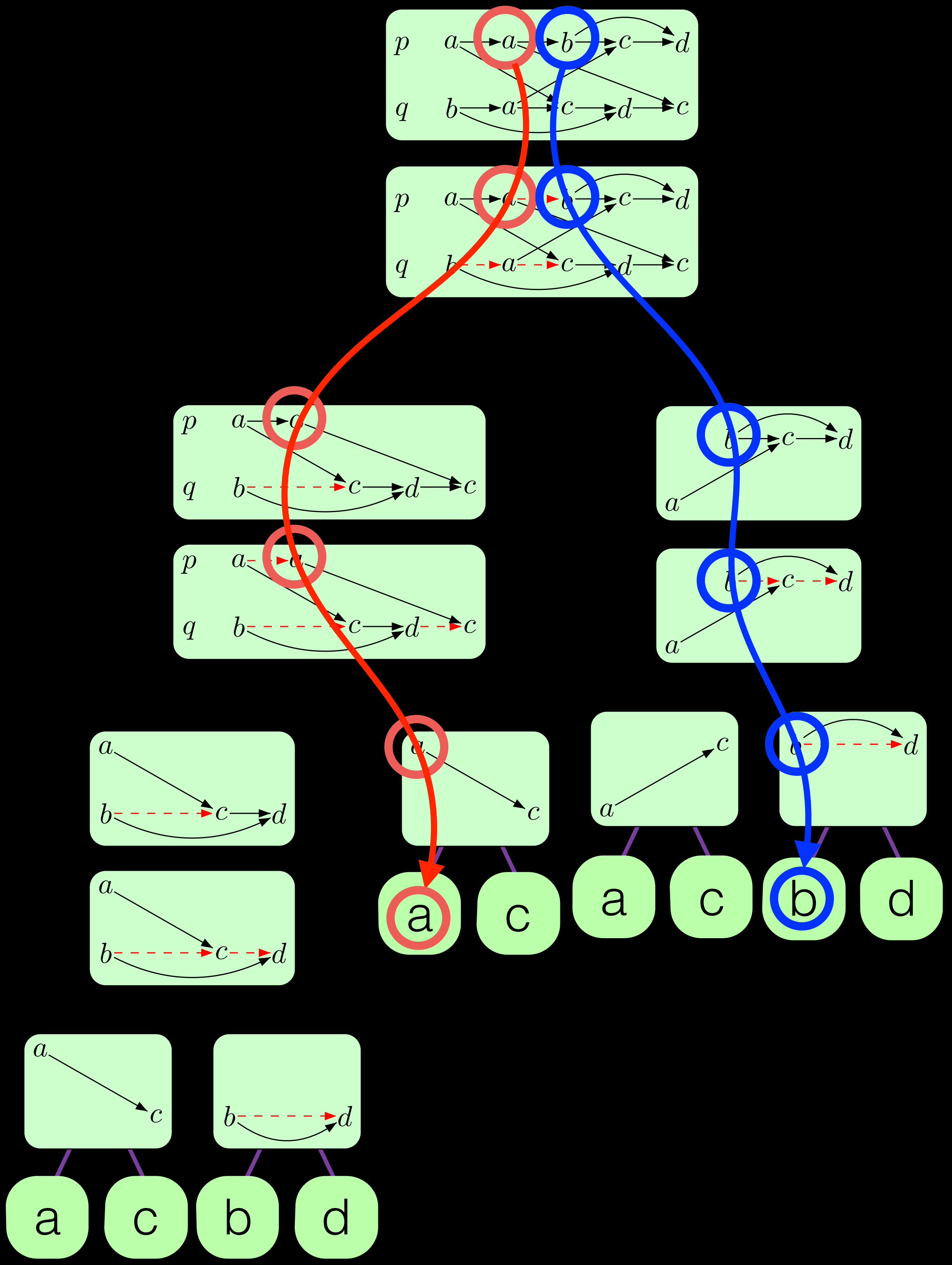

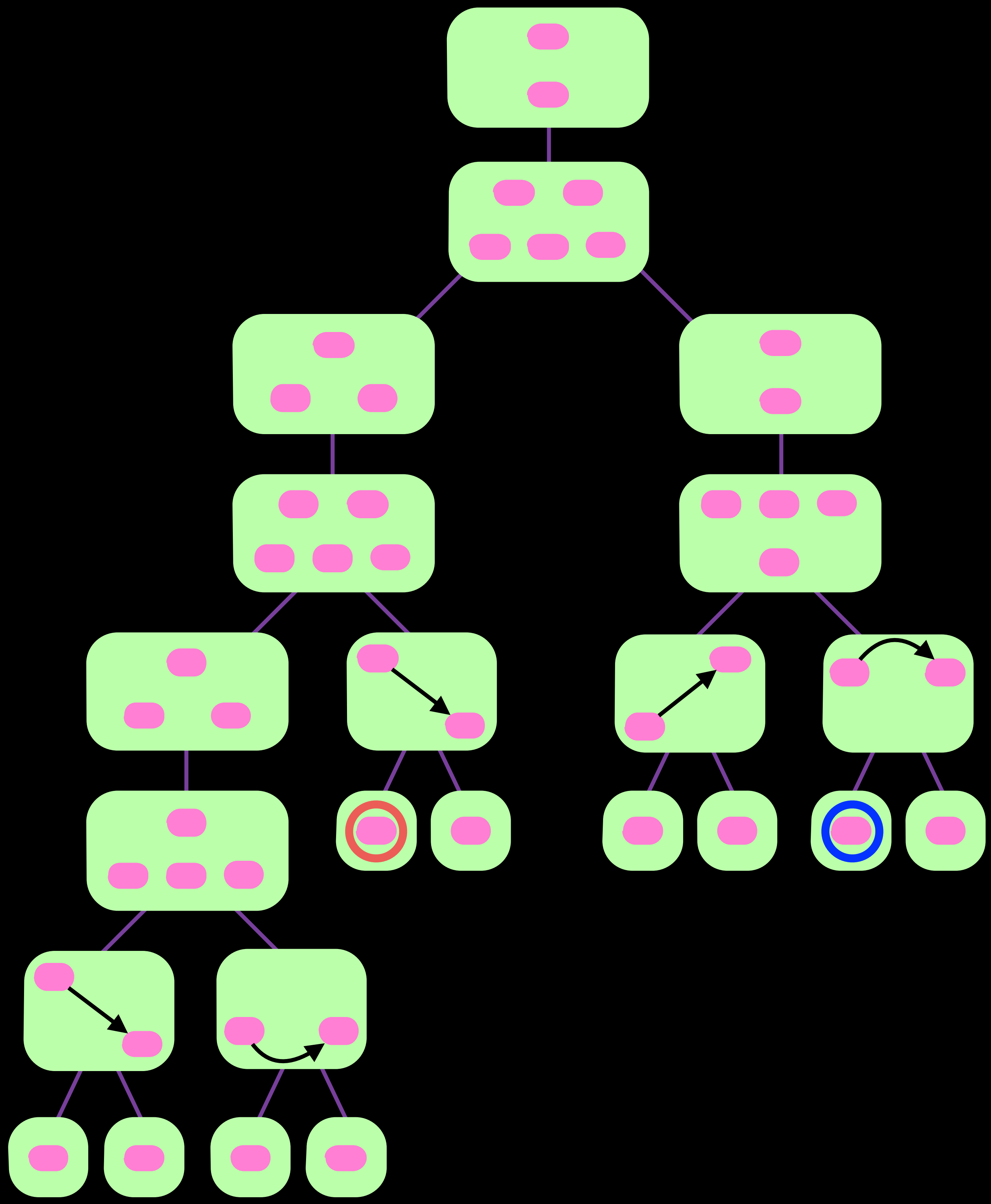

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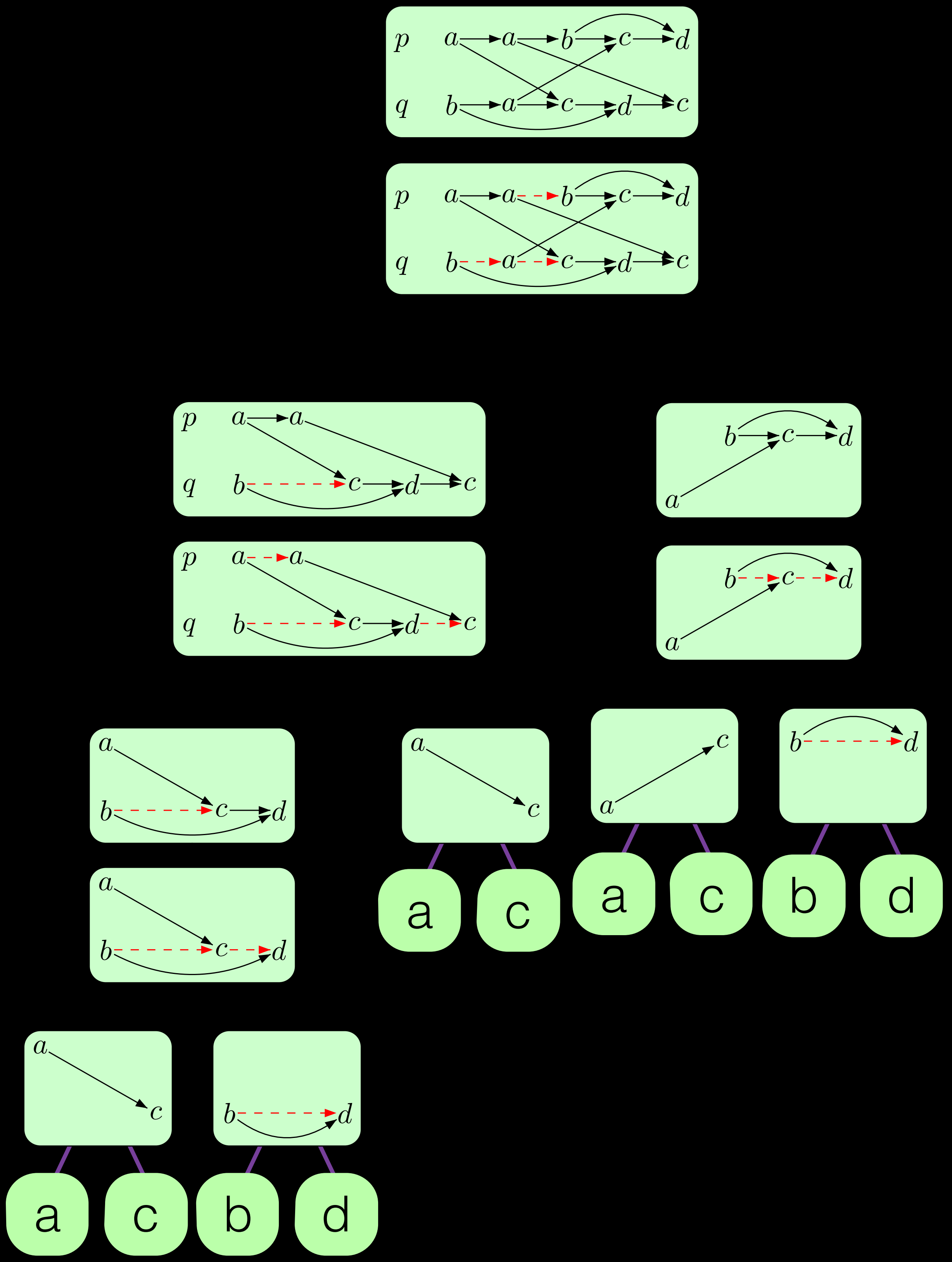

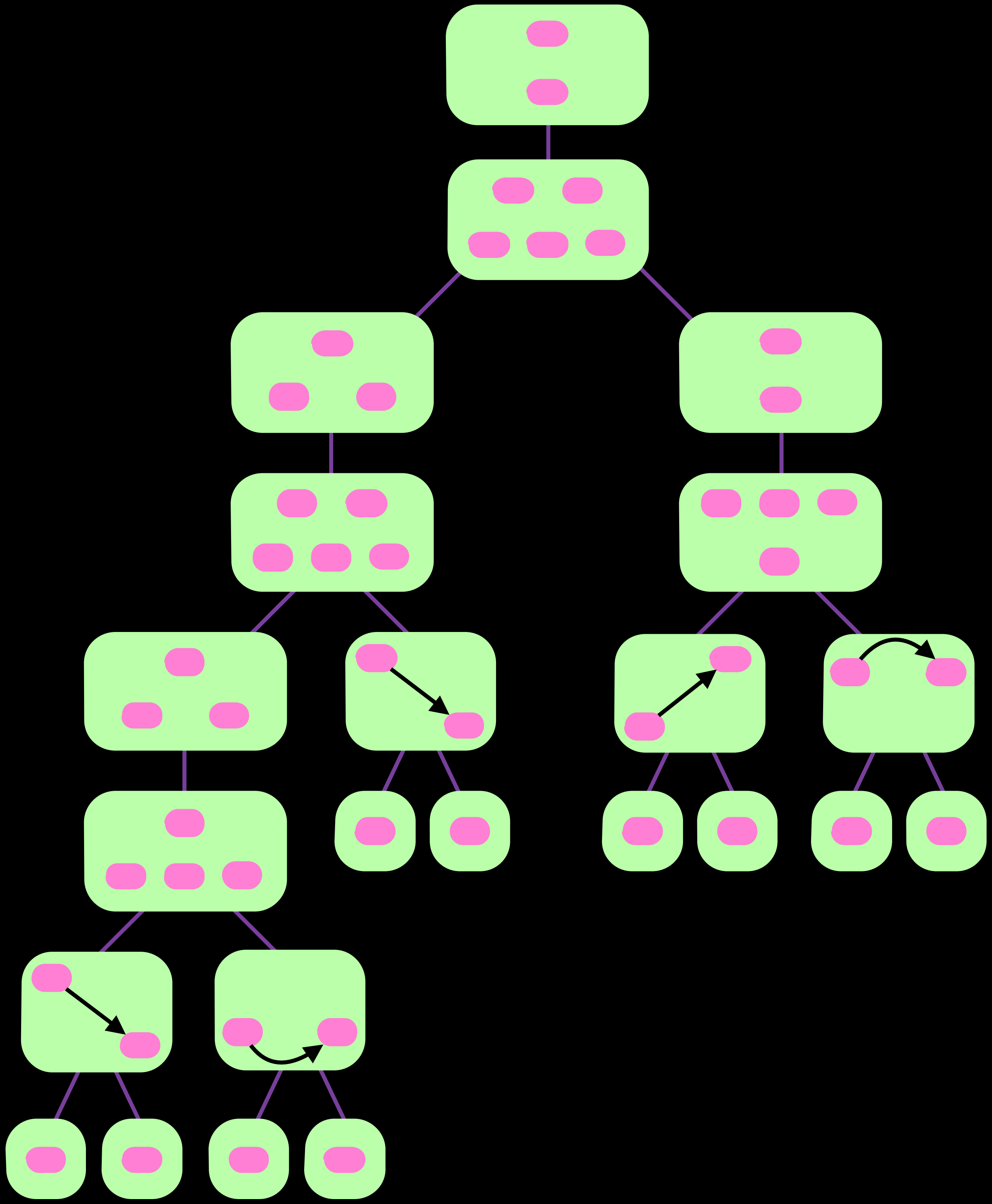

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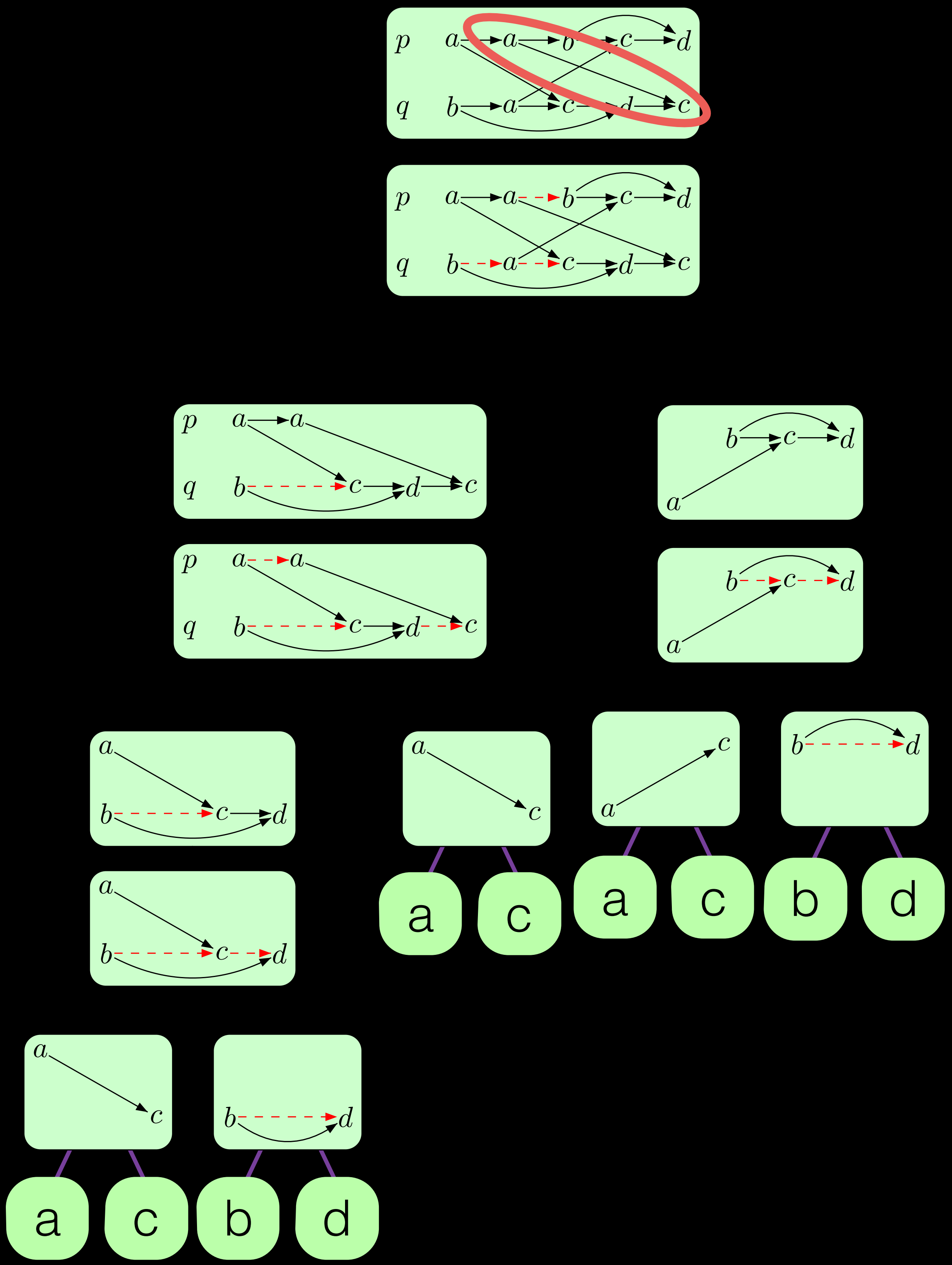

