Analysis

End Semester

December 2005

1. • Prove that the series converges whenever x is not an integer.

$$X = \frac{1}{x} + \sum_{n \in \mathbb{Z} - \{0\}} \left(\frac{1}{n} + \frac{1}{x-n} \right)$$

   • Show that

$$\lim \left(\sum_{-p}^{q} \frac{1}{x-n} \right) - X = -\log k$$

   where p, q tend to ∞ in such a way that $\lim(q/p) = k$.

2. For continuous 2π periodic functoins $f, g : \mathbb{R} \to \mathbb{C}$, let $f{*}g$ be the function given by

$$(f{*}g)(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x-t)g(t)dt$$

   Show that $f{*}g$ is continuous, 2π periodic, adn that the fourier coefficients of $f{*}g$ are given by

$$c_n(f{*}g) = c_n(f)c_n(g)$$

3. • Show that

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \int_0^1 \frac{1-(1-t)^n}{t} dt$$

   • Show that the Euler's Constant is given by

$$\lim_{n \to \infty} \left(\int_0^1 \left\{ 1 - \left(1 - \frac{1}{n} \right) \right\} \frac{dt}{t} - \int_1^n \left(1 - \frac{1}{n} \right)^n \frac{dt}{t} \right)$$

4. • Let

$$u_n = \frac{(n+a_1)(n+a_2)\cdots(n+a_k)}{(n+b_1)(n+b_2)\cdots(n+b_l)}$$

   . Show that the infinite product $\Pi_{n=1}^{\infty} u_n$ converges, then $k = 1$ and $a_1 + a_2 + \cdots + a_k = b_1 + b_2 + \cdots + b_l$.

- When these conditions are satisfied, show that

$$\Pi_{n=1}^{\infty} u_n = \frac{\Gamma(1+b_1)\Gamma(1+b_2)\cdots\Gamma(1+b_l)}{\Gamma(1+a_1)\Gamma(1+a_2)\cdots\Gamma(1+a_k)}$$