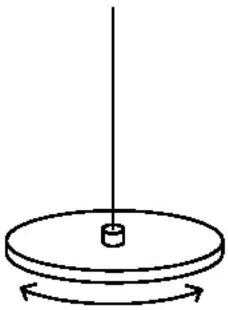
TORSIONAL PENDULUM

RAVITEJ UPPU

1. Introduction

A torsional pendulum, or torsional oscillator, consists of a disk-like mass suspended from a thin rod or wire. When the mass is twisted about the axis of the wire, the wire exerts a torque on the mass, tending to rotate it back to its original position. If twisted and released, the mass will oscillate back and forth, executing simple harmonic motion. This is the angular version of the bouncing mass hanging from a spring. This gives us an idea of moment of inertia. We try to calculate the moment of intertia of a ring given the moment of a disc. We can also verify the perpendicular axis theorem and compare it with theoretically calculated values.



The working is based on the torsional simple harmonic oscillation with the analogue of displacement replaced by Angular displacement θ, Force by Torque τ and the dpring constant by torsinal constant κ. For a given small twist θ (suffciently small), the experienced reaction is given by

$$\tau = -\kappa\theta$$

This is just like the Hooke's law for the springs. If a mass with moment of inertia I is attached to the rod, the torque will give the mass an angular acceleration α according to $\tau = I\frac{d^2\theta}{dt^2}$. Hence we get the relation

$$\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$$

. Hence on solving this second order differential equation we get

$$\omega = \sqrt{\frac{\kappa}{I}}$$

.Hence we have

$$T = 2\pi\sqrt{\frac{Il}{\kappa}}$$

where l is the length of suspension. This is our governing equation of the experiment.

2. Procedure

- Initially we hang the disc alone and give a small angular displacement to the system and leave it to oscillate after fixing a fixed length of suspenstion. Then measure the time period of oscillation of (say) 7 or 10 oscillations and then take the average so as to minimize the error due to our reaction time and precision of the pendulum.Measure the time period for various lengths
- Now hang the ring along with disc and follow the same procedure as before to find the Time Periods for various lengths. We can find the theoretical values of the moment of inertia of ring and disc by knowing their mass and the radii by tthe following equations

$$I_{disc} = \frac{1}{2} M_{disc} R^2$$

$$I_{ring} = \frac{1}{2} M_{ring} (R_{in}^2 + R_{out}^2)$$

  Now, we can take the ratio of Time preiod of disc alone with the time period of disc alone with ring and calculate the ratio of I_{ring} and I_{disc} and tally it with the theoretical value. From this we can assume one of the value of Inertia and find the other.'
- From the graph of T^2 and l we can find the value of κ by knowing the values of I. Hence , we can use different wires in the experiment and repeat the experiment and find the values of κ of the material of the wire.
- We can repeat the first step with jus the ring alone hung along it's diameter instead of the radial axis and find the time periods for various lengths. From this we can compare the value of Moment of Inertia to that predicted by perpendicuar axis theorem. Here, we expect the I along diametrical axis to be half that of I in the perpendicular direction to the plane of the ring(because $I_x + I_y = I_z$, and here $I_x = I_y$ as they are the same axis.

3. Observations and Results

- **The dimensional measurements made :**
  (1) Radius of the Disc $R_{disc} = 6.1cm$
  (2) Outer Radius of the Annulus $R_{out} = 6.1cm$
  (3) Inner Radius of the Annulus $R_{in} = 5.0cm$
  (4) Diameter of the Brass Wire $d_{brass} = 0.68mm$
  (5) Diameter of the Steel Wire $d_{steel} = 0.44mm$
  (6) Mass of the Disc $M_{disc} = 919gm$
  (7) Mass of the ring $M_{ring} = 327gm$
  (8) Moment of Inertia of the Disc(theoretically)=$I_{disc} = 17097.995gm-cm^2$
  (9) Moment of Inertia of the Ring(theoretically)=$I_{ring} = 10171.335gm-cm^2$
  (10) Theoretically $\frac{I_{ring}}{I_{disc}} = 0.595$.

