Classical Mechanics I

End-Semester Examination, Semester I

November, 2004

1. Check if the follwing forces are conservative, if they are find their corresponding potentials.
   (a) $\vec{F} = \vec{a} \wedge \vec{x}$, $\vec{a}$ is a constant vector.
   (b) $F_1 = x_1 x_3 + \beta x_2^2; F_2 = a x_3 + 2\beta x_1 x_2; F_3 = \alpha x_2 + \beta x_3^2$, where α, β are constants.
2. How many degrees of freedom do the following have?
   (a) an infinitely thin rigid rod moving in
      - 2 dimensions
      - 3 dimensions
   (b) a generic ($n-$dimensional) rigid body moving in n dimensions, $n > 3$

   Describe the configuration space of of 2(ii)
3. The isotropic harmonic oscillator in 3 dimensions is a particle of mass m moving in the potential given by:

$$V(r) = \frac{1}{2} k r^2, = |\vec{x}|, k > 0$$

   (a) Is the angular momentum conserved?
   (b) Write down the effective potential and its graph as a function of r
   (c) For given fixed L determine the values of the energy for which the orbit is circular and determine its radius.
   (d) Show that the equation of motion can be solved in the form

$$\vec{x}(t) = \vec{x} \cos \omega t + \frac{\vec{v}(t=0)}{\omega} \sin \omega t$$

   where $\omega^2 = \frac{k}{m}$, and $\vec{x}, \vec{v}$ are vectors in the plane normal to $\vec{L}$.
   (e) Calculate angular momentum for the above solution and verify that it is conserved.
   (f) Calculate the total energy for the above solution and verify that it is conserved and greater than minimum value of the effective potential.