

Probability

Mid-Semester Examination, Semester VI

February, 2007

Answer any 5 questions, choosing any two from the first 3. Time: 3 hrs

1. For some $n > 3$, three numbers are from the set $\{1, 2, \dots, n\}$ are chosen randomly. Find the probability that no consecutive integers occur.
2. If the random variable X takes the values $\{0, 1, \dots, m\}$ (all equally likely) and if Y takes $\{0, 1, \dots, n\}$ (all equally likely) independent of X , then find the probability mass function of the sum $X + Y$.
3. There are b blue balls and r red balls in a bag. Balls are pulled out randomly and removed until a blue ball comes up, at which point we stop the trial. Prove that the expected number of balls removed from the bag is $(b + r - 1)/(b + 1)$.
4. A real number m is said to be a medial of a distribution function F if $\lim_{y \uparrow m} F(y) \leq 1/2 \leq F(m)$. Prove that the set of all medians of F is a non-empty closed interval.
5. For a continuous random variable X taking non-negative values, prove the following:

$$E(X^r) = \int_0^{\infty} r x^{r-1} \Pr[X > x] dx$$

6. In a symmetric random walk (i.e, $p = q = 1/2$) with initial condition $S_0 = 0$, prove that for $r \geq 0$

$$\Pr[M_n = r] = \Pr[S_n = r] + \Pr[S_n = r + 1]$$

where M_n is the maximum height attained in the first n steps.