- Measurements of Time period for various lengths using a disc hung on a brass wire is listed below

S.No	Length (cm)	Time for 7 osc.(s)	Time Period (s)
1	43.4	43.40	6.20
2	42.5	42.92	6.13
3	36.7	39.31	5.61
4	33.4	37.81	5.40
5	28.7	35.28	5.04
6	27.3	34.35	4.91
7	14.7	25.75	3.68

- Measurements of the Time period for various lengths of a brass wire with the ring with annulus are listed below

S.No	Length (cm)	Time for 7 osc.(s)	Time Period (s)
1	46.5	56.50	8.07
2	40.8	53.49	7.64
3	34.8	48.82	6.97
4	30.4	45.81	6.54
5	25.3	41.72	5.96
6	19.9	37.28	5.33

- Measurements of Time Period for various lengths of steel wire with the disc hung are listed below

S.No	Length (cm)	Time for 7 osc.(s)	Time Period (s)
1	50.1	67.28	9.61
2	47.5	66.83	9.55
3	42.8	62.55	8.94
4	38.3	61.07	8.72
5	32.5	56.13	8.02
6	28.9	51.84	7.41
7	19.8	43.54	6.22

- Measurements of Time period for various lengths of steel wire with the disc and the annulus hung are listed below

S.No	Length (cm)	Time for 7 osc.(s)	Time Period (s)
1	50.5	87.50	12.50
2	41.1	80.56	11.51
3	34.5	72.81	10.40
4	21.7	57.96	8.28

- (1) Slope of the graph of T^2 versus l for the first table of data is (given by GNUPLOT for a linear fit) =1.13791
  (2) Slope of the graph of T^2 versus l for the second table of data is (by GNUPLOT for linear fit)=0.709374
  (3) Experimentally, the ratio $\frac{I_{ring}}{I_{disc}} = 0.623$
  (4) Slope of the graph of T^2 versus l for the third data table is (given by GNUPLOT for a liner fit) = 0.52366
  (5) Slope of the graph of T^2 versus l for fourth data table is(given by GNUPLOT for a linear fit)= 1.54457
  (6) The ratio of $\frac{I_{ring}}{I_{disc}} = 0.339$. Here, the ratio I expected to be bad because there was a lot of wobbling with the steel wire under the very heavy weigth and also non-centering of the wire in the

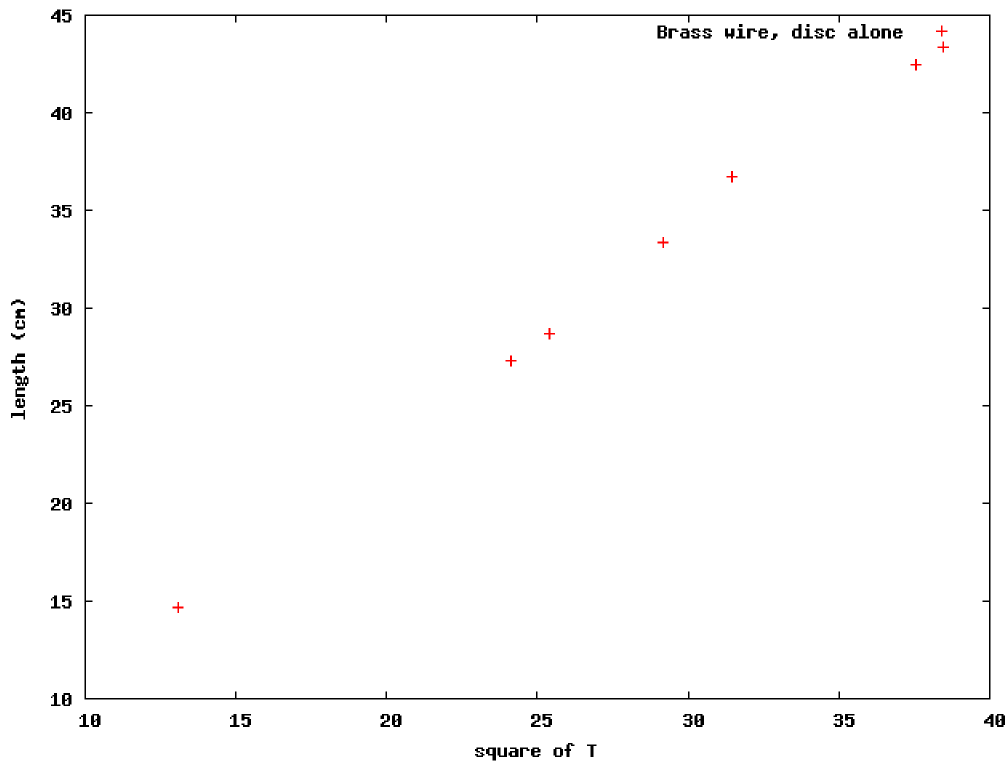
suspension places beacuse the wire was slipping when fixed in the center of the fixtures.