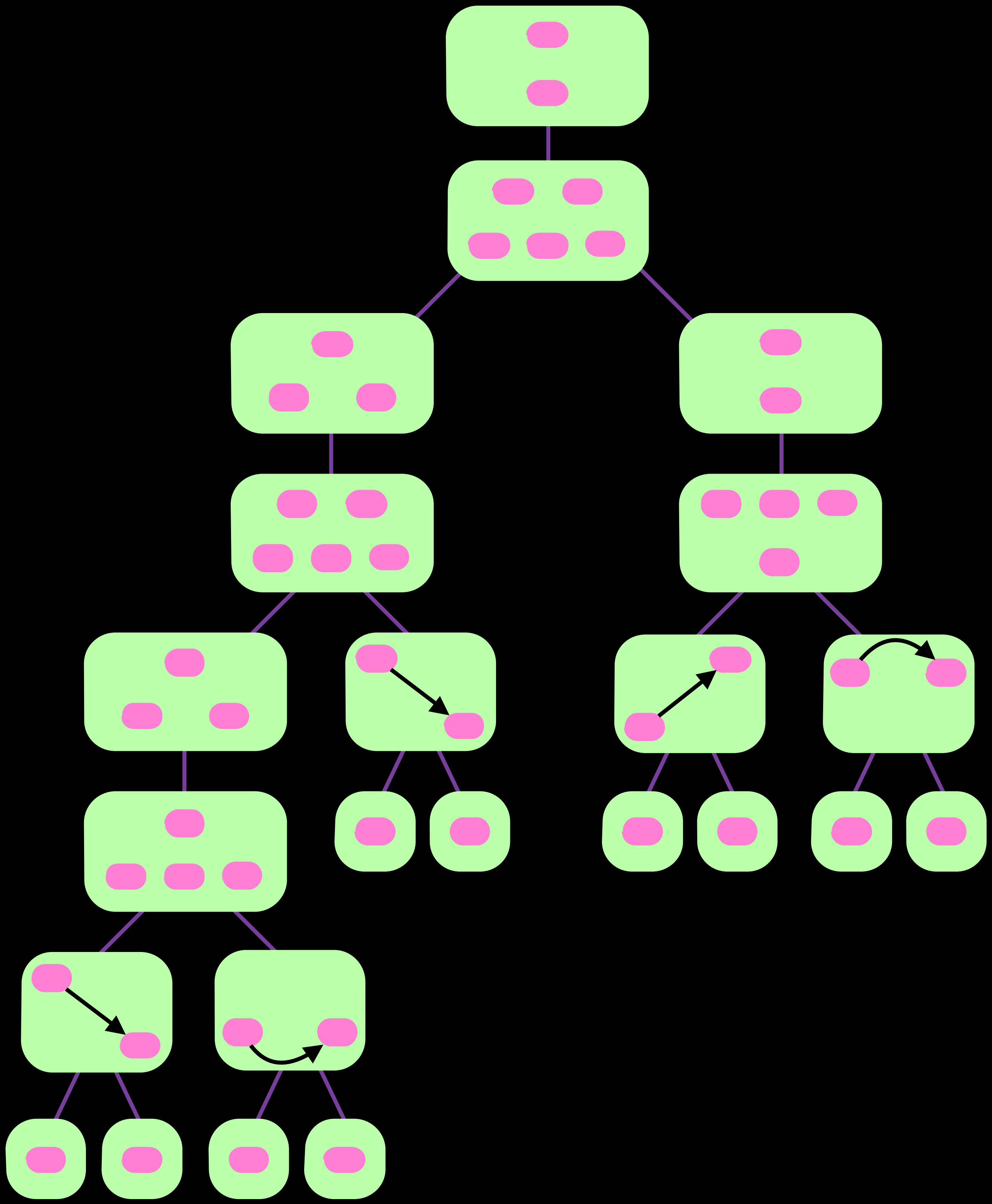

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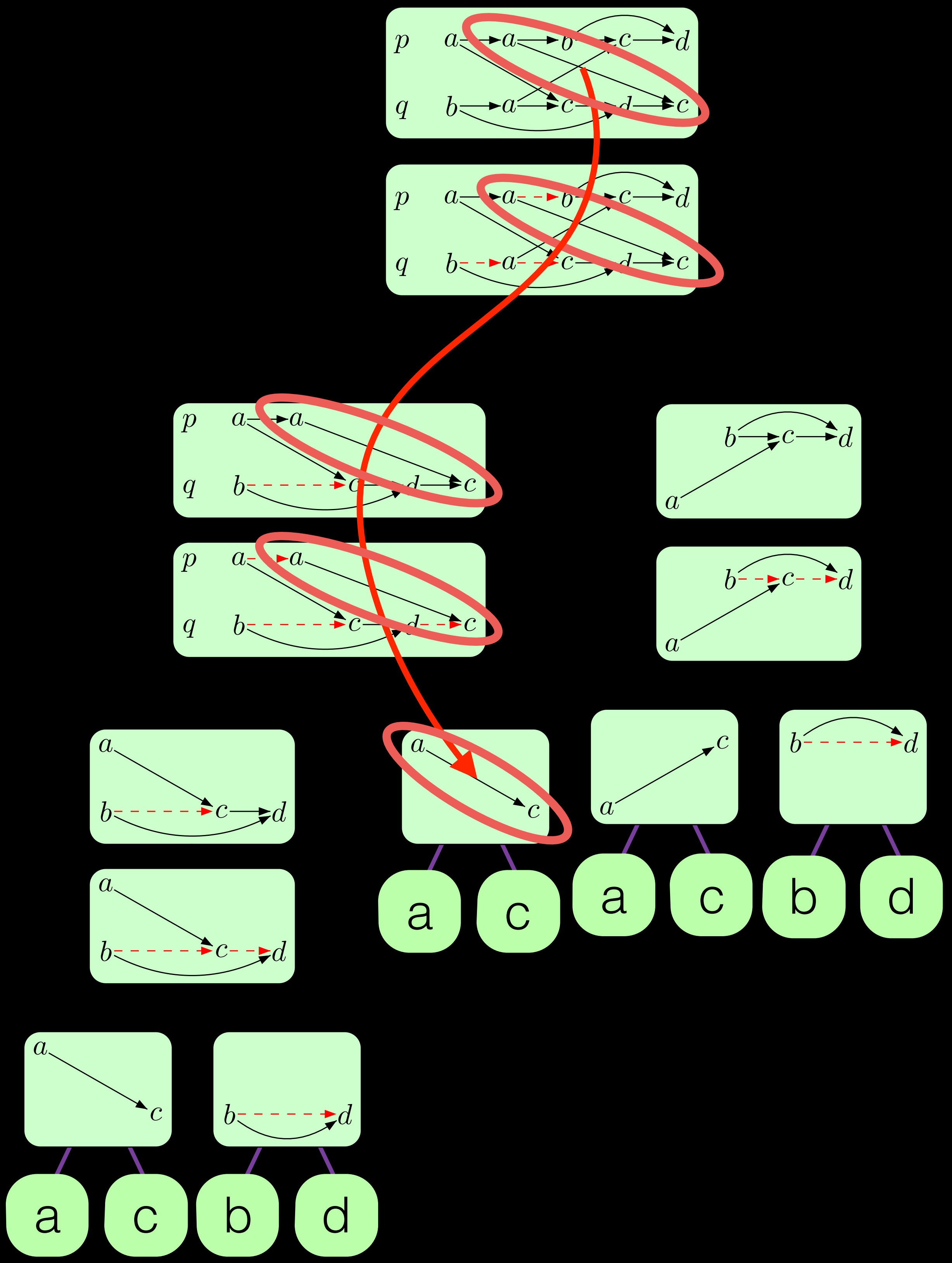

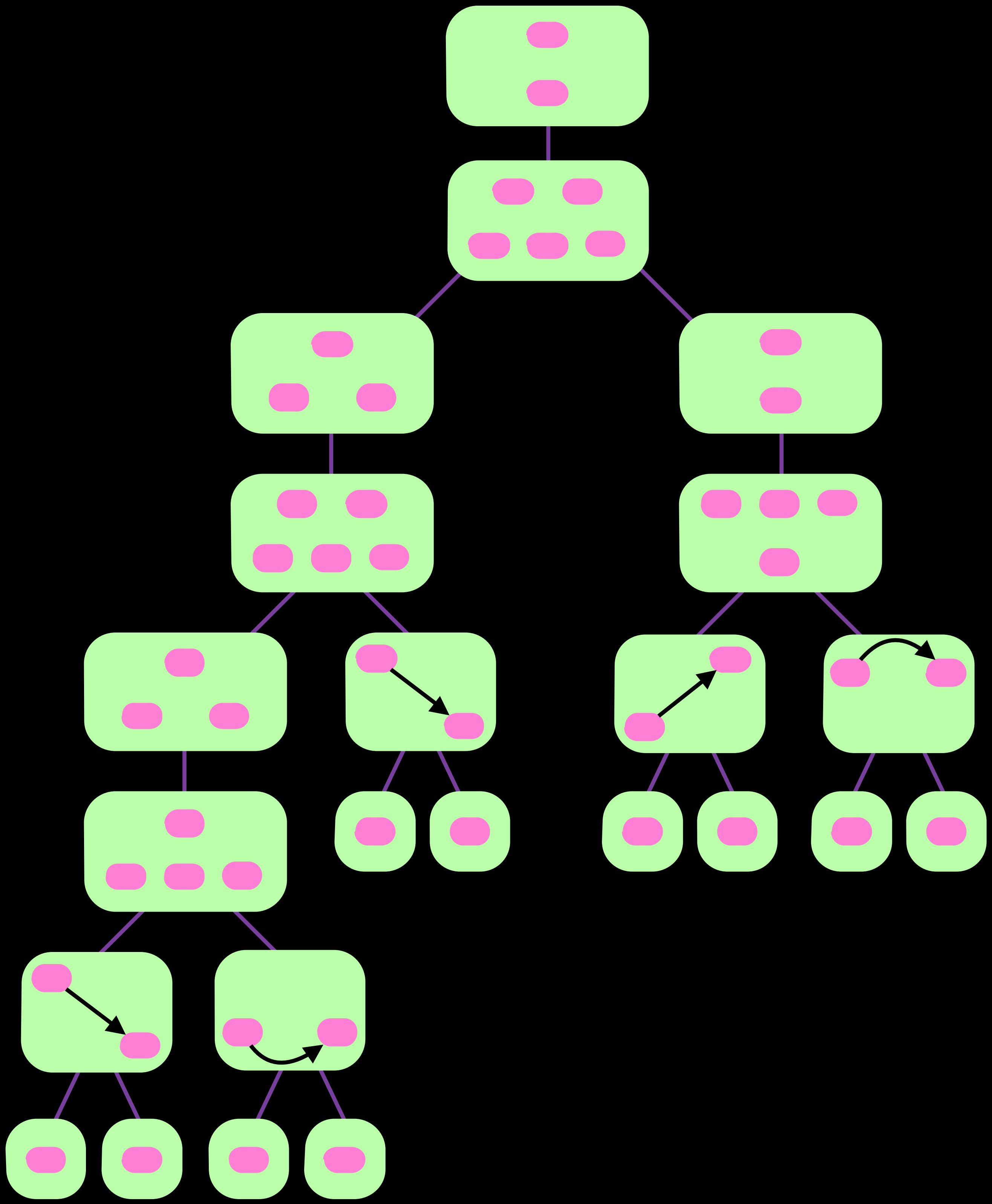

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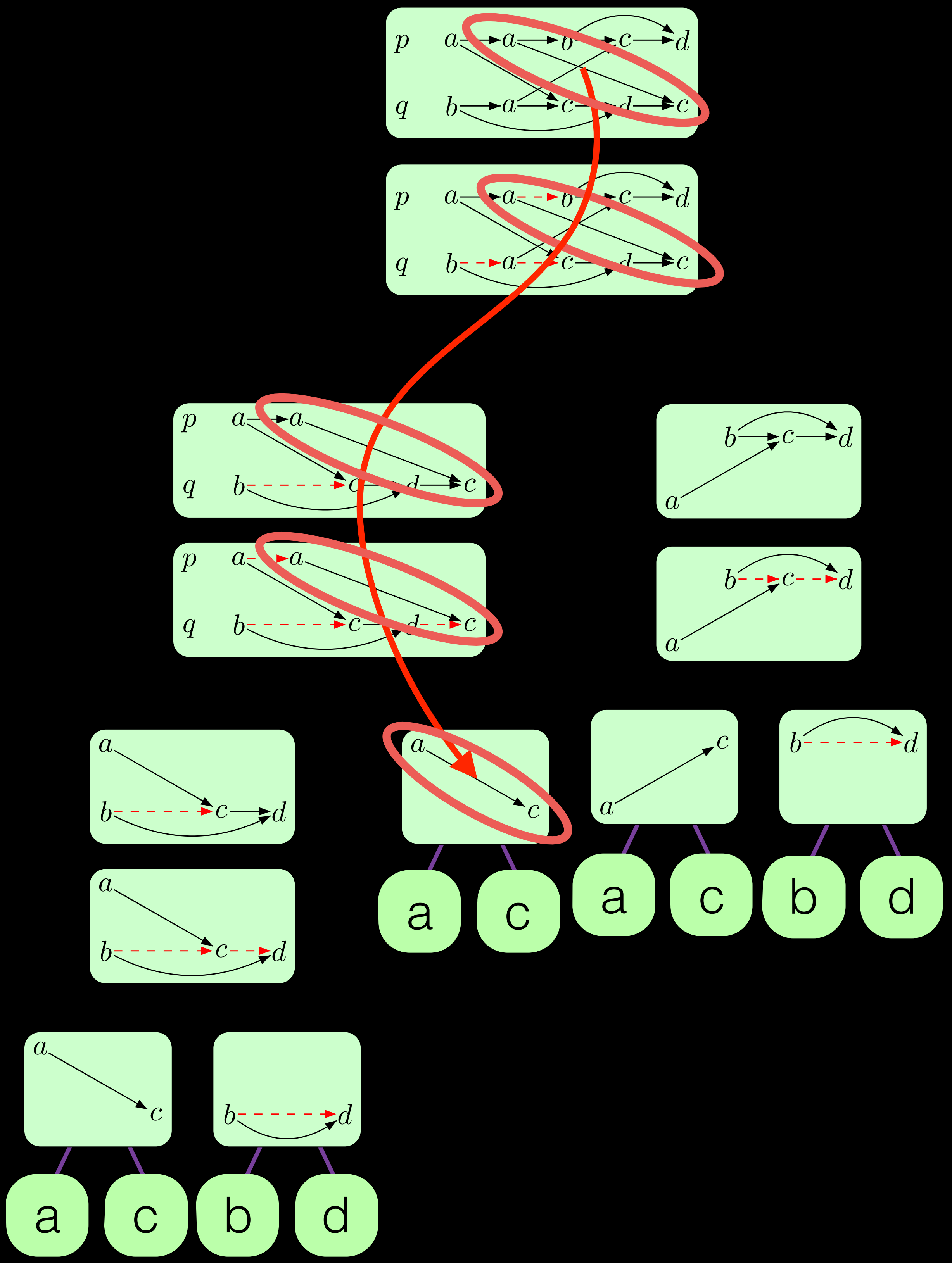

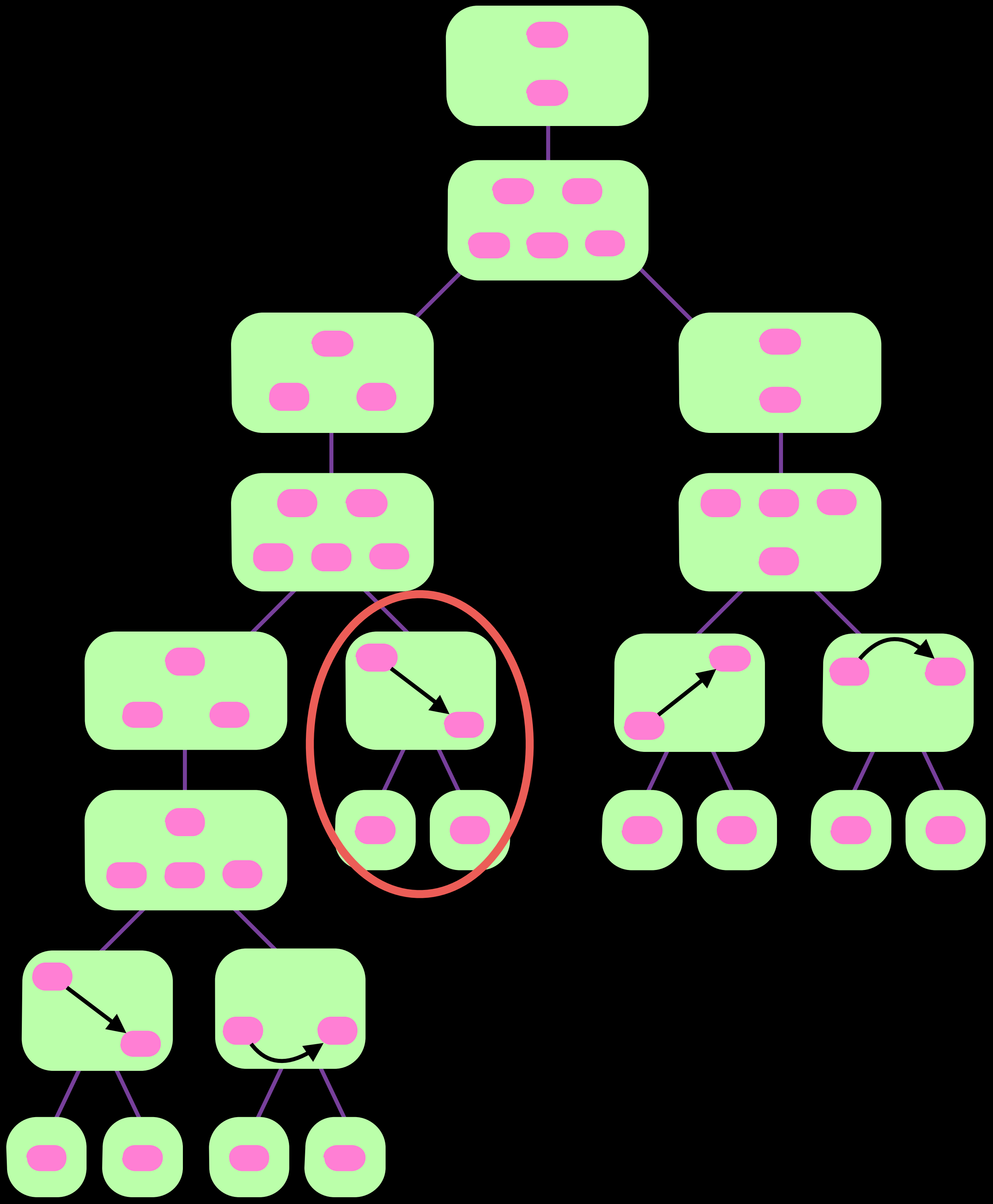

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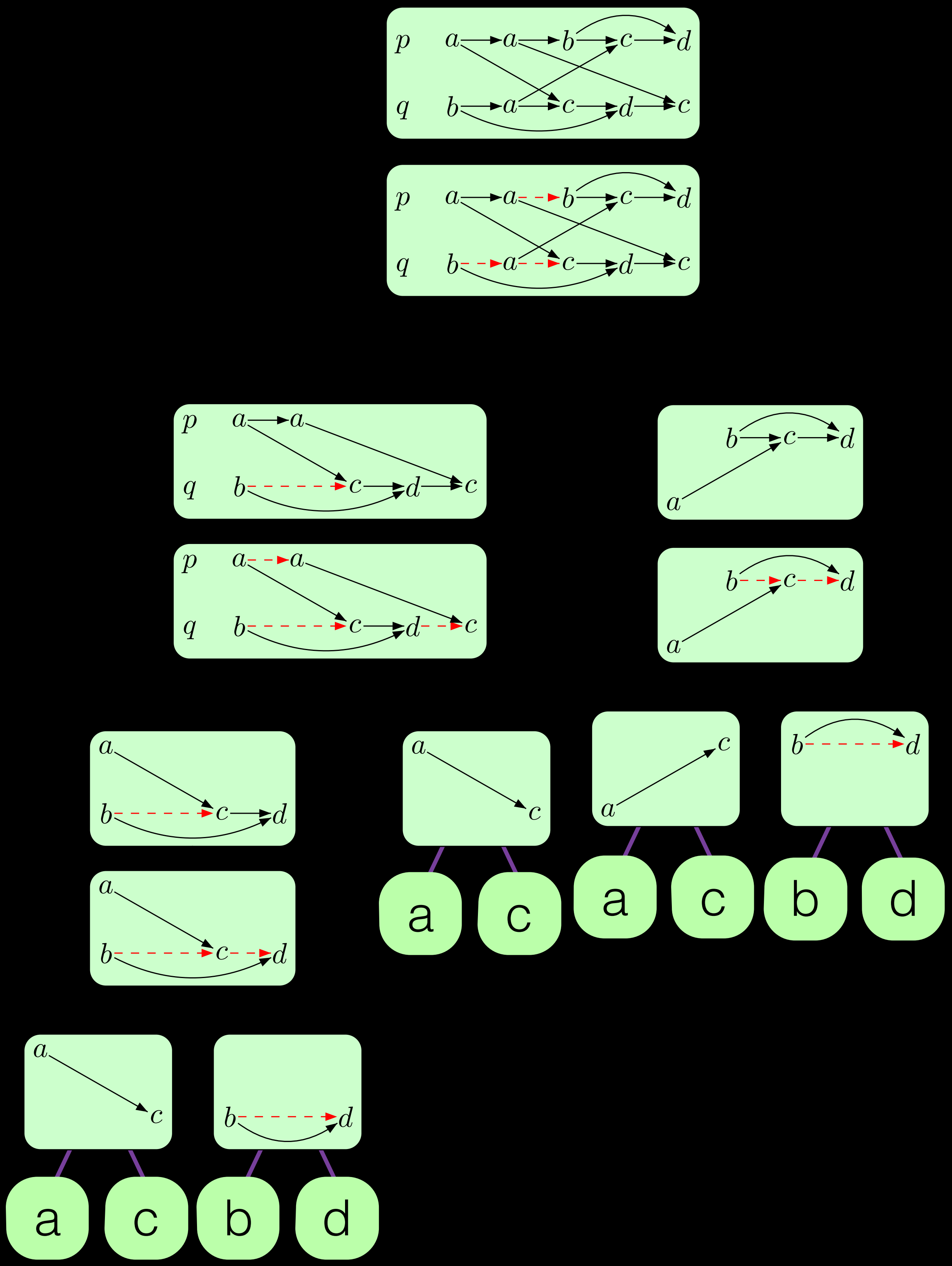

