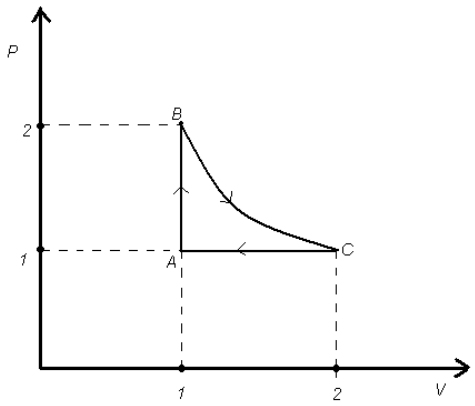
Statistical Mechanics

Mid-Semester Examination, Semester IV

24th February, 2006

100 Marks

1. An ideal gas goes through the cycle $A \to B \to C \to A$ in the P-V diagram shown below. Assume $Nk_B = 1$; the path $B \to C$ is an isotherm



   - Calculate the energy exchanged in the form of heat work during each of the three parts of the cycle.
   - Sketch the T-S diagram for the cycle
   - Calculate the thermodynamic efficiency of the cycle

   (25 marks)

2. Consider an ideal gas of N molecules to which energy Δq is added in the form of heat at constant volume. This results in a change ΔP of pressure and ΔT of temperature. Show that Δq is not a perfect differential and that q is not a state function of pressure and temperature. Let $\Delta S = \Delta q/T$. Show that ΔS is a perfect differential and is a state function of P and T. (20 marks)

3. Derive an expression for the volume, denoted by $V_N(R)$ of an N dimensional sphere of radius R. Let

$$\eta = \frac{V_N(R) - V_N(R - \Delta R)}{V_N(R)}$$

denote the fractional volume contained in an outer spherical shell of width $\Delta R = \epsilon R$ where $\epsilon > 0$ is arbitrarily small. Show that $\eta \to 1$ as $N \to \infty$. From these considerations derive an expression for microcanonical entropy S as a function of energy E, volume V and number of molecules N. Derive an expression for the temperature and pressure of the system. (25 marks)

4. Consider an experiment of tossing N independant fair coins. Derive an expression of the number of microstates having n heads. Find the value of the macrostate n for which entropy is maximum. Show that Gibbs entropy and Boltzmann entropy coincide for $N \to \infty$. (15 marks)

5. 20 moles of an ideal gas is held at temperature $T = 27^\circ C$ while expanding from 10 litres to 100 litres. What is the work done by the gas? ($R =$ 8.3143J/mole- deg. K) (15 marks)