Quantum Mechanics II:
Midsemester examination

Total: 35 marks

Time: 2hrs 30mins.

(1) Consider adding angular momenta $j_1 = 3$ and $j_2 = 1$. Calculate the Clebsch-Gordan $\langle m_1 m_2 | jm \rangle$ coefficients $\langle 20|32\rangle$ and $\langle 11|32\rangle$. Write the $|jm\rangle$ states $|33\rangle$ and $|32\rangle$ in terms of the $|m_1 m_2\rangle$ basis. [8 mks.]

(2) Explicitly calculate the matrix elements $\langle 200|z^2|200\rangle$ and $\langle 210|z|100\rangle$, where $|nlm\rangle$ are the stationary states of the Hydrogen atom, and the Cartesian coordinate $z = r\cos\theta$. [8 mks.]

(3) Consider a system of two non-interacting harmonic oscillators with Hamiltonian

$$H_0 = \omega_1(a_1^\dagger a_1 + \frac{1}{2}) + \omega_2(a_2^\dagger a_2 + \frac{1}{2}),$$

where $a_i, a_i^\dagger$ are the usual creation-annihilation operators for each of the oscillators.

(a) What is the condition on ω_1, ω_2, for the system to have no degeneracy in any of the eigenstates? [1 mk]

(b) Consider the "hopping" perturbation $\lambda V = \lambda v(a_1^\dagger a_2 + a_2^\dagger a_1)$.

Does this conserve parity?

(i) Assume that there are no degeneracies. Calculate the second order energy shifts for the states $|n_1 n_2\rangle$ and the corresponding first order perturbed eigenkets. [7 mks]

(ii) Now say $\omega_1 = \omega_2$. What are the lowest energy degenerate eigenstates? Consider the effect of the perturbation for these states. Find the first order energy shifts and the correct zeroth order eigenkets in this degenerate subspace. [7 mks]

(c) Consider now the perturbation $\lambda V = -\lambda v\ a_1^\dagger a_1 a_2^\dagger a_2$, with $\lambda v > 0$. Find the energy shifts upto second order and the perturbed first order kets $|n_1 n_2\rangle$. From this calculation, what is your intuition for the physical meaning of this perturbation? [4 mks]