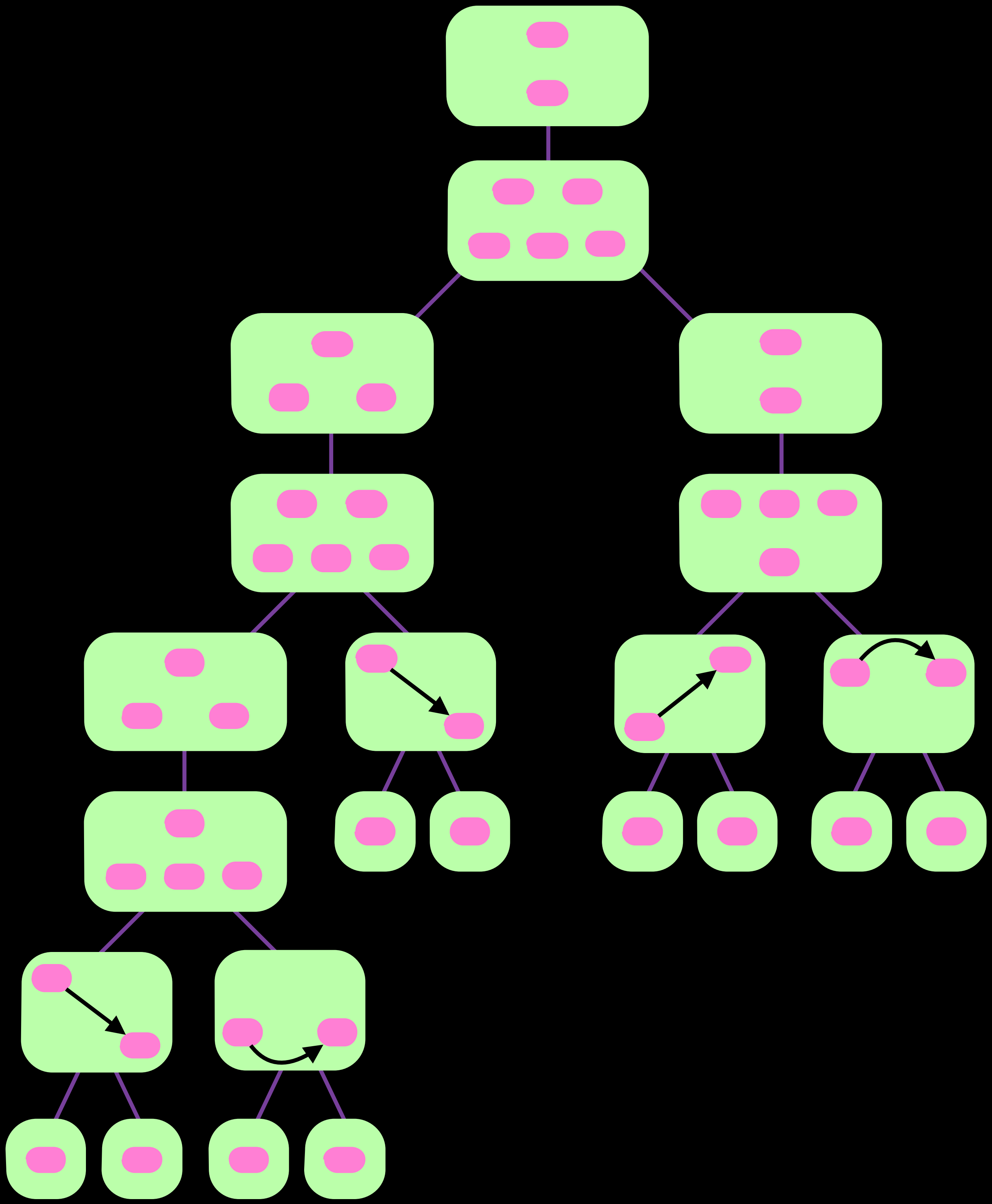

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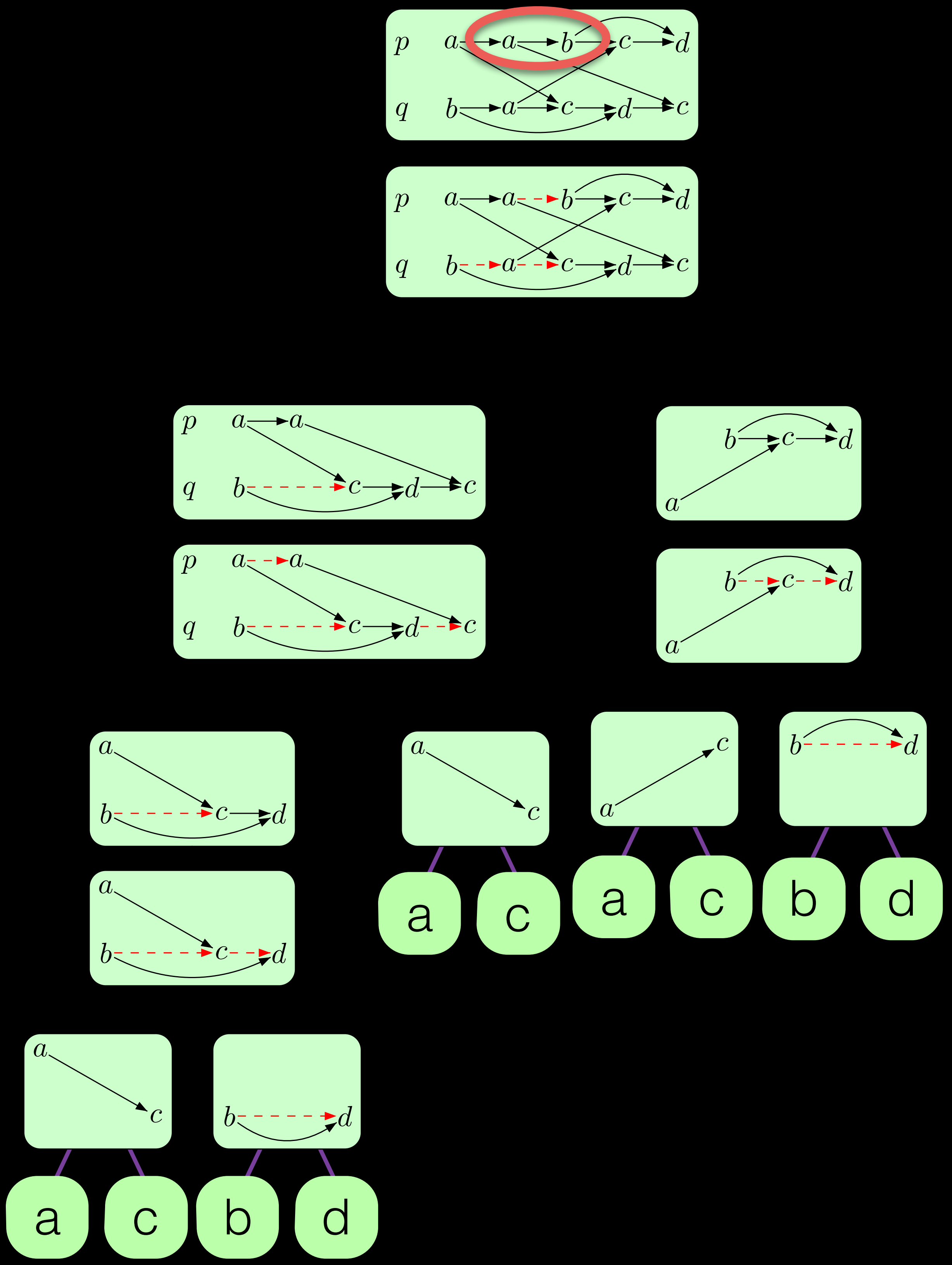

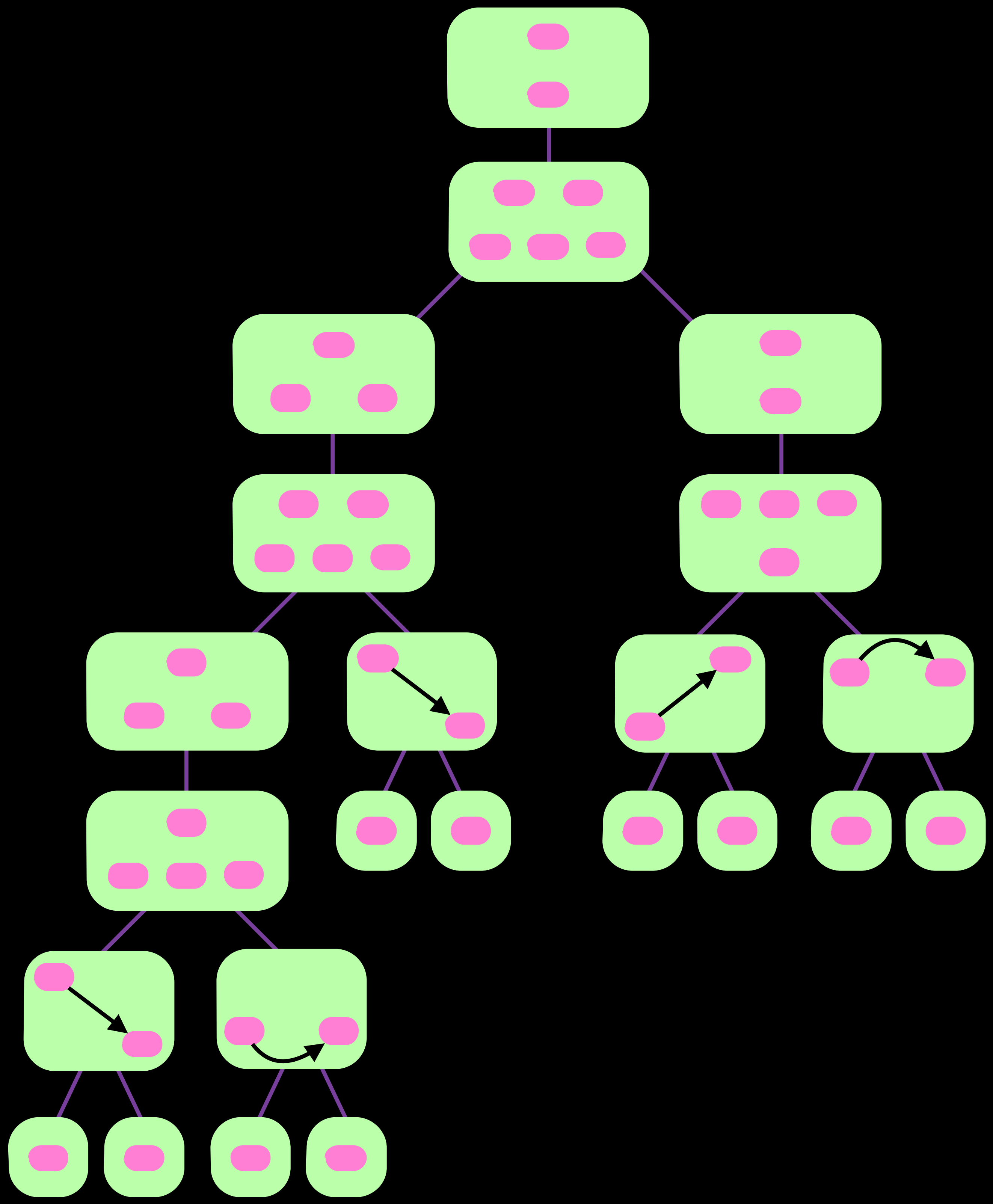

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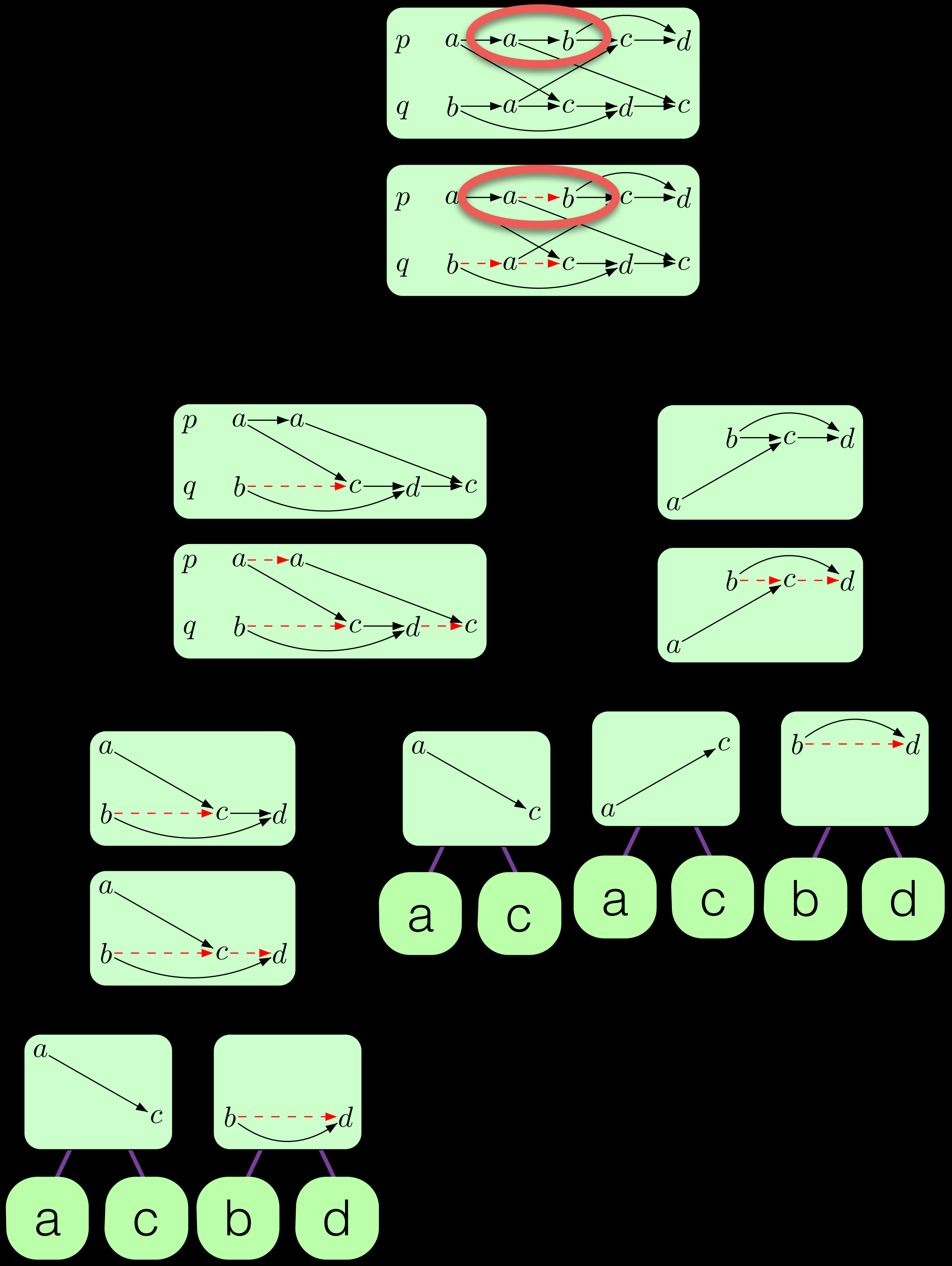

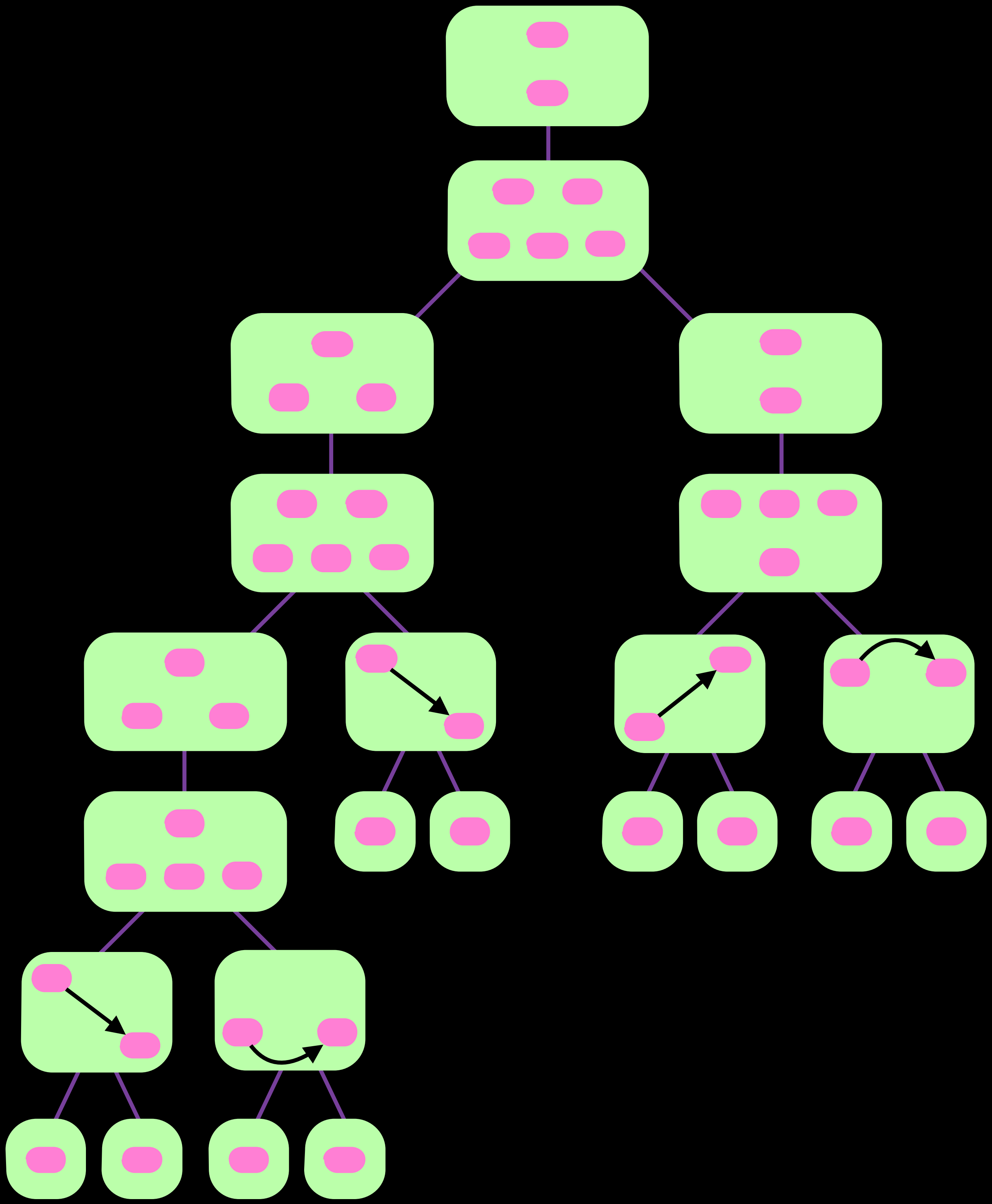

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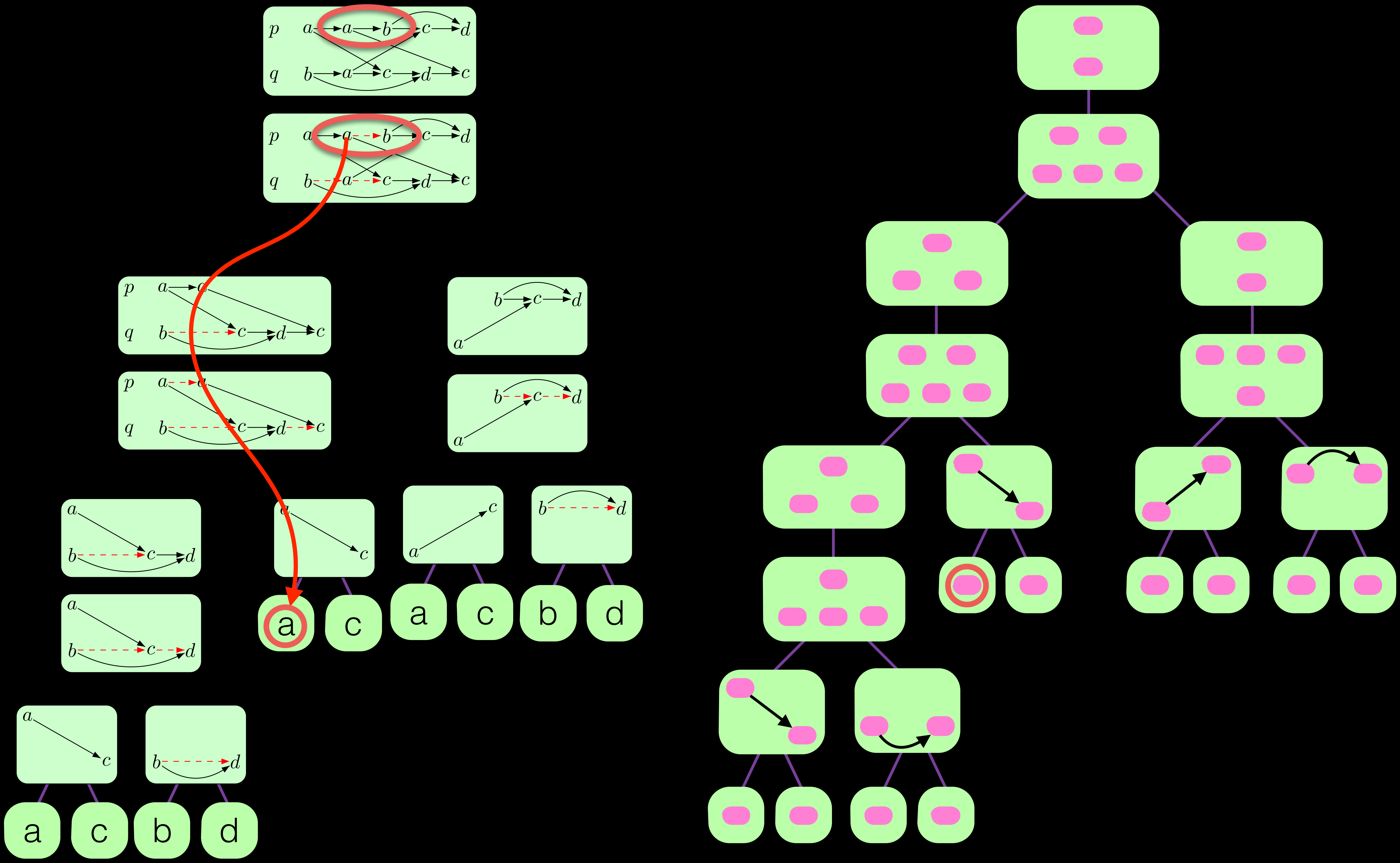

