# General Relativity:
# Midsemester examination

Total: 35 marks

(1) On 10 Sep, 2008, the first (test) proton beams were circulated successfully along the 27km long circular accelerator ring of the Large Hadron Collider at CERN. Protons are accelerated to high speeds, eventually reaching energies of about 10 TeV (= 10000 GeV).
(a) What is the speed of the proton moving at this energy ? [2 mks]
(b) What is the effective length of the ring as seen by the protons ? [2 mks]
(c) What proper time has elapsed for the proton when it goes around the ring once ? [3 mks]

(2) Hypothetical particles moving faster than light are called tachyons.
(a) Show, using the velocity addition law, or the spacetime interval (metric), or spacetime diagrams or any other way, that these are consistent with Lorentz invariance in the sense that if they move faster than light in one frame, they'll move faster than light in any other frame as well. [2 mks]
(b) Show also that the curve representing a tachyon trajectory in spacetime is spacelike. [2 mks]

(3) The spacetime $ds^2 = -dt^2 + 2dtdx + dy^2 + dz^2$ is flat. Find an explicit coordinate transformation that puts the metric in the canonical flat form
$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2$. [4 mks.]

(4) Consider the 2-dim spacetime $ds^2 = -dt^2 + t^{2p}dx^2$, with $p < 1$.
(a) Draw a $t$-$x$ spacetime diagram showing the structure of lightcones in this spacetime. [4 mks]
(b) Describe the timelike trajectories for this spacetime, by analysing the proper-time-normalized velocity vector for a timelike worldline and conserved quantities stemming from symmetries of this spacetime (or alternatively of the timelike particle's Lagrangian). [4 mks]

(5) Consider the Schwarzschild spacetime

$$ds^2 = -(1 - \frac{2m}{r})dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

corresponding to the geometry outside a spherically symmetric stellar object (the coordinate $r$ takes values $r > 2m$). Consider particles moving along null (lightlike) worldlines in this spacetime, in the equatorial plane $\theta = 0$.

(a) Identify the symmetry directions of this spacetime (or alternatively of the action for test particles moving in it), and the corresponding conserved quantities. [3 mks]

(b) Using the above conserved quantities, simplify the equation for the proper-time-normalized velocity vector for a circular light orbit, reducing it to quadratures. What is the effective potential for these light orbits ? What is the radius of the unstable extremal light orbit around the stellar object ? [4 mks]

(c) Consider a timelike particle moving radially on the equatorial plane, i.e. moving at fixed $\theta, \phi$. How much coordinate time $\Delta t$ elapses when the particle falls inward from $r = \infty$ to $r = r_0$? (You do not need to evaluate the integral.) [5 mks]