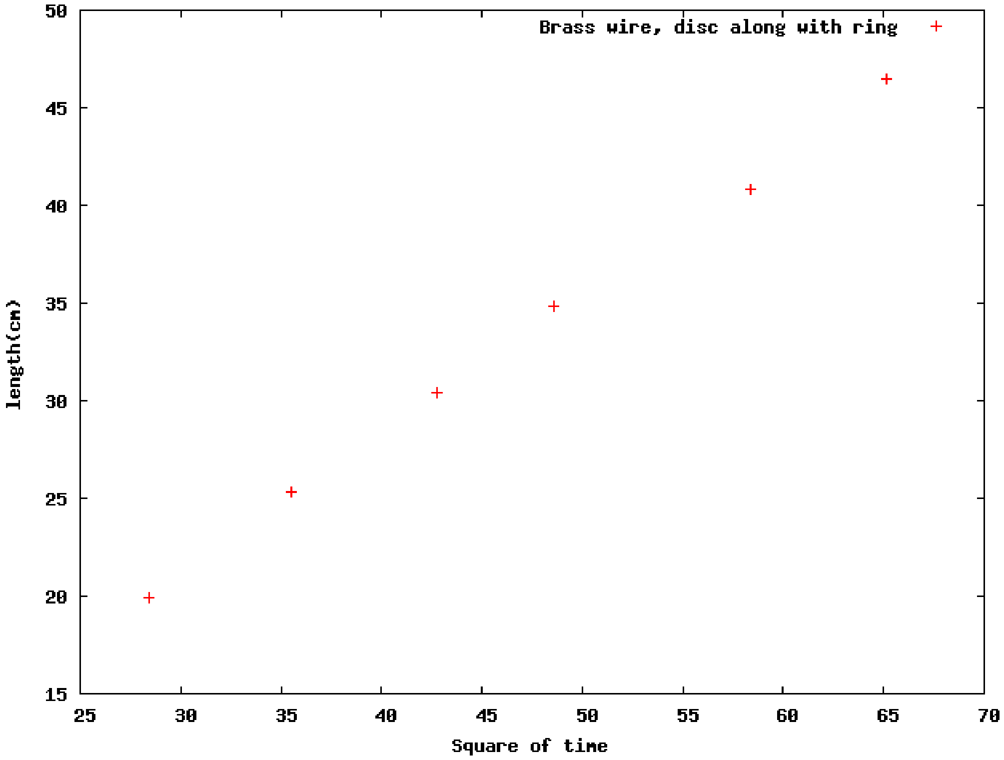
- Hence, if we assume the theoretical value of the disc then the first ratio will ge me the I_{ring} **as** $10658.057 gm - cm^2$ which is around a 5% error.
- Ratio of the first with the third will give us $\frac{\kappa_{steel}}{\kappa_{brass}} = 1.355$
- Measurements of T for various lenths for a ring alog it's diameter

