Computational Complexity

Mid-Semester Examination, Semester IV

13th March, 2006

50 marks

1. Suppose A is NP-complete and $B \in P$ such that $A \bigcap B = \phi$, prove that $A \bigcup B$ is NP-complete. What can you say when $A \bigcap B \neq \phi$? (5 marks)

2. Classify the following problems into P,NP,co-NP,NP-complete,co-NP complete (as accurately possible)

   - Given a 3-CNF formula F and a positive integer k in unary, does F have atleast $k+1$ satisfying assignments?
   - Given a graph G and a clique C in G, is C a maximum size clique of G?
   - Given a graph G and a clique C in G, is C a maximal clique of G? Here maximal is with respect to the containment partial order.

   (5+3+2 marks)

3. Prove that there exists a language L in Σ_2^p that does not have circuits of size n^2. Hint: In which level of PH can I find a language that is not in P/n^2? Either SAT is in P/poly or is not in P/poly...

   (10 marks)

4. Construct a recursive oracle A such that $E^A \neq NE^A$, where $E = \bigcup_{c \geq 0} \text{DTIME}(2^{cn})$ and NE is defined respectively. (10 marks)

5. Let $f : \Sigma^* \rightarrow \Sigma^*$ be a polynomial time honest function (i.e there is a polynomial p such that $|x| \leq p(|f(x)|)$). Consider teh following language: $L_f = \{< x, y > | y \text{ is a prefix of some } z \in f^{-1}(x)\}$. Show that L_f is in NP. Suppose L_f is polynomial time reducible to a tally set, show that there is a polynomial time algorithm that computes $f^{-1}(x)$ on input x from Range(f). (5 marks)

6. Show that 2-SAT is NL-complete under logspace many one reductions. (10 marks)