Design and Analysis of Algorithms
Class Test 2

April 9, 2012

1. (10 marks) State if each of the following is true or false. Justify.

   (a) If X is a problem in NP, and if we find a polynomial-time algorithm for X, then there is a polynomial-time algorithm for the circuit satisfiability problem.

   (b) If X has a polynomial-time algorithm, then X is in NP.

   (c) If X is an NP-complete problem, then a polynomial-time algorithm for X also implies a polynomial-time algorithm for boolean formula satisfiability (SAT).

   (d) If X is an NP-hard problem then every problem in NP can be reduced to X in polynomial time.

   (e) If X is in NP, then there should be a certificate and a polynomial time verifier for YES as well as NO instances of X.

2. A set-cover problem is defined as follows: We are given a finite set X and a set F of subsets of X. If an element x is picked from X, it is said to *cover* those sets in F which contain x. The goal is to pick as few elements from X as possible, so as to cover all the sets in F.

   Example: $X = \{1, 2, 3, 4, 5\}$, $F = \{\{1, 2\}, \{1, 3, 4\}, \{3, 4\}, \{4, 5\}\}$ Then picking 1 from X covers the sets $\{1, 2\}$, and $\{1, 3, 4\}$. A set cover for F is $\{1, 4\}$.

   (a) (2 marks) State the set cover problem as a decision problem.

   (b) (6 marks) Prove that the set cover problem is NP-complete.

3. (6 marks) In the max-flow problem, prove that for any pair of vertices (u, v), we have $c_f(u, v)+c_f(v, u) = c(u, v) + c(v, u)$ where $c(u, v)$ is the initial capacity of the edge from u to v, and $c_f(u, v)$ is the residual capacity after setting up a flow of f units.

4. (6 marks) There are n children and m schools. The children need to be assigned to a school that is within 10 km of their house. Each school can take at most k children. We need to find an assignment of children to schools so as to maximize the number of children going to school.

   State this problem as a network flow problem.