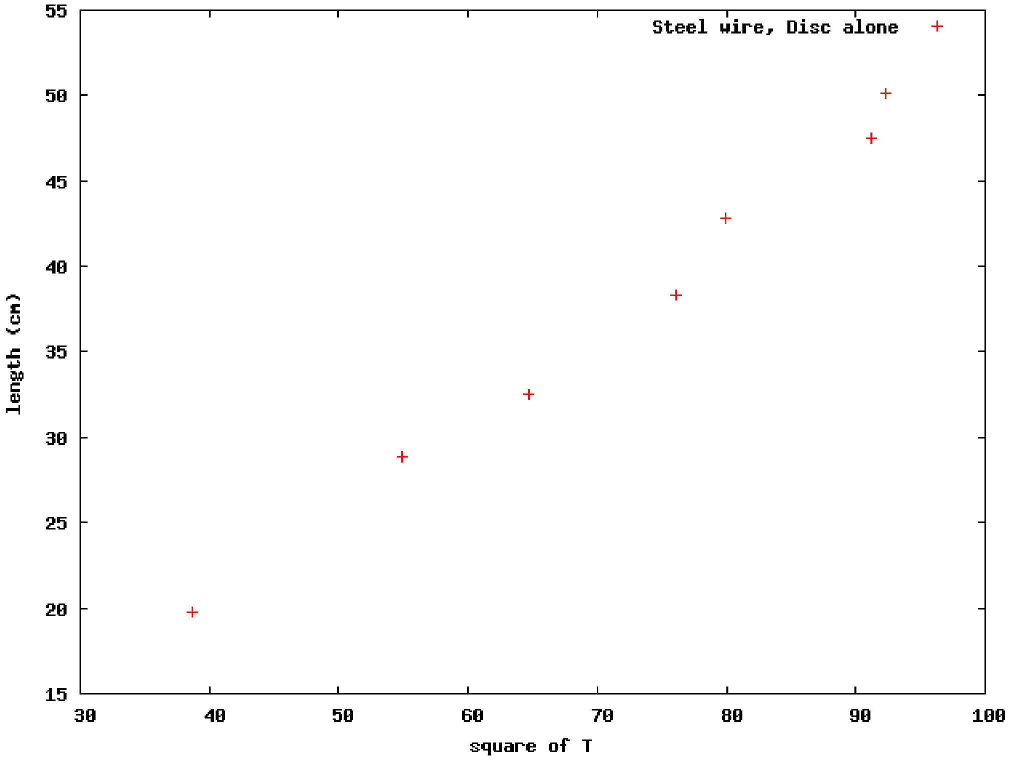
S.No	length (cm)	Time for 7 osc (s)	Time period(s)
1	45.2	24.97	3.57
2	41.7	24.00	3.43
3	36.9	22.72	3.25
4	32.3	21.32	3.05
5	27.8	19.77	2.82
6	21.6	17.77	2.52

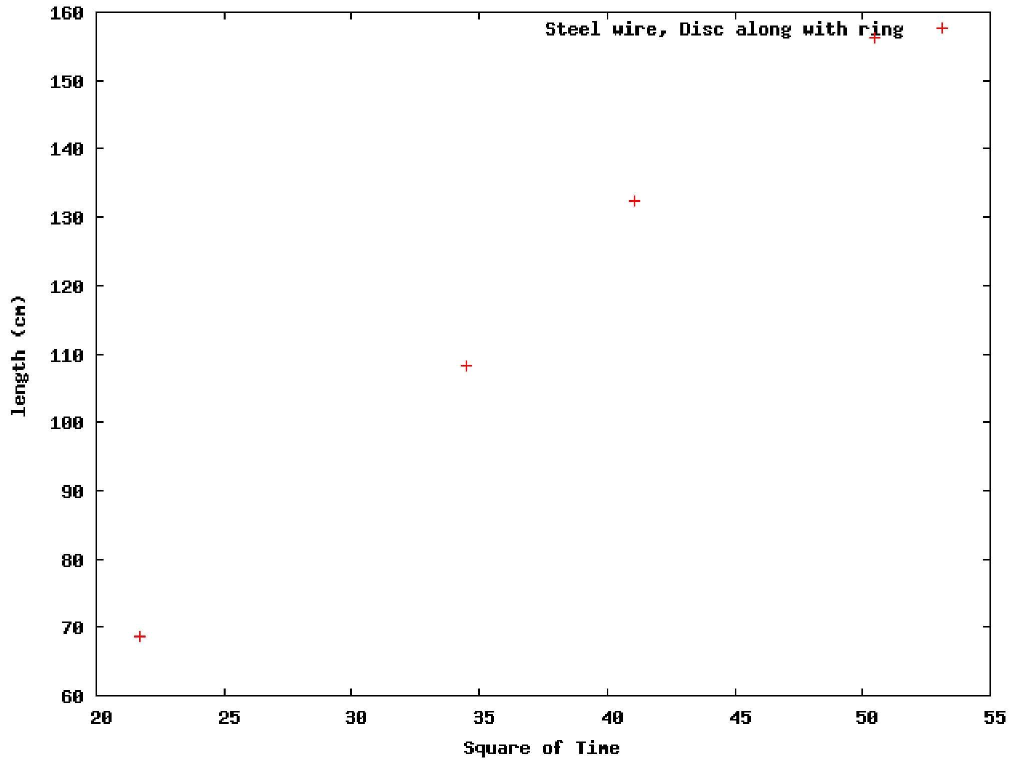
- slope of the above table for the linear fit of T^2 versus l is 3.51075. This will give the value of the $\frac{I_{ringdia}}{I_{disc}} = 0.324$ which is the same as expected from the perpendicular axes theorem. Assuming the I_{disc}, we get the $I_{ringdia} = 5539.750 gm - cm^2$.

