Quantum Mechanics I: Midsemester examination

Total: 35 marks

(1) The wavelength of light emitted when an electron transits between the first and second Bohr orbits of the Hydrogen atom is given to be λ. Find the de Broglie wavelength of an electron in the first Bohr orbit in terms of λ and the electron mass m and charge e (and fundamental constants).
(You do not need to evaluate this numerically.) [3 mks.]

(2) The work function for a particular metal is 2.5 eV.
(a) Find the threshold wavelength of light for producing the photoelectric effect. [1 mk.]
(b) Find the frequency of light required to induce photoelectric emission of electrons with maximum kinetic energy $4 \times 10^{-19} J$ from the substance. [2 mks]

(3) A 1-dimensional spin-$\frac{1}{2}$ system is described by the state ket

$$|\psi\rangle = \cos\left(\frac{2\pi x}{L}\right)|+\rangle + \sin\left(\frac{2\pi x}{l}\right)|-\rangle \ ,$$

where x labels the 1-dimensional x-position coordinate and L is a constant of dimension length. $|\pm\rangle$ are the eigenkets of the spin S_z operator.
(a) Is this state ket normalized appropriately ? If not, find the normalization constant. [2 mks]
(b) Find the expectation value $\langle S_z\rangle$ in this state. Does $\langle S_z\rangle$ vanish at some position x ? [4 mks]
(c) Calculate the probability of finding the system in the state $|-\rangle$ at $x = L$. [2 mks]
(d) Find the uncertainty ΔS_z in this state by calculating $\langle \Delta S_z^2\rangle_\psi = \langle S_z^2\rangle - \langle S_z\rangle^2$. At what position x is this uncertainty minimum ? [5 mks]
(e) Find the matrix representation of the operator $|\psi\rangle\langle\psi|$ in the S_z eigenstate basis. [4 mks]

(4) A certain quantum system is described by the wave function $\psi(x) = \frac{1}{\sqrt{\pi}}e^{-x^2}$. Evaluate the expectation value $\langle \hat{p}^2\rangle$ ($\hat{p}$ being the momentum operator) in the state $\psi(x)$ using the

position space wave function representation and the corresponding expression for the momentum operator. *(Hint: You might find differentiation under the integral sign useful.)* [6 mks]

(5) A variant of the double slit interference experiment has a source (at location $x = 0, y = 0$) followed by two screens with slits followed by a detector screen. Screen 1 immediately after the source is at location $x = x_1$ and has two slits at locations $y = \pm y_1$. The second screen 2 (at location $x = x_2$) has again two slits at locations $y = \pm y_2$. Calculate the probability (or intensity) on the detector screen at location $x = D, y = 0$,

(a) when all four slits are open, [3 mks]

(b) when an additional detector placed near the slit (x_2, y_2) registers quanta passing through this slit. [3 mks]

You will find it convenient to use position space kets possibly labelled by their coordinates as $|x, y\rangle$ for calculating the amplitudes.