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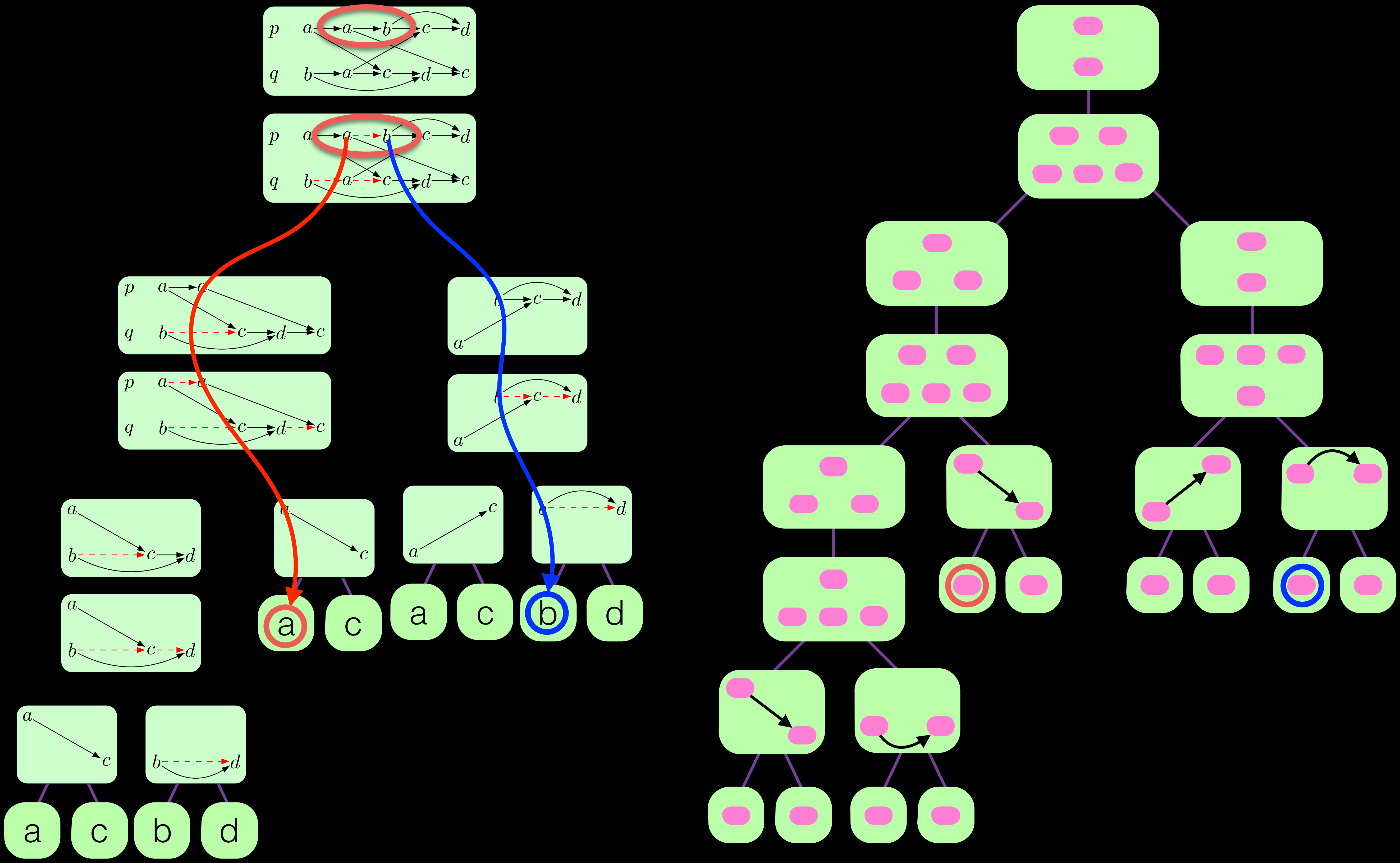

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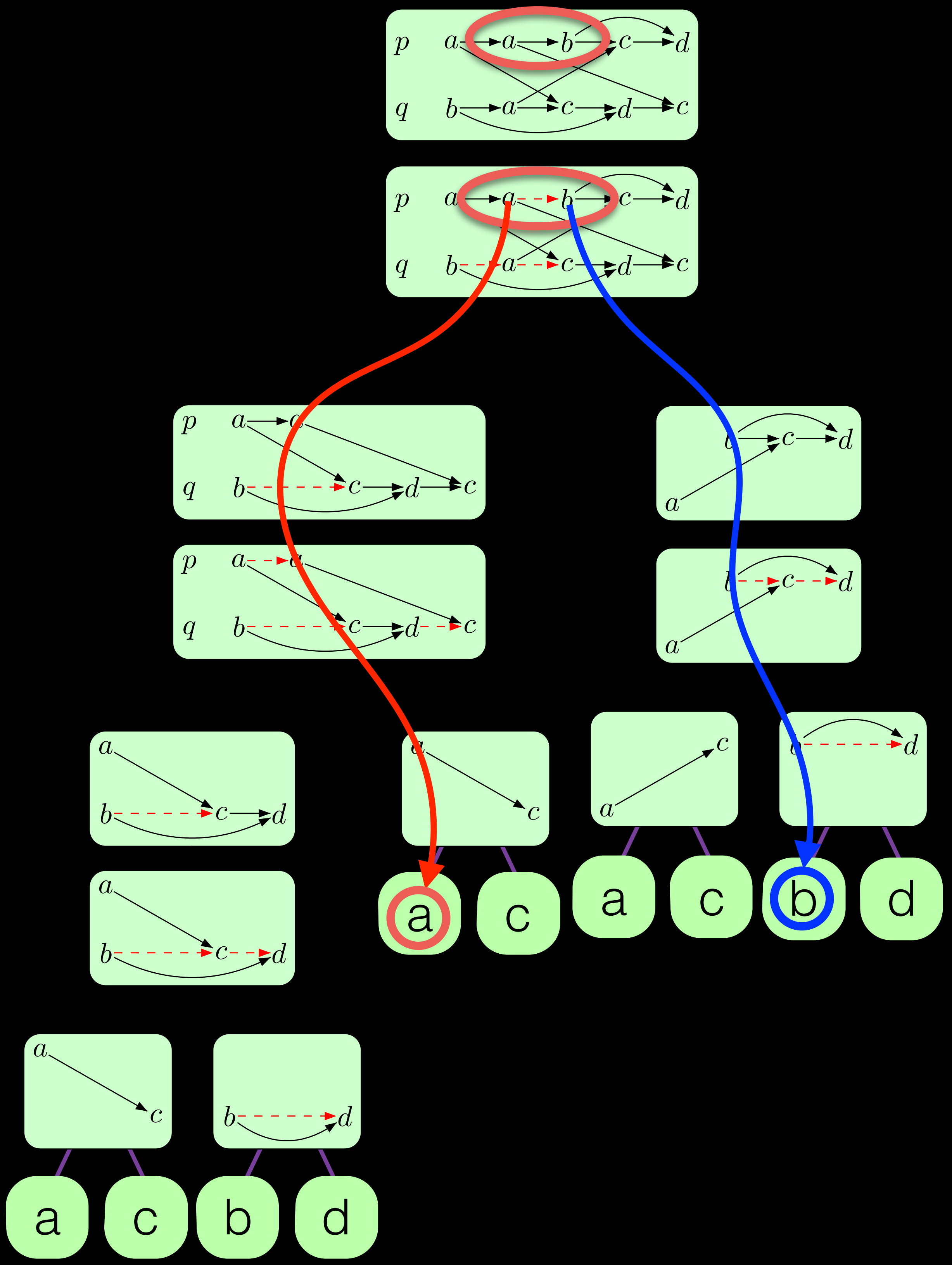

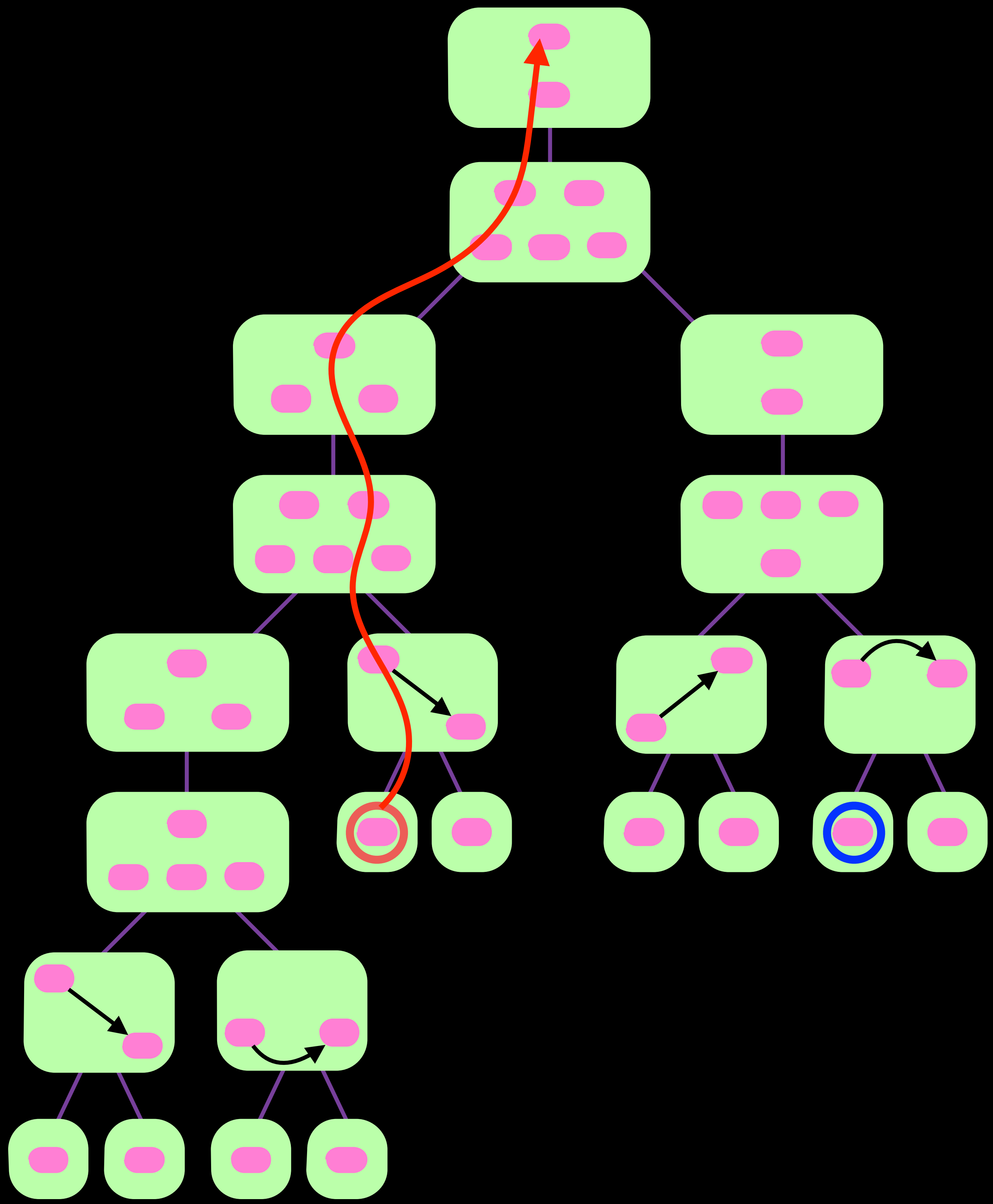

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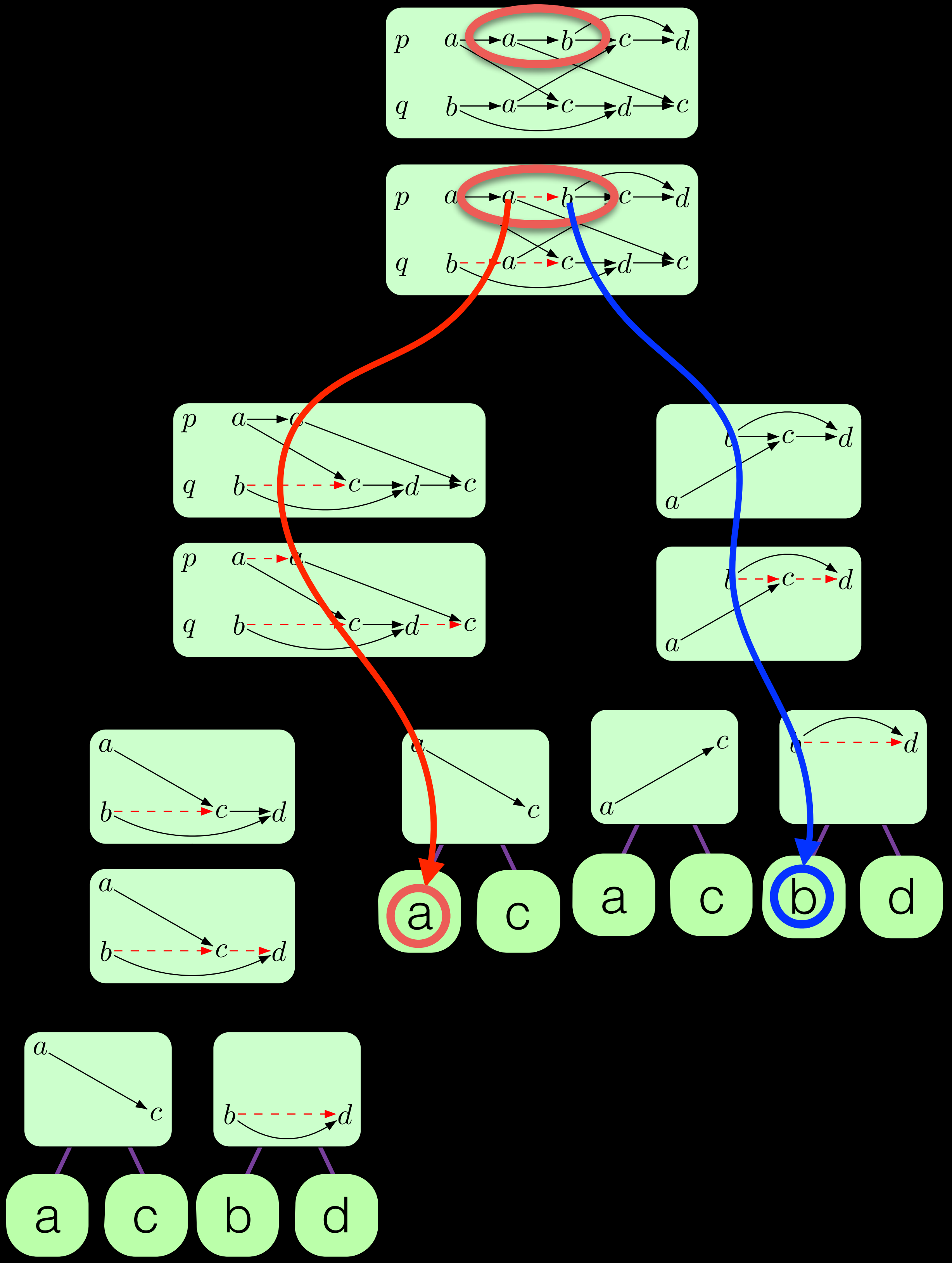
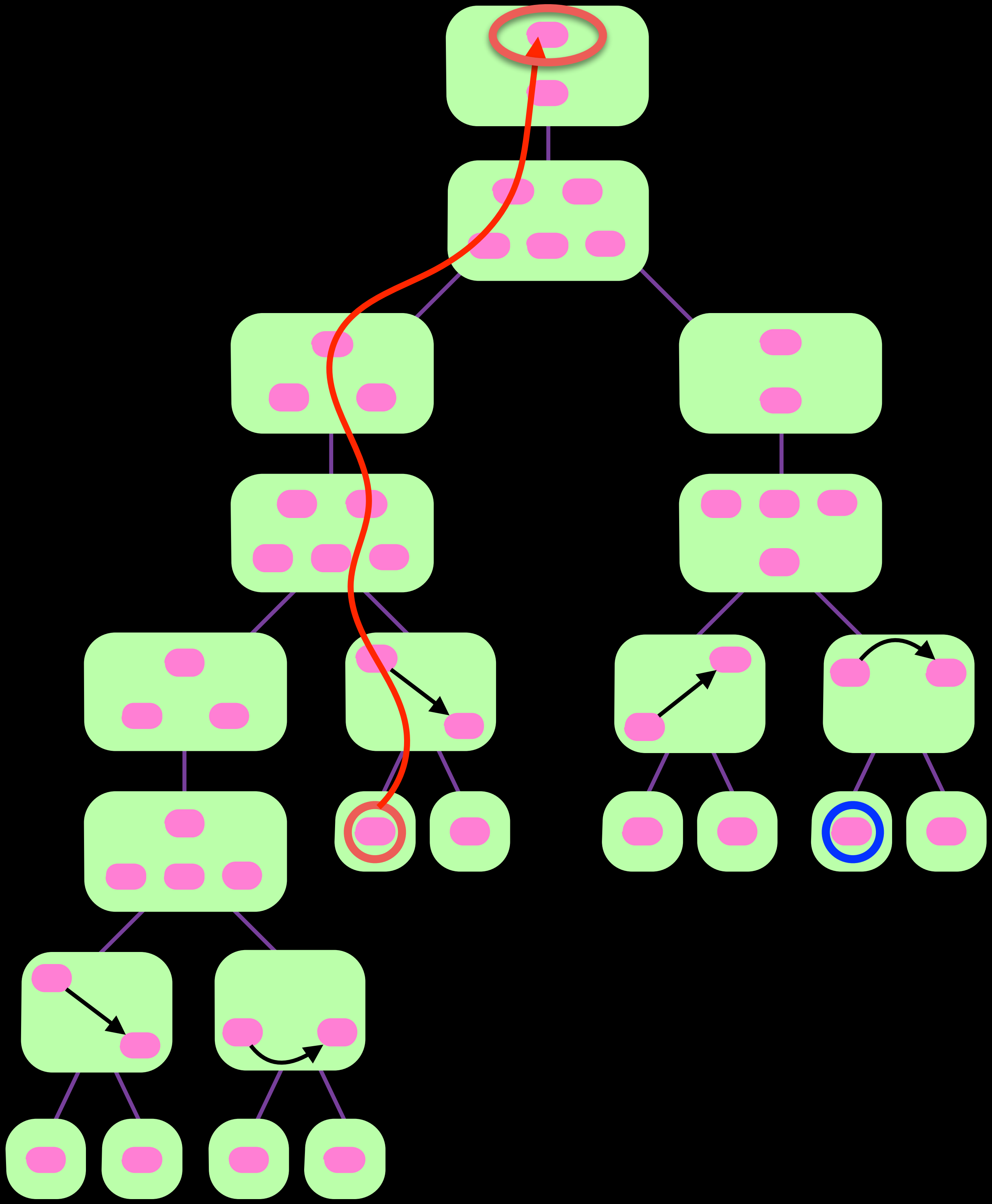
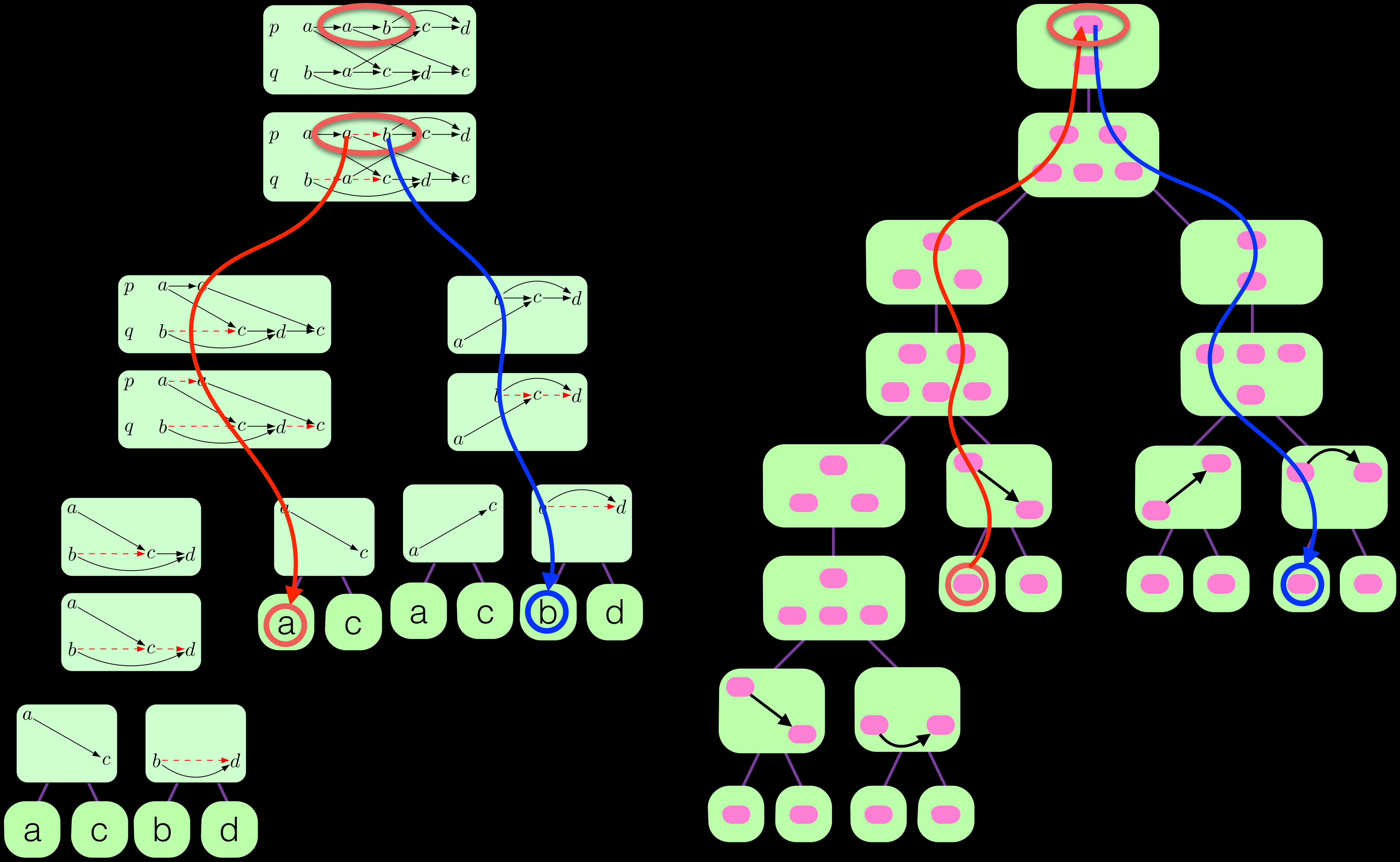


