Classical Mechanics II:
Quiz

Total: 10 marks

You may find it useful to use Newton's laws as a guide/check.

(1) Consider a rotating pendulum, i.e. a mass point m hanging from a fixed point on the ceiling by a string (length l).
(a) Define convenient generalized coordinates and write the Lagrangian for this system. [1 mk]
(b) Work out the inertia tensor components, I_1, I_2, I_3, for this symmetric top for motion about the fixed ceiling point. [1 mks]
(c) Draw a figure defining Euler angles for this system, choosing the moving x_1, x_2 axes in a convenient way, and explain clearly your choice. [1 mks]
(d) Using Euler angle variables, write the Lagrangian for this system. [2 mks].

(2) Consider the two conserved quantities, energy E and angular momentum L^2, expressed in terms of the angular velocities Ω_i, for free rotation. Express Ω_1, Ω_3 in terms of Ω_2 from the expressions for E, L^2. Write these expressions for a symmetric top with say $I_1 = I_2$. Now use the Euler equation $I_2 \frac{d\Omega_2}{dt} + (I_1 - I_3)\Omega_3\Omega_1 = 0$, and reduce the expression for $\Omega_2(t)$ to an integral. If possible, solve this for a closed form expression. [5 mks]