Topics in Topology
(Homework 6)
February 16, 2015

- Due date - March 2, 2015.

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1. Let π denote the finite group of order 2, denote by $\mathbb{Z}_+$ the trivial π-module and by $\mathbb{Z}_-$ the non-trivial π-module (i.e., the generator acts by multiplication by -1). Suppose A is a finitely generated left π-module which is finitely generated and torsion free as an abelian group. Then prove that A is direct sum of modules of the form $\mathbb{Z}_+, \mathbb{Z}_-$ and $\mathbb{Z}[\pi]$. [10 points]

2. Compute the abelian group $\mathbb{Z}_+ \otimes_{\mathbb{Z}/2} \mathbb{Z}_-$. [10 points]

3. Let X be a connected space with the fundamental group π and X' be a cover corresponding to the subgroup π' of π. Then prove that

$$C_n(X') \cong C_n(\widetilde{X}) \otimes_\pi \mathbb{Z}[\pi/\pi']$$

as abelian groups, here $\widetilde{X}$ is the universal cover. Make sure that you justify every argument used to prove this. [10 points]