CANTILEVER BEAMS

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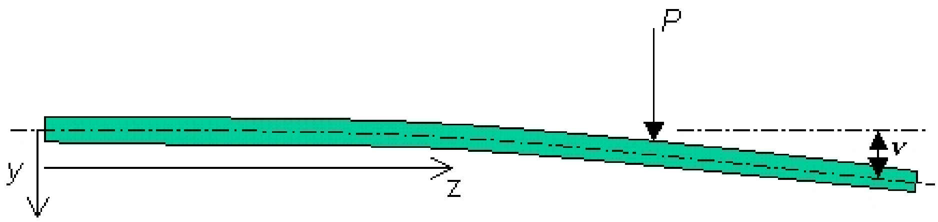
[1]

1. Introduction

In this experiment we try to measure the Elasticity modulus of scales made of different materials with the cantilever beam arrangement.If we consider the beam in the position given in the figure below



We define coordinate z along the length of the beam, coordinate y vertically from the centerline of the beam and coordinate x widthwise across the beam so as to complete a right handed system. Beam deflections (in the y direction) are denoted using the variable ν which in general will be a function of z , i.e. $\nu = \nu(z)$. The bending moment $M_x(z)$ is positive if the upper fibers of the beam are in compression and the bottom fibers are in tension.

For a symmetric cross section beam made of a linear elastic material, whose displacements and slopes under load are small relative to its undeformed configuration, the relationship between the displacement and bending moment is

$$EI\{-\frac{d^2\nu}{dz^2}\} = M_x$$

where EI designates the bending moment of the beam; E is the modulus of elasticity and I is the second area moment of the cross section (L^4) about the x-axis. For a rectangular cross section $I = \frac{bh^3}{12}$. We can get the relation after some manipulation as

$$\nu = \frac{mgz^2(3l - z)}{6EI}$$

From which if $z \simeq L$ we get, for

$$E = \frac{4mgl^3}{bh^3\nu}$$

[1]Physics Lab Report-V

In this, I vary the mass suspended m and get the Elasticity modulus of various materials. We can also vary the length of beam and get the behaviour. Then we can try to gauge how good our approximation og the formula works with a change in the point of suspension of the weights.

2. Procedure

- Firstly, i use a wooden scale and get the relation of the depression from the unloaded position for various loads attached at a particular point on the scale and get the relation between the two and compare with the theoretically predicted relation. We can also get the Young's modulus from this. We use a pin and a travelling microscope measurement to find the depression by subtracting the height from the unloaded reading on a travelling microscope
- Next, we can hang a plastic scale and repeat the same procedure to calulate the youn's modulus of the plastic scale.
- For the wooden scale the length of the scale was aried keeping the point of suspension to be same and a similar experiment was carried as before for various lengths to get the relation between the length of beam and the depression can be checked experimentally.
- Here, the point of suspension was varied for a particular length and the relation between length and the depression was to to be checked how the parameters are related to each other.

3. Observations and Results

- **Dimensional Measurements**
  1. Thickness of wooden scale $h_{wood} = 0.640cm$
  2. Width of the wooden scale $b_{wood} = 2.68cm$
  3. Thickness of the plastic scale $h_{plastic} = 0.582$
  4. Width of the plastic scale $b_{plastic} = 4.47$
- Tabular Data of the Stress to the depression of a wooden scale for $l = 39.8cm$

S.No	Mass suspended (gm)	Measured height (cm)	Depression ν (cm)
1	0	5.807	0
2	50	5.335	0.472
3	100	4.865	0.942
4	150	4.381	1.426
5	200	3.895	1.912
6	250	3.406	2.401

  For a linear fit on the data for m versus l the data given by GNU-PLOT was A1 = 0.00955891 (0.2576%) where $a1$ is the slope the graph from which we can calculate $E = 3.68x10^{10} dyne/cm^2$.

- For the plastic scale a similar collection of data reads as follows :

S.No	Mass suspended (gm)	Measured height (cm)	Depression ν (cm)
1	0	4.981	0
2	50	3.040	1.941
3	100	1.045	3.936

The slope of this is approximately 0.0377. Hence the young's modulus can be calulated given the length $l = 35cm$; and it is $E = 0.506x10^{10} dyne/cm^2$.