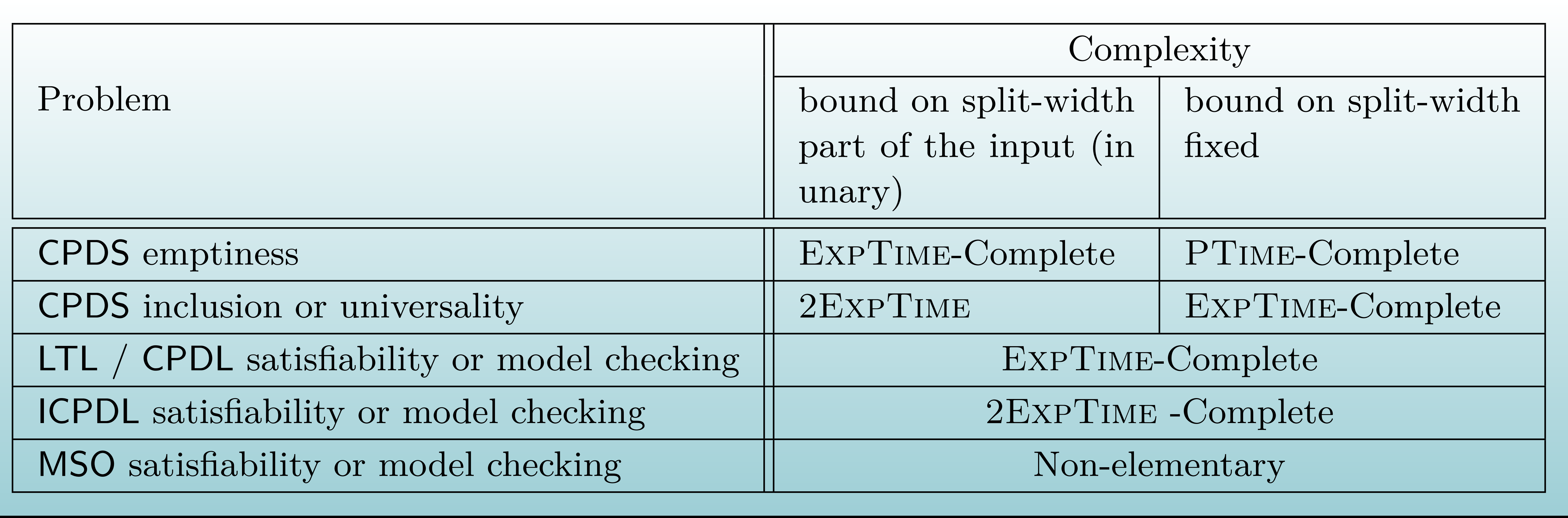


TREE INTERPRETATION



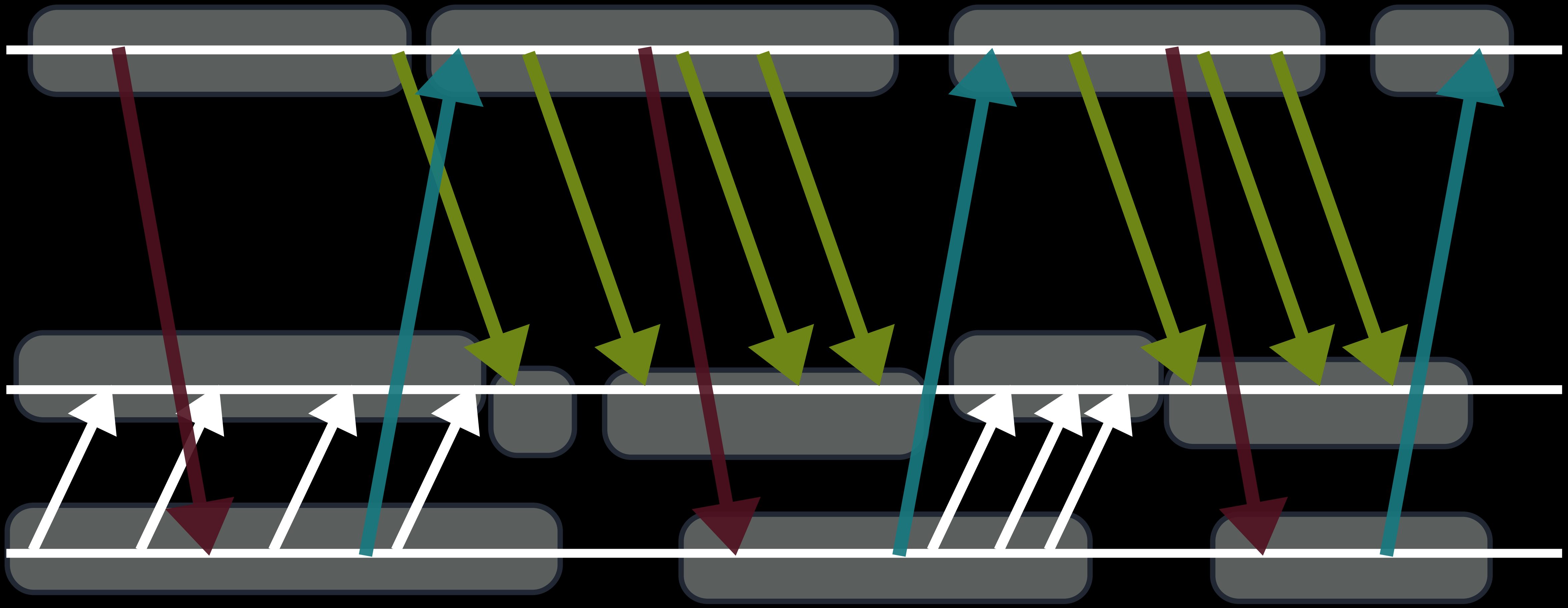
TREE INTERPRETATION

Split-width

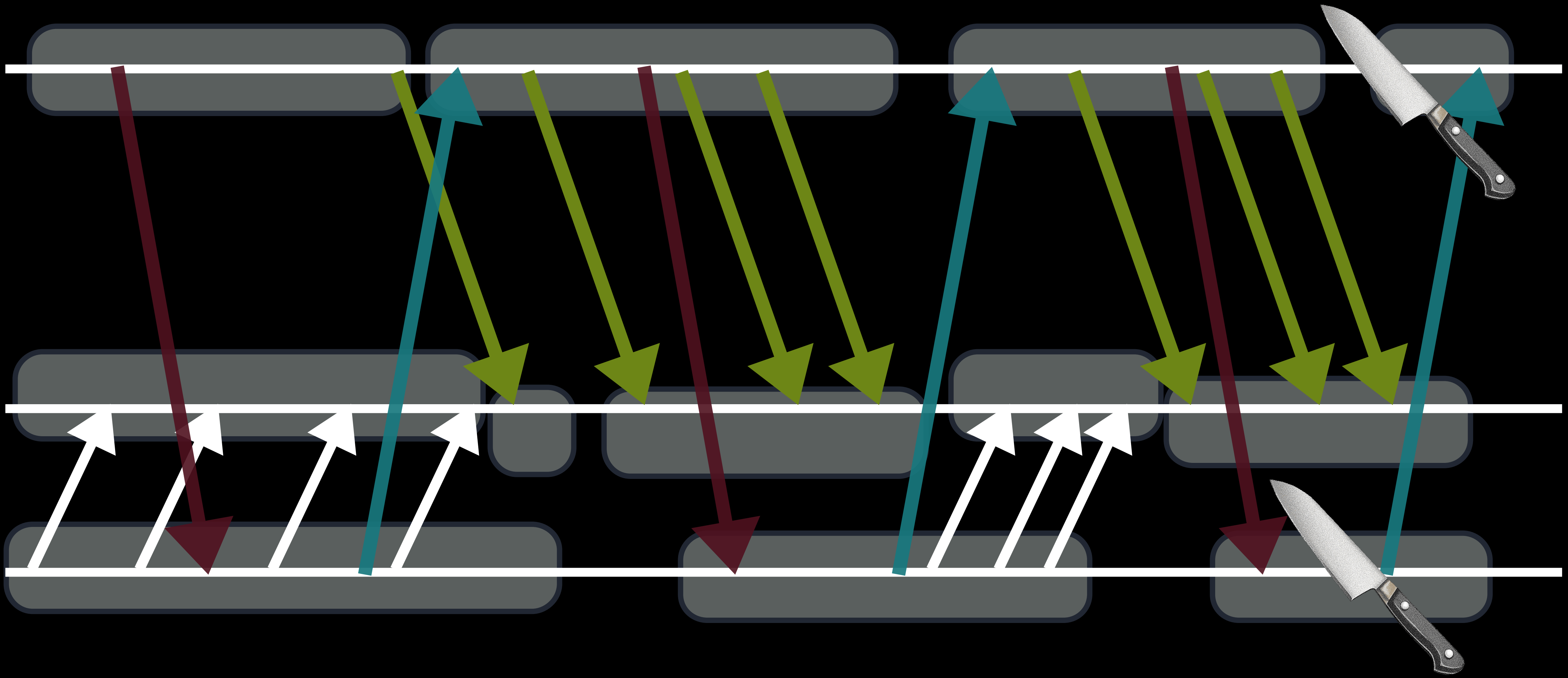


Problem	Complexity	
	bound on split-width part of the input (in unary)	bound on split-width fixed
CPDS emptiness	ExpTime-Complete	PTime-Complete
CPDS inclusion or universality	2ExpTime	ExpTime-Complete
LTL / CPDL satisfiability or model checking	ExpTime-Complete	
ICPDL satisfiability or model checking	2ExpTime -Complete	
MSO satisfiability or model checking	Non-elementary	

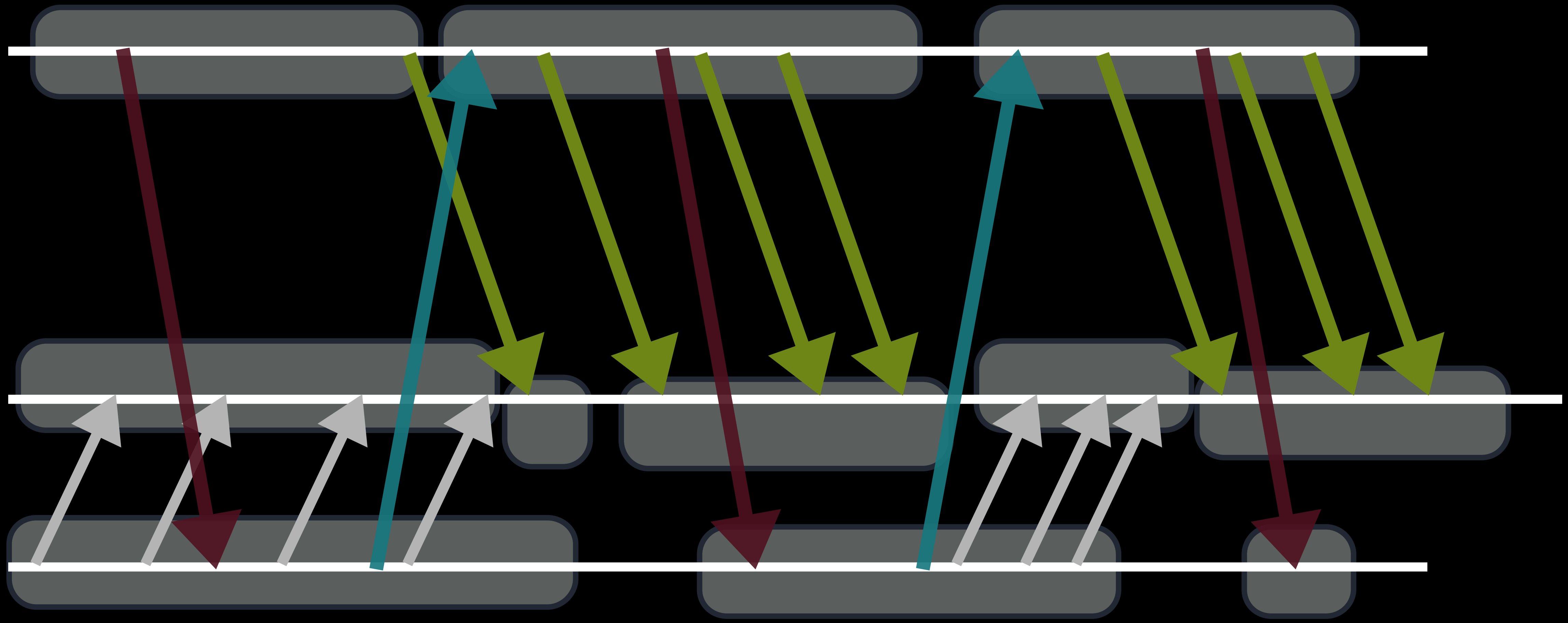
SPLIT-WIDTH



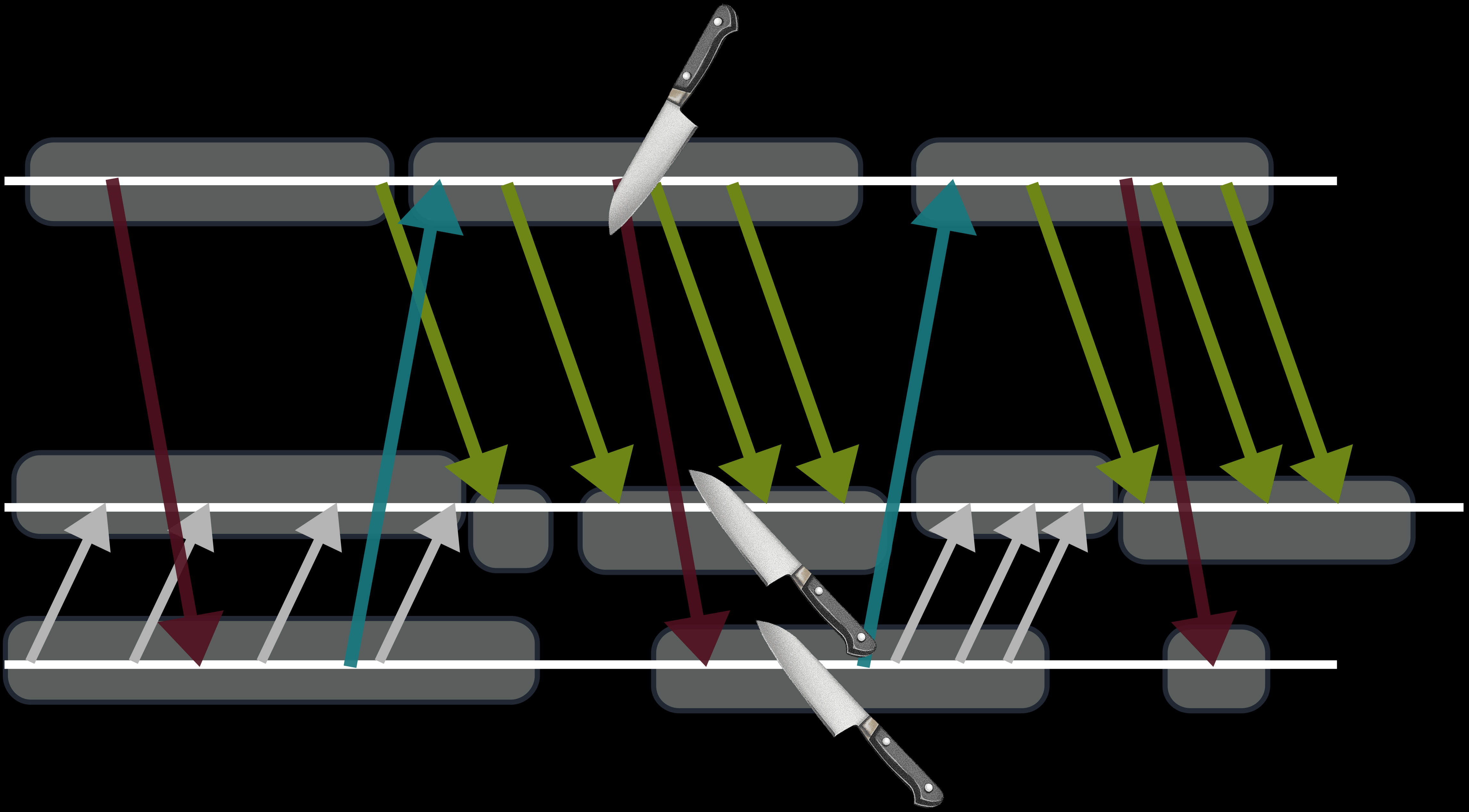
SPLIT-WIDTH



SPLIT-WIDTH 3



SPLIT-WIDTH 3



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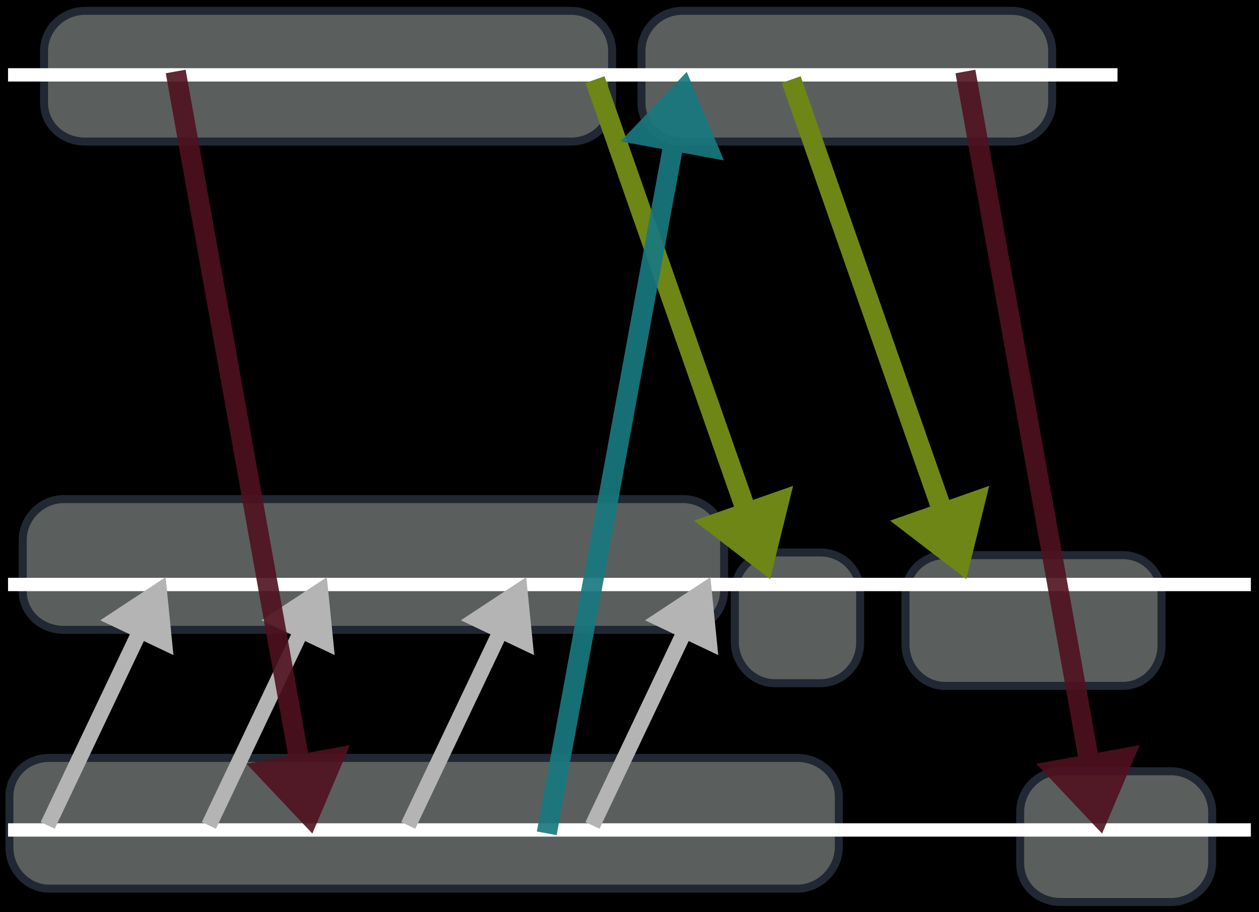