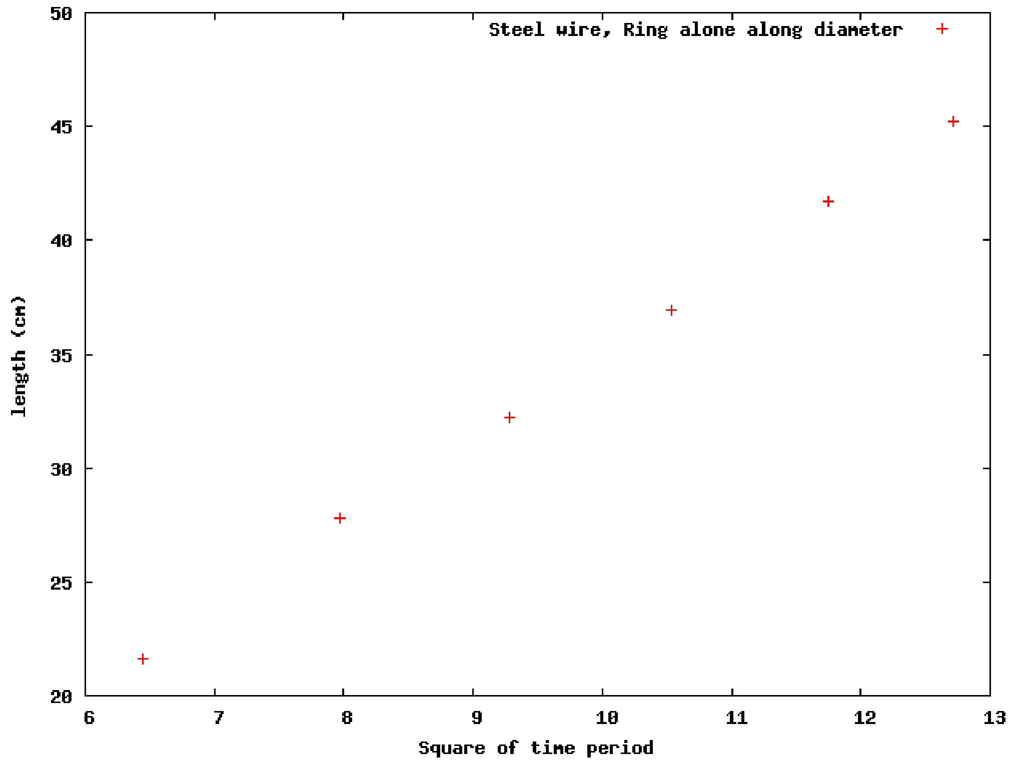
- The graphs of T^2 versus l of the above data are given in the same rder as th data tables.




Brass wire, disc along with ring
length(cm)
Square of time
50
45
40
35
30
25
20
15
25
30
35
40
45
50
55
60
65
70


Steel wire, Disc alone
length (cm)
square of T
55
50
45
40
35
30
25
20
15
30
40
50
60
70
80
90
100


Steel wire, Disc along with ring
length (cm)
160
150
140
130
120
110
100
90
80
70
60
20
25
30
35
40
45
50
55
Square of Time


Steel wire, Ring alone along diameter
length (cm)
50
45
40
35
30
25
20
6
7
8
9
10
11
12
13
Square of time period