- For various lengths of beam the data collected is tabulated below

S.No	length(cm)	Mass suspended(gm)	Measured height(cm	Depression(cm)
1	30.5	0	5.971	0
2	30.5	50	5.705	0.266
3	30.5	100	5.495	0.476
4	30.5	150	5.182	0.789
1	24.5	0	6.185	0
2	24.5	50	6.040	0.145
3	24.5	100	5.900	0.285
4	24.5	150	5.766	0.419
1	19.5	0	6.170	0
2	19.5	50	6.105	0.065
3	19.5	100	6.030	0.140
4	19.5	150	5.946	0.224

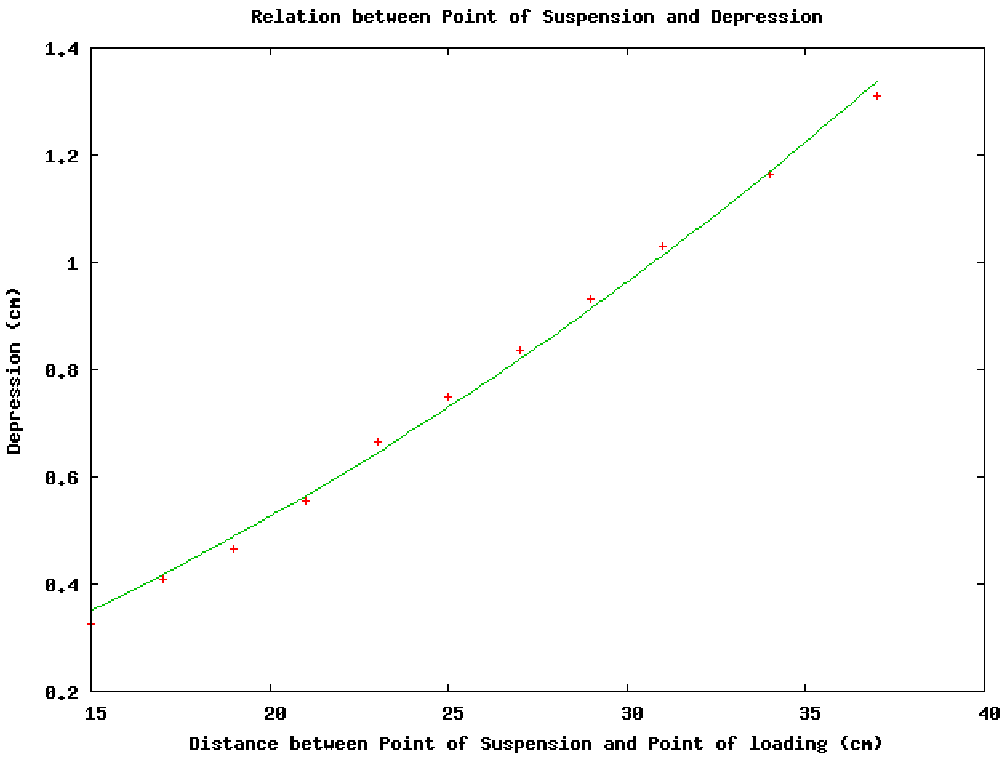
From this we can see the relation between length and the depression for a given weight as that the depression is preportiona lto the cube of the length.

S.No	Ratio of l^3	ν at 50gm	ν at 100gm	ν at 150gm
1	$(\frac{30.5}{24.5})^3$ =1.929	1.834	1.670	1.883
2	$(\frac{24.5}{19.5})^3$ =1.983	2.231	2.036	1.870
3	$(\frac{31.5}{19.5})^3$ =4.215	4.092	3.400	3.522

- The relation for point of suspension to the depression is to be mado out of the follwing data for 150gm weight and with a clamp length of the beam as 40cm.

S.No	Dist of point of loading(cm)	Measured height(cm)	Depression (cm)
1	unloaded	5.915	0
2	37	4.604	1.311
3	34	4.750	1.165
4	31	4.885	1.030
5	29	4.985	0.930
6	27	5.080	0.835
7	25	5.165	0.750
8	23	5.250	0.665
9	21	5.360	0.555
10	19	5.450	0.465
11	17	5.505	0.410
12	15	5.590	0.325

As expected a *quadratic function* fit on the data gave the best curve which is given in the figure with the function given as $\nu = a_1 x^2 + b_1 x$, where the fit gave $a_1 = 0.002; b = 0.03$.


Relation between Point of Suspension and Depression
Depression (cm)
1.4
1.2
1
0.8
0.6
0.4
0.2
15
20
25
30
35
40
Distance between Point of Suspension and Point of loading (cm)