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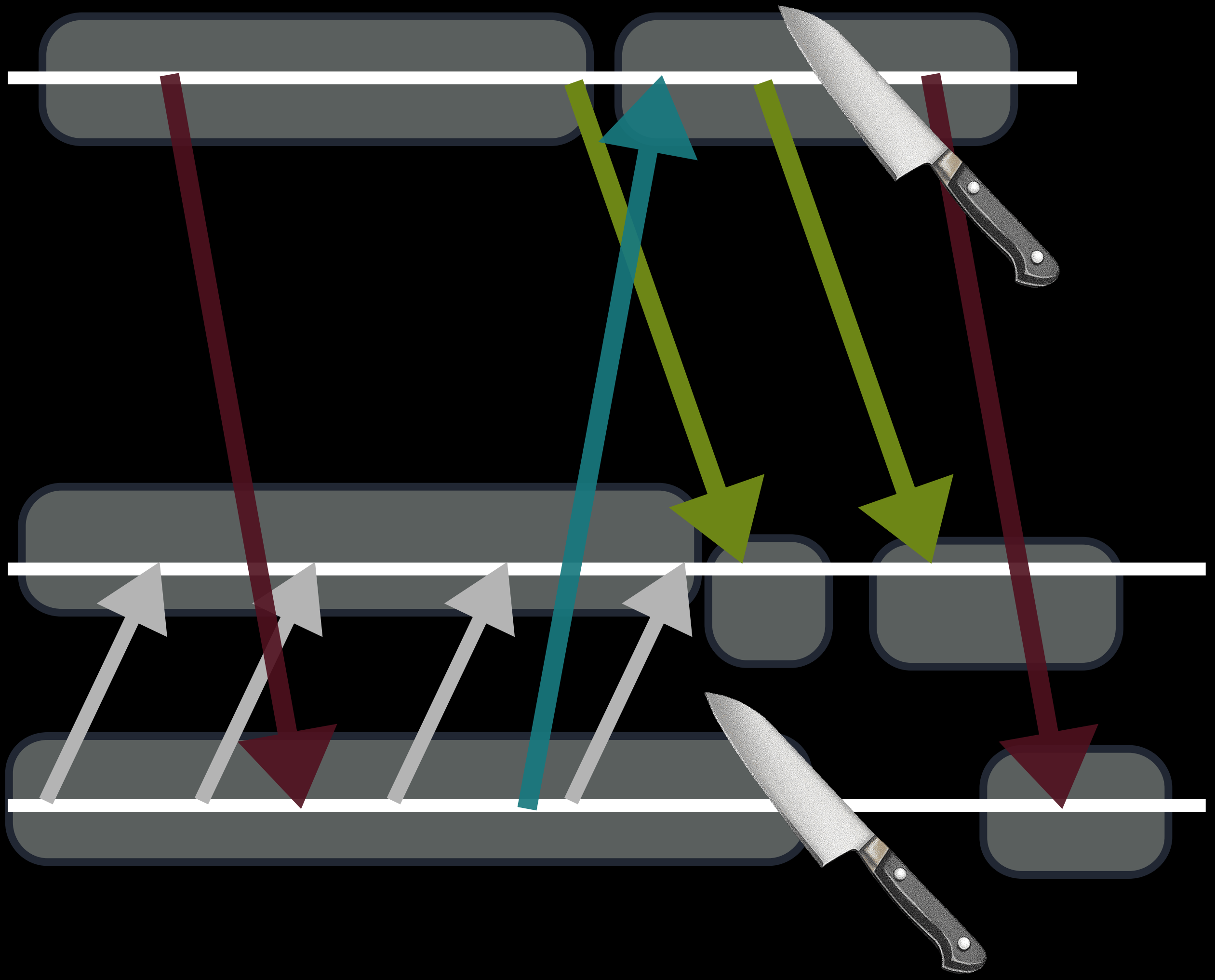

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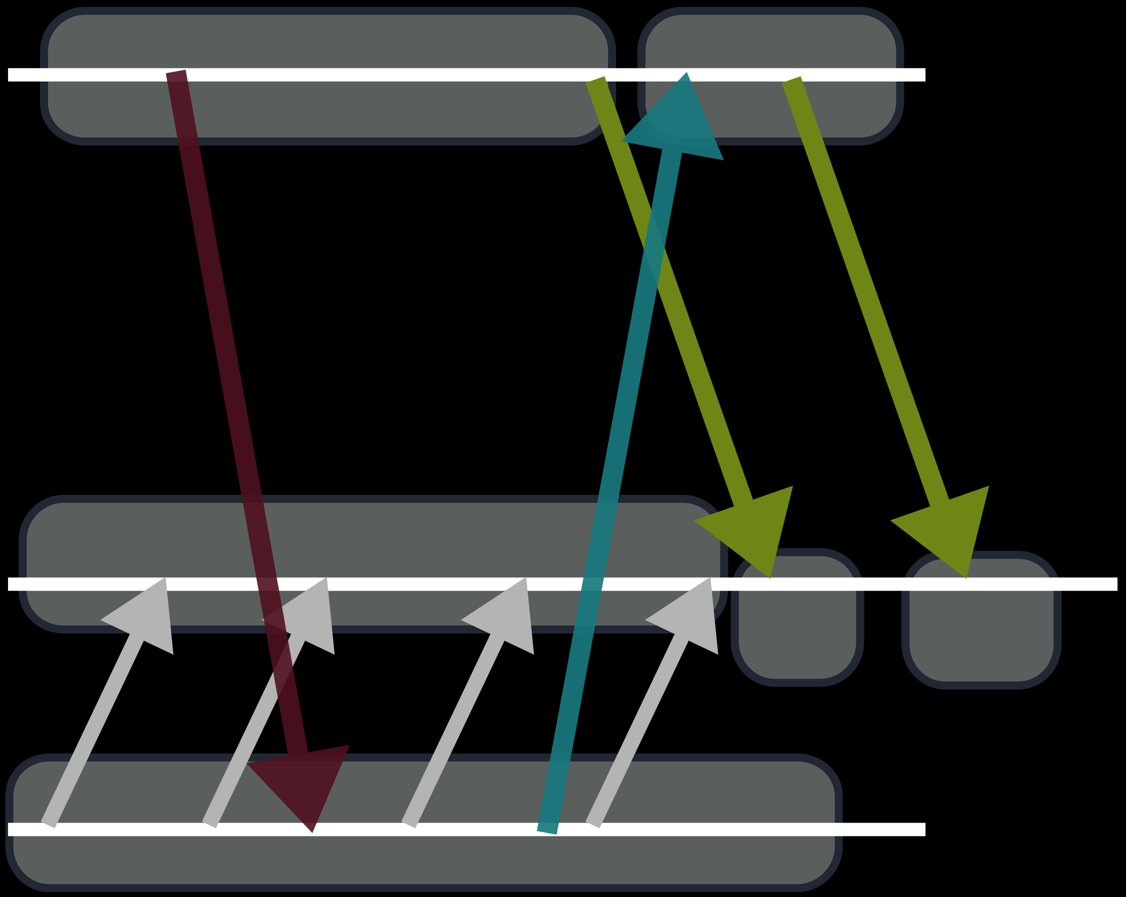

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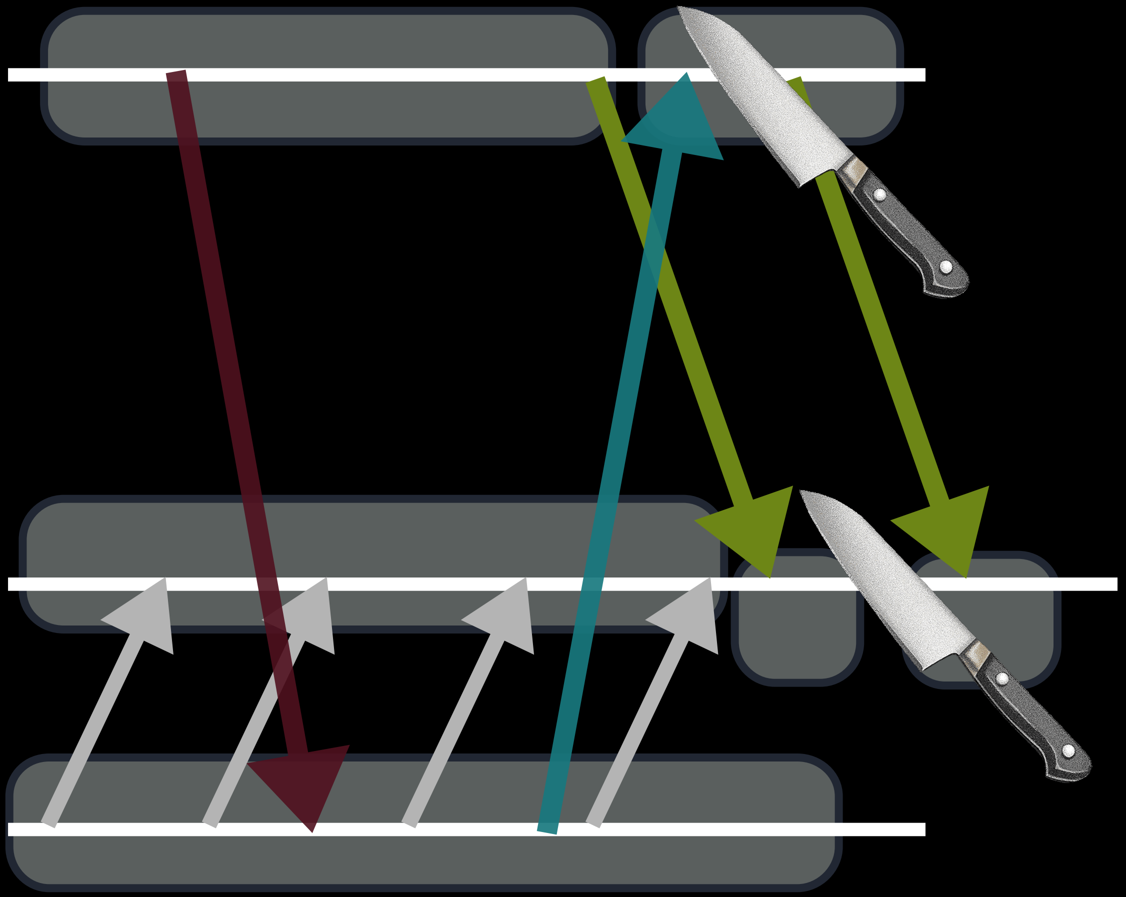

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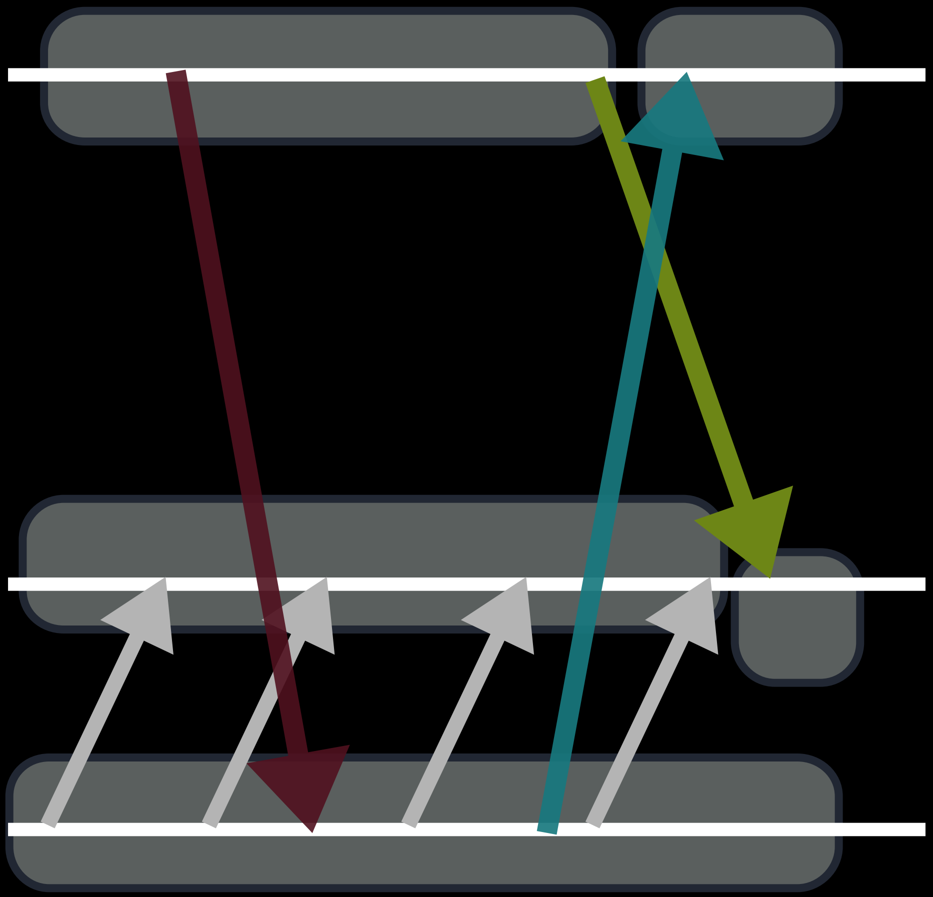

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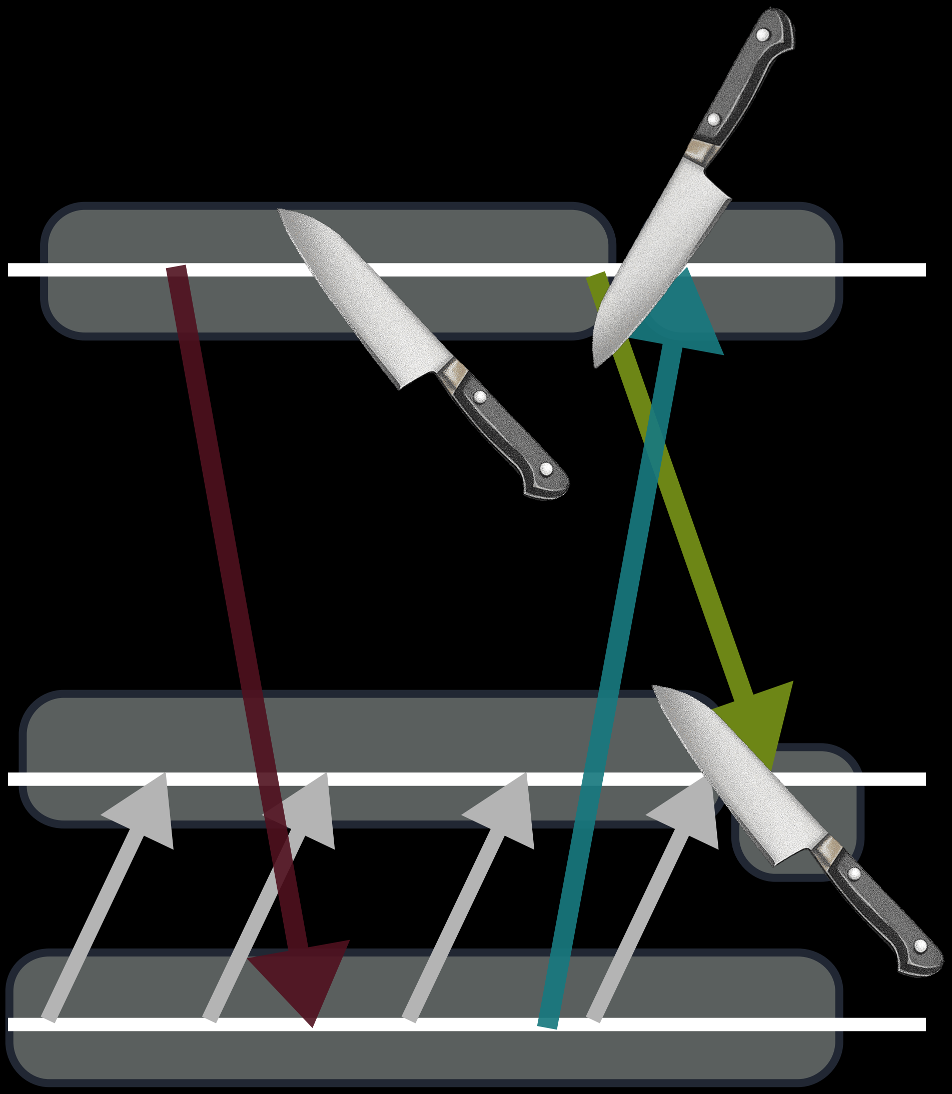

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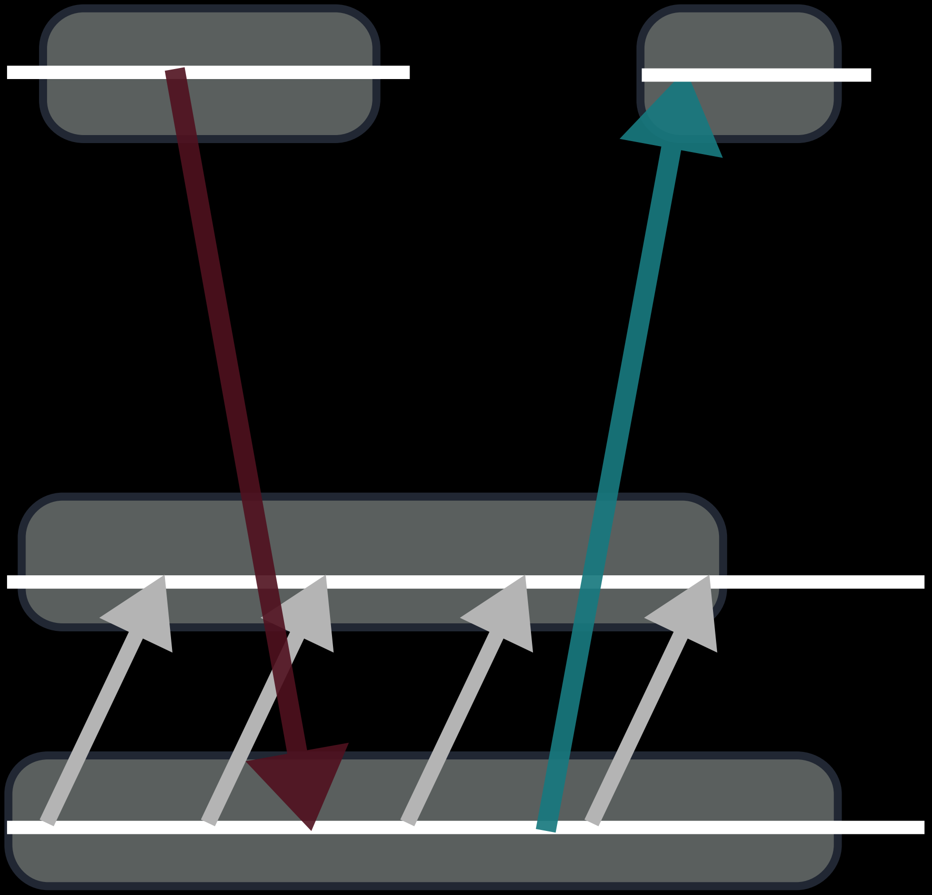

