Classical Mechanics I

Mid-Semester Examination, Semester I

September, 2004

Marks: 25

1.
   - Let A be a $m \times 4$ matrix which has the property that the space of solutions of the system of linear equations $AX = 0$ is generated by $\{1,1,0,1\}^t, \{2,3,1,0\}^t, \{-1,4,1,1\}^t$. FInd the dimension of the rowspace of A.
   - Suppose $\det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = \det \begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix}$. Find a basis of the solution space of the system of linear equations:

$$a_1x_1 + a_2x_2 + a_3x_3 = 0$$

$$b_1x_1 + b_2x_2 + b_3x_3 = 0$$

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2.
   - In teh set of $n \times n$ matrices, $M_n(F)$ where F is any field, for any two matrices A, B, define ρ by $A\rho B \Leftrightarrow \exists$ non-singular matrices P, Q such that $A = PBQ$. Prove that ρ is an equivalence relation and show that there are exactly $n+1$ equivalence classes.
   - State and prove Lagrange's theorem on subgroups of finite groups.

3.
   - Let G be a cyclic group of order n and let d be a divisor of n. Show that there is a unique subgroup of G of order d and prove that the subgroup is cyclic.
   - Find all the elements of $GL_2(F)$ where F is a field of 2-elements. Identify the group.

4.
   - Let V be a vector space of dimension 3 with basis $\{e_1, e_2, e_3\}$. Suppose v and w are vectors with coordinates $\{1,1,0\}$ and $\{0,1,1\}$ with respect to B. Find a new basis B' so that v and w will have coordinates $\{0,1,0\}$ and $\{1,0,1\}$ with respect B'.
   - Let V be the vector space of all polynomials of order ≤ 4 in an indeterminate X over the field F. Find a basis $\{e_1, e_2, e_3, e_4\}$ where e_1 is taken as $X + X^2 + X^3$.

5. Let V be a vector space and $B = \{v_1, v_2, \ldots, v_n\}$ is a subset of V. Prove that the following are equivalent.

   - B is a basis
   - B generates V and no proper subset of B generates V.
   - V has dimension n and B generates V.
   - V has dimension n and $\{v_1, v_2, \ldots, v_n\}$ are linearly independant