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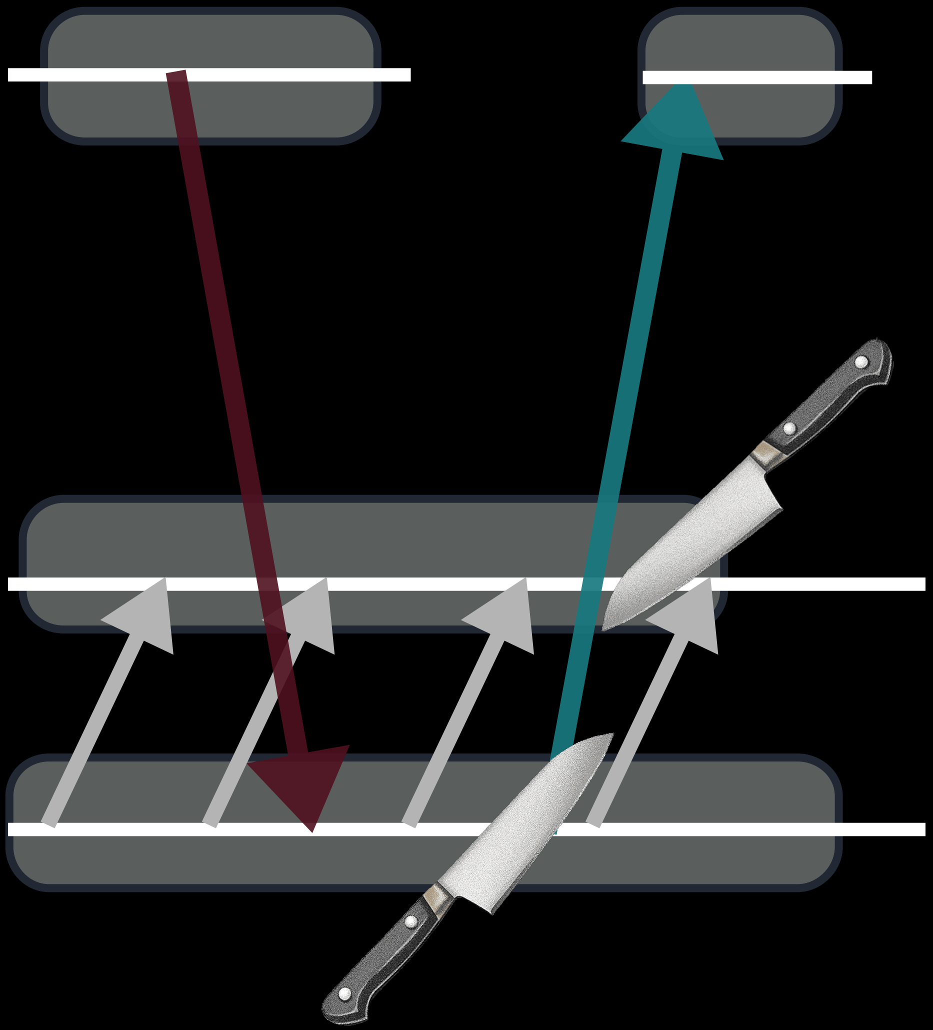

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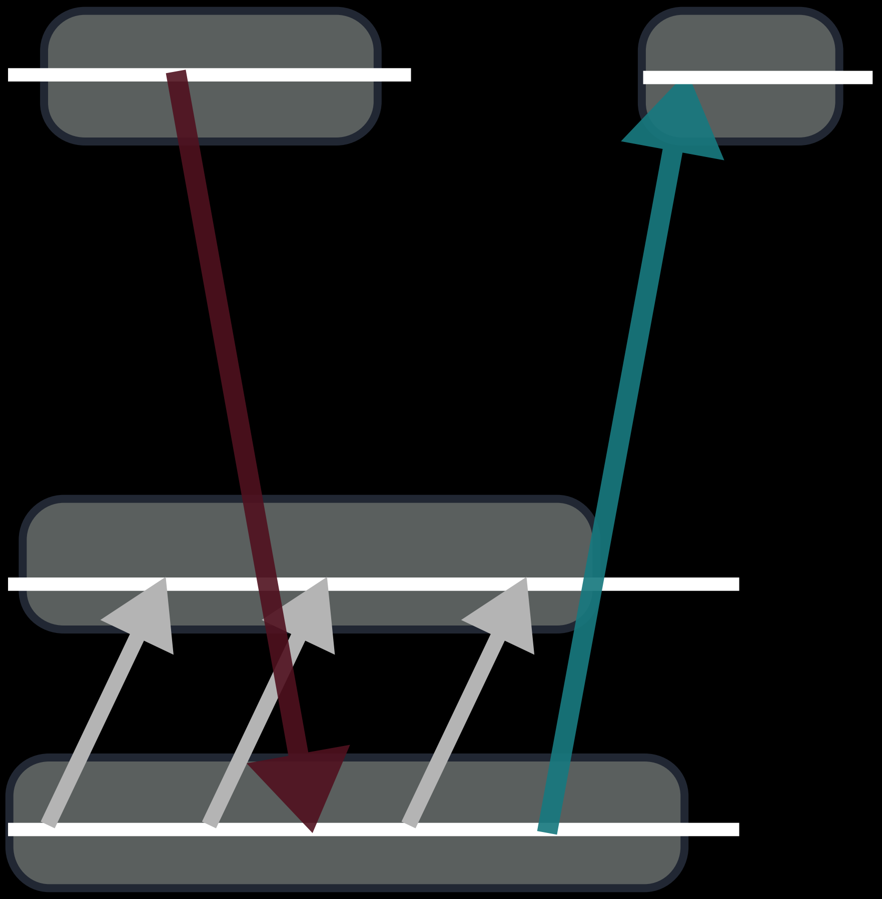

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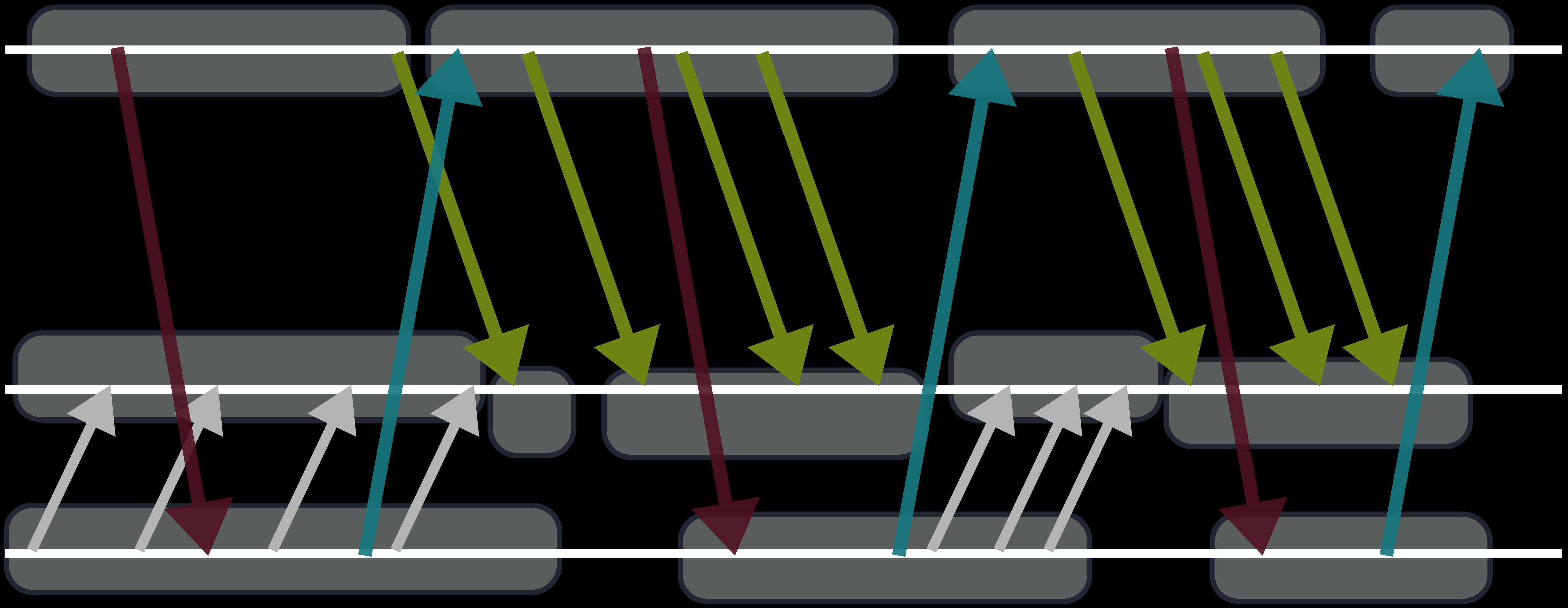
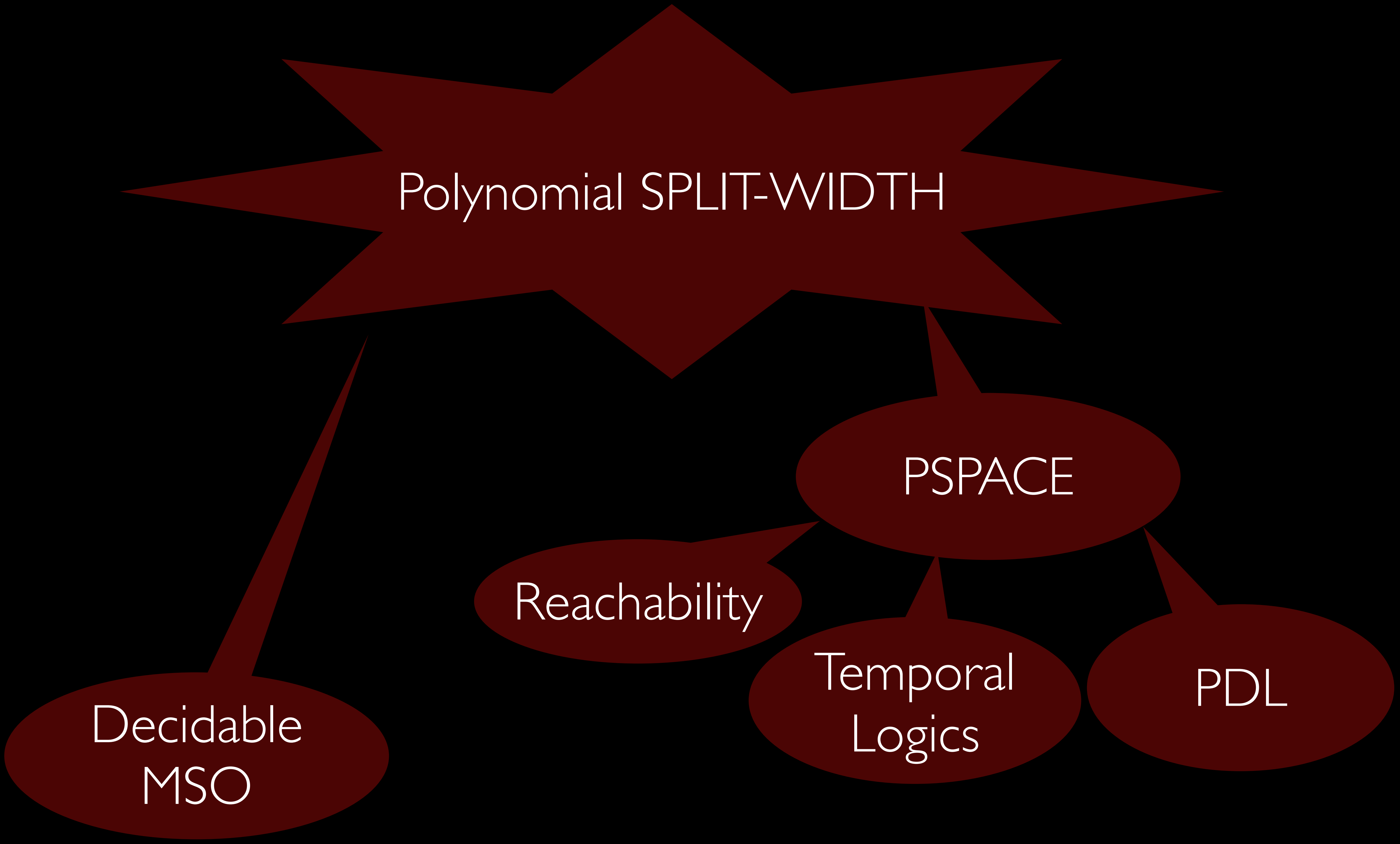




Polynomial SPLIT-WIDTH

PSPACE

Reachability

Temporal Logics

PDL

Decidable MSO

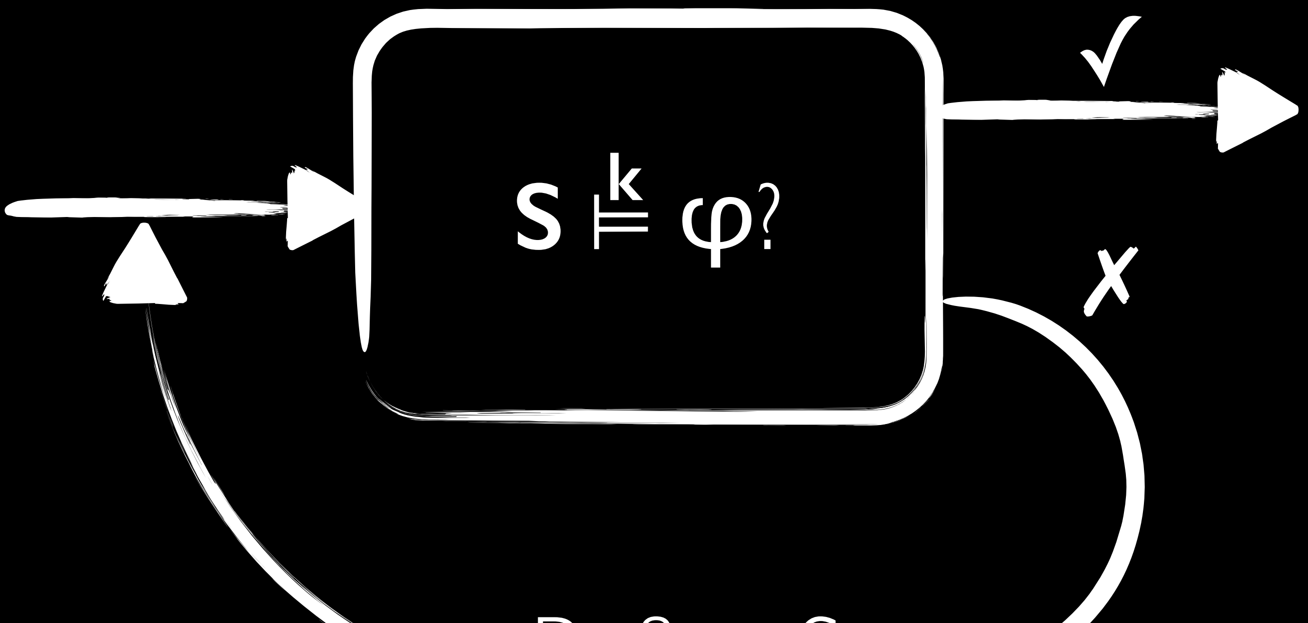
UNDER-APPROXIMATE VERIFICATION

Model Checking

> **Decidable**

System **S**

Specification **φ**



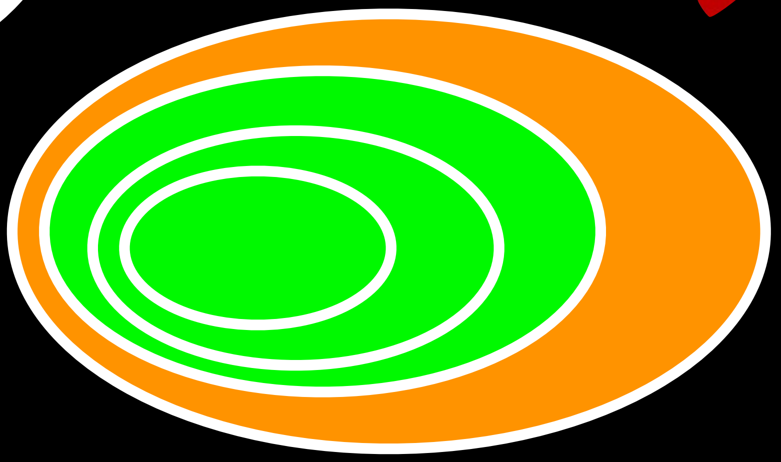
$\mathbf{S} \overset{k}{\models} \varphi$?

✓

✗

Refine S

(Fix bugs)



OTHER UNDER-APPROXIMATIONS

- Bounded channel size
- Existentially bounded [Genest et al.]
- Acyclic Architectures [La Torre et al., Heußner et al. Clemente et al.]
- Bounded context switching [Qadeer, Rehof], [LaTorre et al.], ...
- Bounded phase [LaTorre et al.]
- Bounded scope [LaTorre et al.]
- Priority ordering [Atig et al., Saivasan et al.]

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Tree-width

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Tree-width

- Many of the above classes have bounded tree-width [Parlato, Madhusudhan]

OTHER UNDER-APPROXIMATIONS

Split-width

* Acyclic Architectures	Constant
* Bounded channel size * Existentially bounded * Bounded context switching * Bounded scope	Bound + 2
* Bounded phase * Priority ordering	2^{Bound}
* Bounded Tree-width	Linear

Width: split vs tree vs clique

Split-Width k

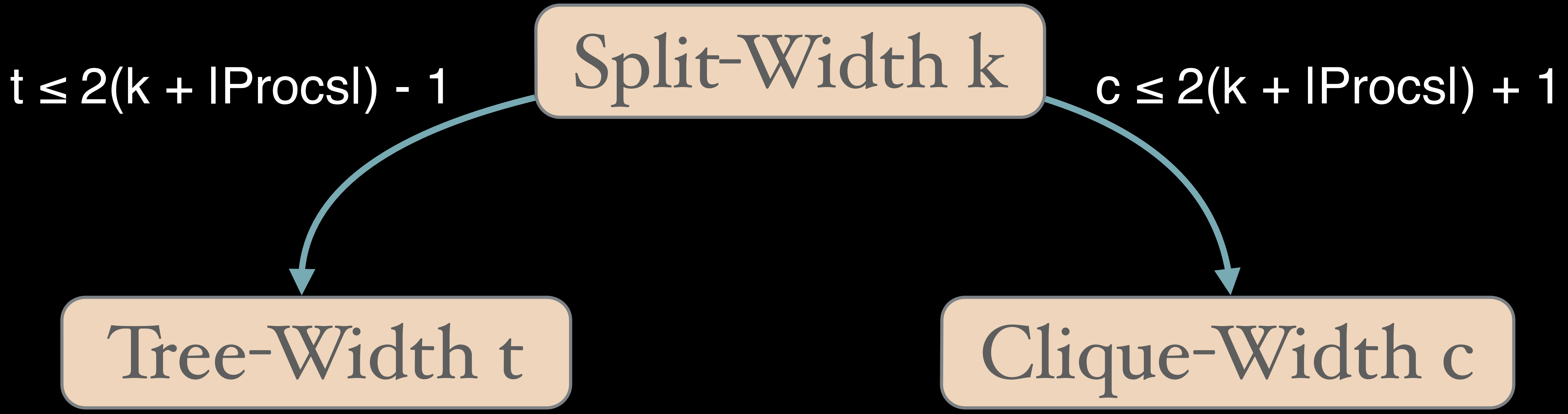
Tree-Width t

Clique-Width c

Let C be a class of bounded degree MSO definable graphs.
TFAE

1. C has a decidable MSO theory
2. C can be interpreted in binary trees
3. C has bounded tree-width
4. C has bounded clique-width
5. C has bounded split-width (for concurrent recursive behaviors)

Width: split vs tree vs clique


Split-Width k

$t \leq 2(k + |Procs|) - 1$

$c \leq 2(k + |Procs|) + 1$

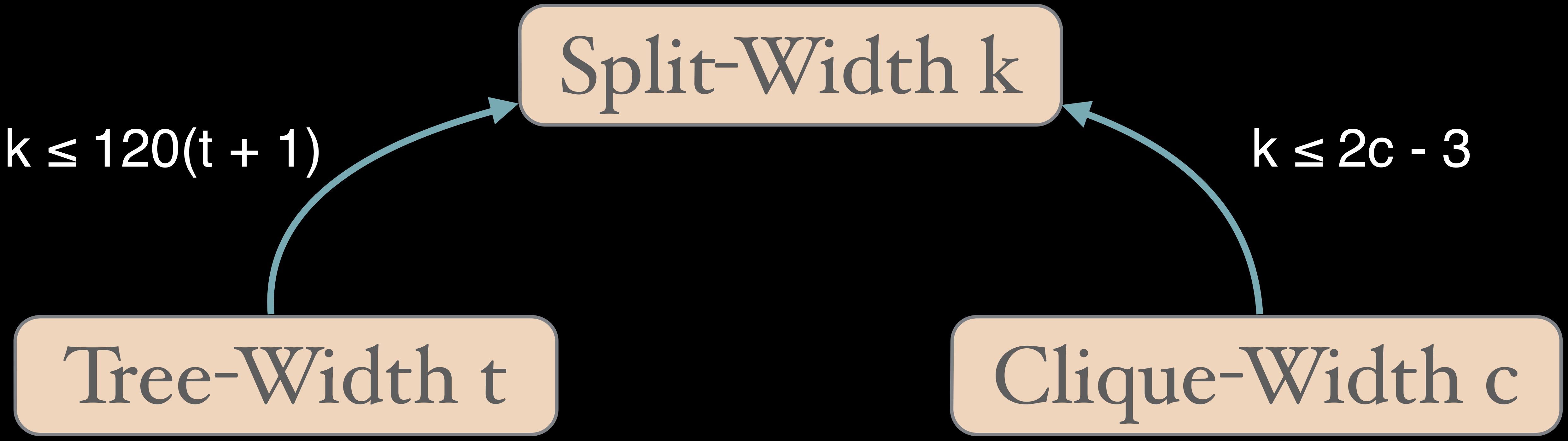
Tree-Width t

Clique-Width c

Let C be a class of bounded degree MSO definable graphs. TFAE

1. C has a decidable MSO theory
2. C can be interpreted in binary trees
3. C has bounded tree-width
4. C has bounded clique-width
5. C has bounded split-width (for concurrent recursive behaviors)

Width: split vs tree vs clique



Split-Width k

k ≤ 120(t + 1)

k ≤ 2c - 3

Tree-Width t

Clique-Width c

Let C be a class of bounded degree MSO definable graphs. TFAE

1. C has a decidable MSO theory
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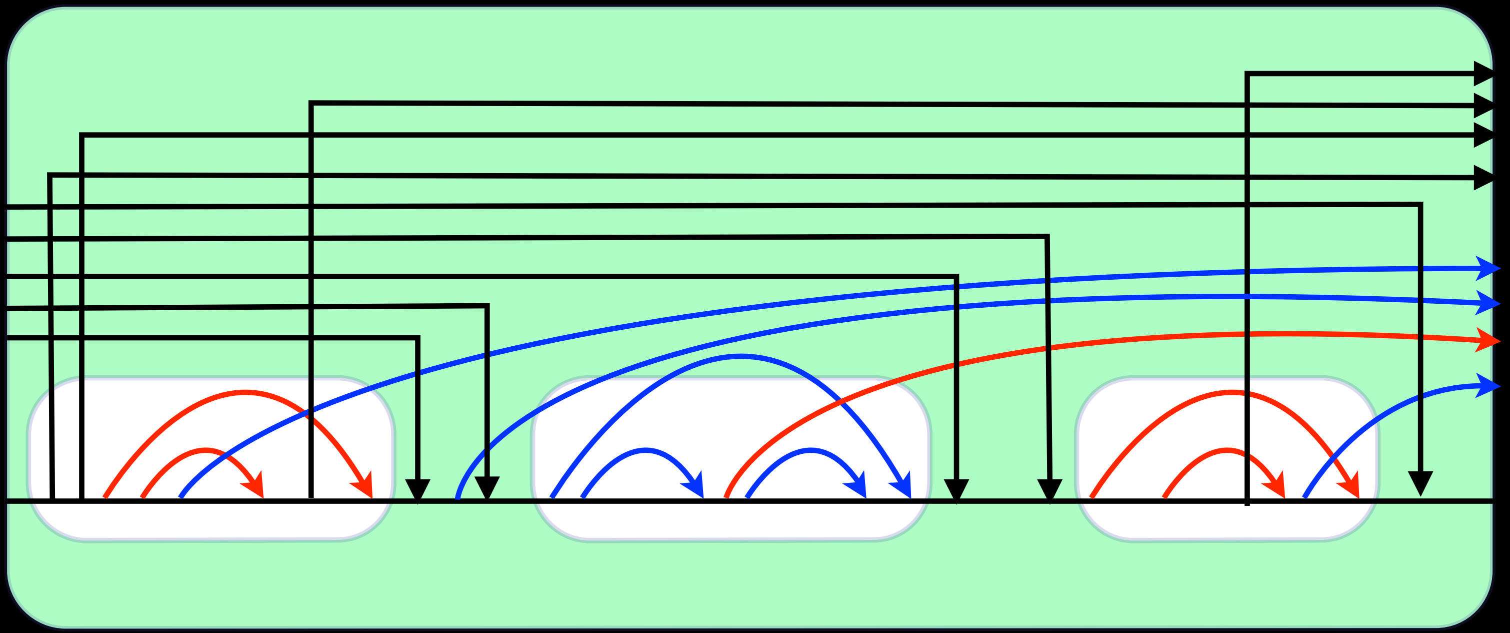
COMMUNICATING RECURSIVE PROGRAMS: CONTROL AND SPLIT-WIDTH

AUTONOMOUS COMPUTATIONS

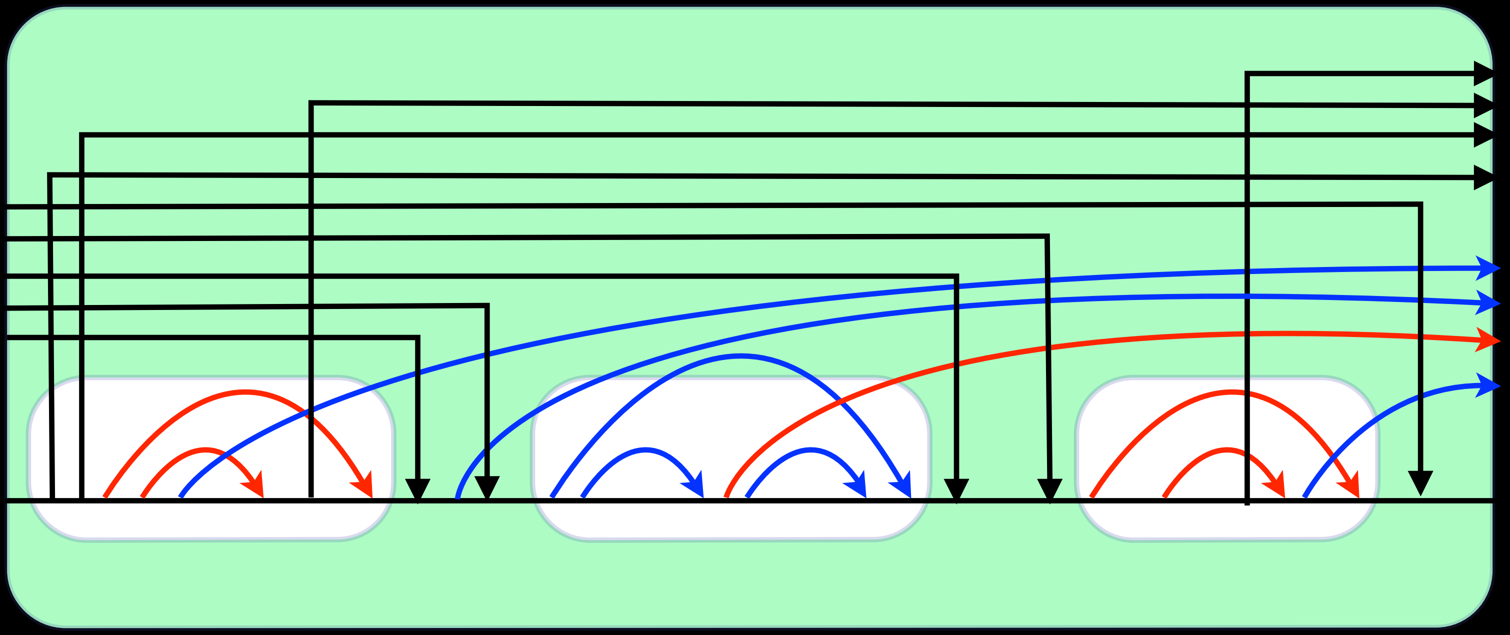
- Recursive computations which does not read from other stacks/queues.
- A stretch of computation in which all incoming edges are on a single stack

AUTONOMOUS COMPUTATIONS

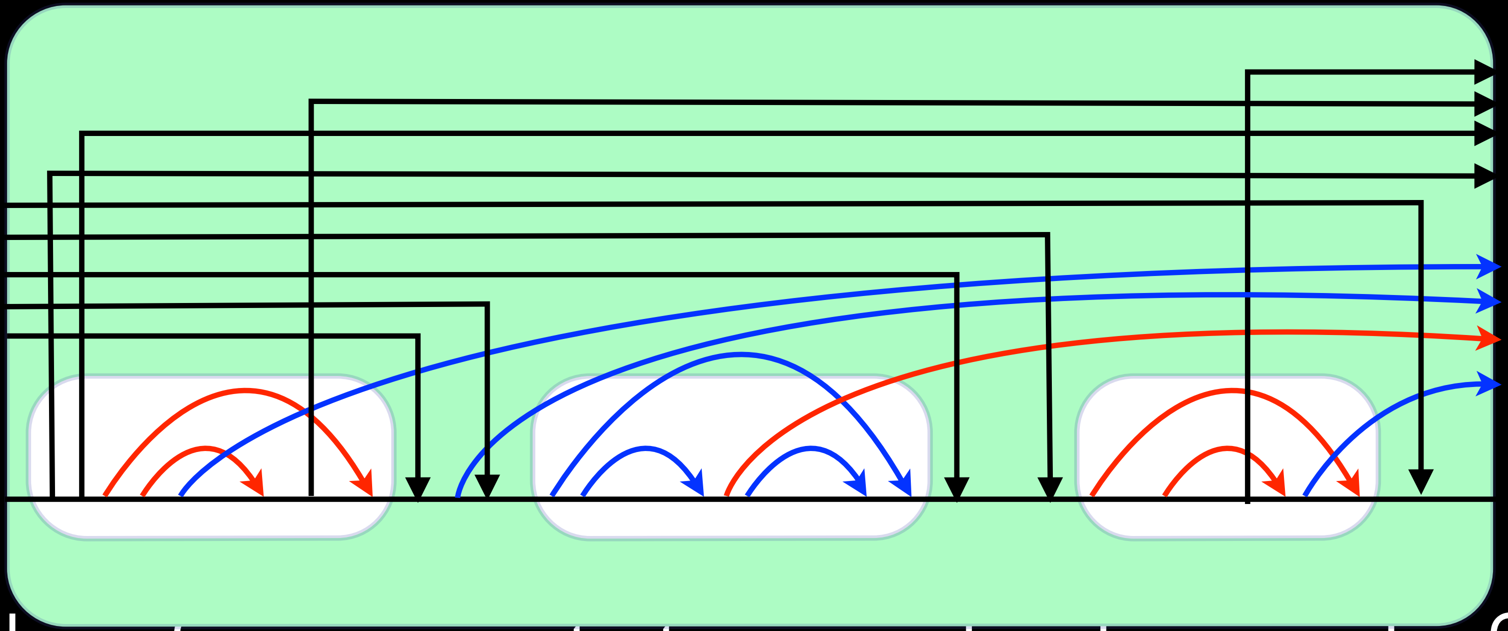


- Recursive computations which does not read from other stacks/queues.
- A stretch of computation in which all incoming edges are on a single stack

PHASE

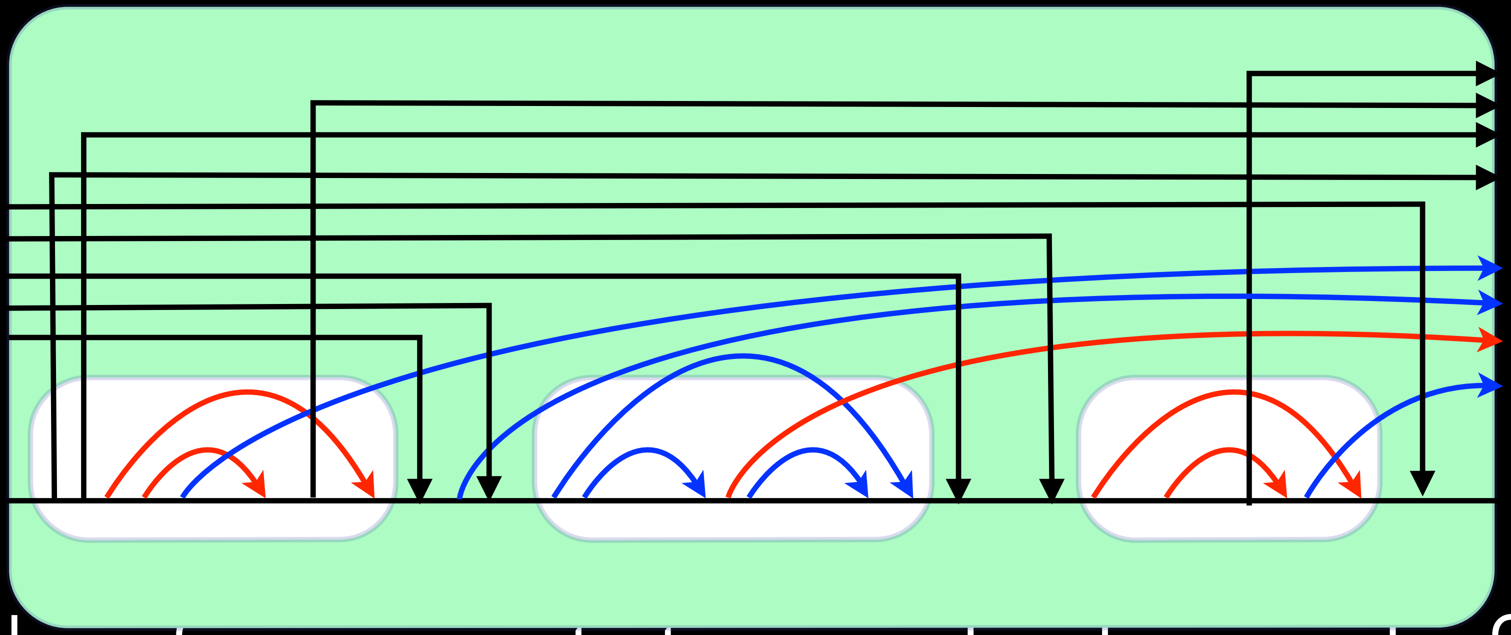


PHASE



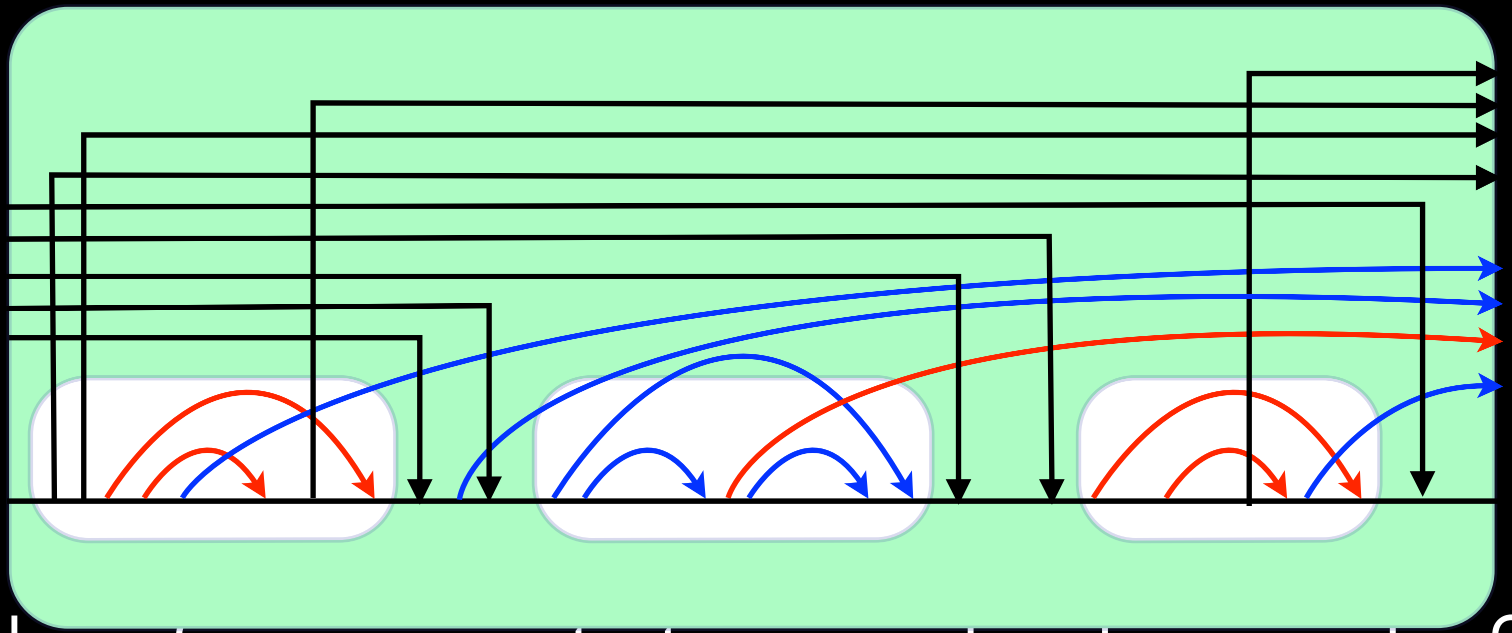
- A stretch of computation which reads from at most one stack/queue

PHASE



- A stretch of computation which reads from at most one stack/queue
- free (unlimited) autonomous computations

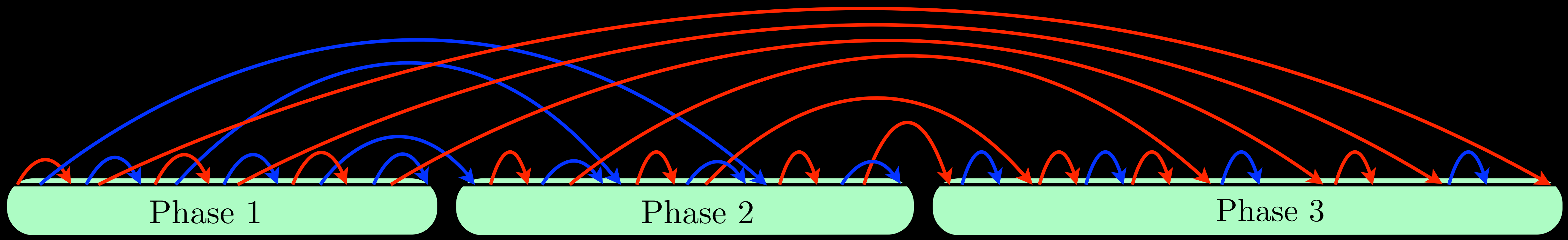
PHASE



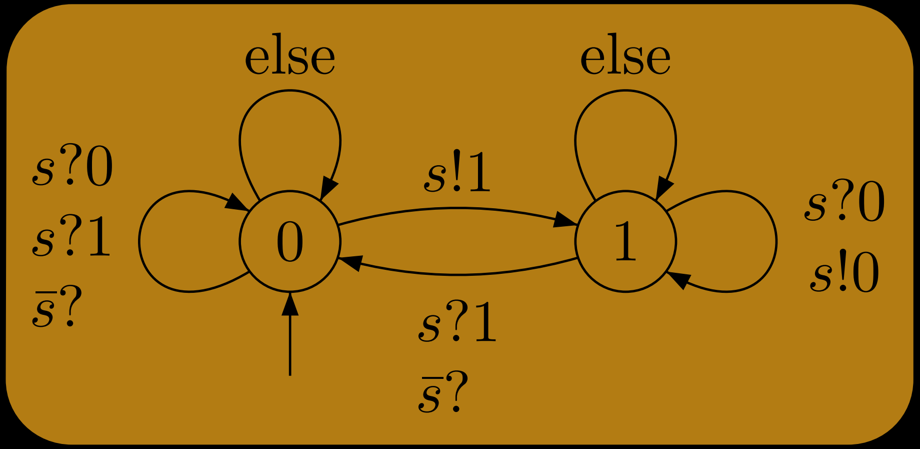
- A stretch of computation which reads from at most one stack/queue
- free (unlimited) autonomous computations
- no loops

K-BOUNDED PHASE

K-BOUNDED PHASE



IDENTIFYING AUTONOMOUS POPS



- Possible by tagging the values on stacks
- Deterministic controller for each stack
- The phase controller simulates one such automaton for each stack.

COMMUNICATING RECURSIVE PROGRAMS: CONTROL AND SPLIT-WIDTH

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- A. C., Paul Gastin, and K. Narayan Kumar. MSO decidability of multi-pushdown systems via Split-width. In *CONCUR* 2012.
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